Radiometric Observations of Liquid Water in Thunderstorm Cells

Martin T. Decker and Evan J. Dutton
ESSA Research Laboratories, Boulder, Colo.

(Manuscript received 27 January 1970, in revised form 15 May 1970)

Abstract

An estimate of the integrated liquid water content along radio rays passing through a thunderstorm has been made by using the thermal emission from the storm at 10.7 GHz. The radiometric data, together with supplementary radar, radiosonde, and surface meteorological data are used to determine absorption in the storm, which in turn is used to determine the liquid water content. The results of the technique are illustrated by contour diagrams showing liquid water content of a small storm system and its changes over a period of 2 hr.

1. Introduction

Microwave thermal emission from condensed water in the atmosphere is a potential source of significant information about that water. In the case of thunderstorm cells the emission is easily observed with simple radiometer systems. This remote sensing technique can be used to deduce various properties of the radiating cell of interest both to communication system designers and to cloud physicists. Quantities that can be measured or estimated include noise degradation of sensitive communication systems, total attenuation of an electromagnetic wave passing through the cell, and the integrated liquid water content. The last is emphasized in this paper and is illustrated by observations made of thunderstorm cells during the summer of 1967.

Briefly, the procedure consists of the following steps: 1) measuring the emission from the thunderstorm and the clear sky in terms of brightness temperatures, 2) separating the absorption of the storm from the combined absorption of atmospheric gases and the storm, and 3) estimating the integrated water content from this absorption and certain properties of water drop absorption cross sections.

2. Observations

During the summer of 1967, frequent observations of thunderstorms were made from the ESSA Table Mountain site near Boulder, Colo. These storms were generally small and isolated, typical of the high-plains thunderstorms that build immediately east of the Rocky Mountains during summer afternoons. The isolated nature of the storms is particularly advantageous since it allows a reference observation of essentially clear sky near the storm.

Typical data obtained 18 July 1967 were chosen for examination in this paper. A compact storm system was observed for about 2 hr at distances ranging from 50–75 km from the radiometer. Storm cells were located with a radar operating at 9-cm wavelength.

A conventional Dicke-switching radiometer (Dicke, 1946) operating at 10.7 GHz (2.8 cm) was used for brightness temperature measurement. The principal equipment parameters for the conditions of this experiment are given in Table 1. The reference arm of the radiometer used an ambient temperature termination. The system was operated near the balanced condition by the addition of noise from a gas discharge tube to the signal arm. The antenna was a parabolic reflector 18.3 m in diameter with the horn feed (and the radio frequency portion of the radiometer) mounted at the prime focus. In the usual mode of operation the antenna beam was scanned across the storm at a fixed initial elevation angle from the clear sky on the one side of the storm to the clear sky on the other. The basic measurement then was the difference in noise temperature at the antenna terminals for the two conditions of measurement, i.e., the beam directed toward the clear sky background and then directed toward the storm. A separate gas discharge tube was used to calibrate this change. The scan was repeated in 0.5° increments of elevation angle until all of the storm that could be observed from the radiometer site had been covered. This technique allowed us to draw contour diagrams of the storm; thus, repetitive observations showed changes in storm characteristics with time.

The antenna directivity pattern and main beam gain were measured with the aid of a target transmitter at a distance of 29 km from the 18.3 m antenna. Considerable effort was expended in arranging the target trans-

<table>
<thead>
<tr>
<th>Table 1. Radiometer equipment parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Wavelength</td>
</tr>
<tr>
<td>Antenna diameter</td>
</tr>
<tr>
<td>Antenna half-power beamwidth</td>
</tr>
<tr>
<td>Azimuthal scan rate</td>
</tr>
<tr>
<td>Bandwidth (double side-band)</td>
</tr>
<tr>
<td>Integration time (RC)</td>
</tr>
<tr>
<td>Radiometer noise temperature</td>
</tr>
<tr>
<td>Minimum detectable temperature change</td>
</tr>
</tbody>
</table>
mitter to produce a field over the radiometer antenna aperture that was uniform to about ±1 db. The pattern and gain data were used to derive the beam efficiency (Kraus, 1966), computed here as a function of conical angle measured from the axis of the main beam. This efficiency is defined as the percent of the total energy radiated by the antenna that would be directed into the solid angle described by a cone whose axis coincides with the main beam axis. As the half-angle of the cone increases from 0 to π, this efficiency increases from 0 to 100%.

Meteorological data included surface observations of temperature and relative humidity at the radiometer site and radiosonde data from Denver, the Denver station being ~50 km southeast of the radiometer site. Two radiosonde flights were made for most of the days on which storm measurements were made.

In summary, we now have available for analysis 1) the change in antenna noise temperature as the antenna beam is moved from clear sky to storm, 2) the antenna beam efficiency, 3) the location of the storm cell, and 4) radiosonde and other meteorological data.

3. Analysis

The brightness temperature \( T_{bee} \), observed in the direction of the storm is the sum of emission contributions from the atmosphere between the observer and the storm, from the storm itself, from sources beyond the storm, and from energy scattered by particles within the cloud. Using the microwave equation of radiative transfer (Shkolnovsky, 1960), these four terms are written

\[
T_{bee} = \int_0^{r_1} T(r)\beta_e(r)e^{−τ(r)}dr + \int_{r_1}^{r_2} T(r)[β_e(r) + (1−ω)β_s(r)]e^{−τ(r)}dr + \int_{r_2}^{∞} T(r)β_s(r)e^{−τ(r)}dr + \int_{r_1}^{r_2} T_{bee}(r)ωβ_e(r)e^{−τ(r)}dr, \tag{1}
\]

where the symbols are defined as follows:

- \( T \): kinetic temperature of the absorbing medium along the observed ray path
- \( β_s \): extinction (or absorption) coefficient of atmospheric gases
- \( β_e \): extinction coefficient of liquid particles
- \( τ \): total attenuation
- \( ω \): single-scattering albedo defined as the ratio of scattering coefficient to extinction coefficient
- \( T_{bee} \): mean brightness temperature of the earth
- \( r_1, r_2 \): distances from the observer to the near and far boundaries of the storm

If we assume that “mean radiating temperatures” are known for the first three terms of (1) and are designated \( T_1, T_2 \) and \( T_3 \), respectively, and that \( ω \) and \( T_{bee} \) are constant through the storm, then (1) may be solved for the absorption of the liquid particles, \( τ_e \). The result is

\[
τ_e = -\ln \left( \frac{T_1(1−e^{−r_1}) + T_2(1−ω)e^{−r_1} + T_{bee} + T_{bee}ωe^{−r_1}}{[T_2(1−ω)−T_3(1−e^{−r_1})] + T_{bee}ωe^{−r_1}} \right) - (1−ω)τ_3, \tag{2}
\]

where \( τ_1, τ_2 \) and \( τ_3 \) are the gaseous attenuations from 0 to \( r_1, r_1 \) to \( r_2 \) and \( r_2 \) to \( ∞ \), respectively.

In some observational situations it is possible to simplify (2), and two cases will be considered. First, if the wavelength of observation is long enough with respect to the drop sizes, scattering may be neglected, and \( ω = 0 \). At our measurement frequency \( ω ≈ 0.05 \), i.e., the scattering is only ~5% of the total attenuation in precipitating clouds (Crane, 1966); thus, we have neglected \( ω \) in the results shown here. Second, for cloud heights and ranges great enough so that the total gaseous absorption may be approximated by the absorption between the observer and the storm, \( τ_3 \) is set equal to zero. With these two simplifications (2) becomes

\[
τ_e = -\ln \left( \frac{T_1(T_{bee}−T_{bee})}{T_2(T_1−T_{bee})} \right) \tag{3}
\]

where \( T_{bee} \) is the sky brightness temperature in the absence of cloud. This clear-sky brightness temperature was determined from five years (248 soundings) of Denver radiosonde data for a summer month, using the Van Vleck formulas for atmospheric gaseous absorption (Van Vleck, 1951). The arithmetic mean of data at a given elevation angle is used to represent \( T_{bee} \) at that elevation angle. The possibility of improving this estimate by a regression of the calculated \( T_{bee} \) vs surface temperature was considered, but the standard deviation of \( T_{bee} \) was not large (~5% of \( T_{bee} \)) and the regression analysis did not improve the estimate appreciably. A plot of the resulting \( T_{bee} \) vs elevation angle is shown in Fig. 1.

The five years of radiosonde data used determine \( T_{bee} \) were also used to estimate \( T_1 \). A regression of the 248 resulting values of \( T_1 \) with their corresponding surface temperature and the surface temperature observed at the radiometer site was then used to obtain an estimate of \( T_1 \) for the time of observation. Values of \( T_1 \) are shown in Fig. 2.

\footnote{That is to say, for example,}

\[
\int_0^{r_1} T(r)β_e(r)e^{−τ(r)}dr = T_1 \int_0^{r_1} β_e(r)e^{−τ(r)}dr = T_1(1−e^{−r_1}).
\]
The value of $T_2$ is assumed to be equal to the kinetic temperature at the height at which the ray passes through the storm. This temperature is estimated from the surface temperature at the radiometer site and the average of the two temperature lapse rates taken from that day's 1200 and 1800 MDT Denver radiosondes.

The brightness temperature $T_{bs}$ in the direction of the storm is derived from the radiometer measurements from

$$T_{bs} = \frac{\Delta T_a}{\epsilon} + T_{bs},$$

where $\Delta T_a$ is the difference in antenna temperature measured by the radiometer as it moves from clear sky to storm. An example of a chart recording of this temperature difference is shown in Fig. 3. The antenna beam efficiency $\epsilon$ is computed for a conical angle of 0.5° with a value of 0.62 resulting for $\epsilon$. This choice of beam angle is a compromise; it is chosen to be large enough so that the contribution of noise sources outside the beam does not change much as the antenna is moved horizontally, and small enough so that the brightness temperature as weighted by the beam pattern and integrated over the beam will represent the brightness temperature in the direction of the main beam.

Having obtained values for integrated liquid absorption, we proceed to estimate the corresponding line integral of liquid water $L$, given by

$$L = \int_{r_1}^{r_2} M(r) dr,$$

where $M$ is liquid water content (gm m$^{-2}$). For very small drop-size-to-wavelength ratios, i.e., non-precipitating clouds in the microwave region, the absorption coefficient at a given temperature is proportional to liquid

---

**Fig. 1.** Clear sky brightness temperature at 10.7 GHz as a function of radiometer elevation angle as obtained from five summers of Denver, Colo., radiosonde data.

**Fig. 2.** Linear regressions of clear air mean radiating temperature vs surface temperature using 248 summer radiosonde profiles from Denver, Colo.

**Fig. 3.** Example of chart recording of radiometer sweeps across a thunderstorm.
water content, so that

$$\tau_e = \int_{r_1}^{r_2} \beta_e(r) dr = \int_{r_1}^{r_2} aM(r) dr,$$

or

$$L = \frac{\tau_e}{a}.$$

However, precipitating storms contain drop sizes large enough so that we cannot use this simple relation, and we modify it as follows. Crane (1966) has computed attenuation coefficients $\beta_e$ for a large sample of drop size distributions and at a number of frequencies. At each frequency he has plotted attenuation coefficient vs liquid water content $M$ and computed a fit to the data of the form

$$\beta_e = aM^b.$$

(4)

Letting $(1-\omega)\beta_e = \beta_e$ as before, we have

$$\tau_e = a \int_{r_1}^{r_2} [M(r)]^b dr.$$

(5)

Eq. (5) implies that we must have a knowledge of the spatial distribution of $M$ in order to solve for the total liquid water content. However, Hölder’s inequality (Munroe, 1953) can be used to show that

$$\int_{r_1}^{r_2} M(r) dr \leq (r_2 - r_1)^{(b-1)/b} \left[ \int_{r_1}^{r_2} [M(r)]^b dr \right]^{(1/b)},$$

or

$$L \leq (r_2 - r_1)^{(b-1)/b} \left( \frac{\tau_e}{a} \right)^{(1/b)},$$

(6)

with the restrictions that $b > 1$ and that $M(r) > 0$, both of which are true in our case. The equality in (6) occurs when $M(r)$ is a constant, i.e., the liquid water is uniformly distributed between $r_1$ and $r_2$. Since $M(r)$ is undoubtedly not a constant, $L$ will be somewhat less than the maximum, depending on the value of $b$. For our frequency of 10.7 GHz, a simple linear interpolation between Crane’s computations at 9.35 and 15.5 GHz results in constants $a = 0.12$ and $b = 1.28$ for (4), where $\beta_e$ is in nepers per kilometer and $M$ in grams per cubic meter. Since $b$ is not very much greater than 1, $L$ is rather close to the right-hand side of (6), as can be seen by taking some simple distributions of $M(r)$ for comparison. For a “triangular” distribution that is symmetrical about the center of the ray path between $r_1$ and $r_2$, the $L$ resulting here is 95% of the maximum given by (6). A second example is a symmetrical exponential distribution of the form

$$M(l) = M_0 \exp \left( -\frac{l}{l_e} \ln K \right),$$

where $M_0$ is the liquid water density at the “center” of the storm, $l$ the distance from this point, $l_e$ the distance to the edge of the storm, and $K$ the ratio of $M_0$ to the density at the edge of the storm. Fig. 4 shows the ratio of integrated liquid water, $L_{\text{exp}}$, from this model to the maximum $L$ of (6) as a function of $K$. The figure shows that for a very large value of $K$, indicating almost a spiked distribution, $L$ is reduced by only $\sim 25\%$ from the maximum.

Eq. (6) also requires an estimate of $r_2 - r_1$. This estimate of storm size is made from the location of relative “hot spots” (maximum $L$ value, estimated) and the outer edge (zero contour) of the storm. The outer boundary of the storm is taken to be a semicircle in a plane perpendicular to the contour diagram and containing the line from the hot spot through the measurement point to the zero contour. The radius $r_g$ of this semicircle is the distance from the hot spot to the zero contour. If the distance from the hot spot to the data point (in the contour plane) is $d$, then

$$r_2 - r_1 = 2\sqrt{r_g^2 - d^2}.$$

Since $b$ in (6) is near 1, the solution for $L$ is not very sensitive to errors in this quantity, again indicating that the measurement is not strongly dependent on the spatial distribution of $M$.

4. Results

The relationship (6) is used to plot the contours of $L$ shown in Fig. 5. The contours are plotted on an azimuth angle-elevation angle grid as seen from the radiometer site. The horizontal scans necessary to produce the contour diagrams were made during the time interval indicated. The units of (6) are grams of liquid water per cubic meter times kilometers, which is equivalent to the number of kilogram of liquid in a column 1 m² in cross section along the entire path through the storm. Alternatively, one can estimate the average liquid water density (gm m⁻³) over the path length by dividing the contour values by an estimated length (km) through the storm.

Eq. (6) is used with the implicit assumption that (4) is applicable at all cloud temperatures encountered in addition to the 0°C for which it was determined. It can be shown that this is a rather good assumption at our
Fig. 5. Liquid water content (kg m$^{-3}$ column) of an 18 July 1967 thunderstorm near Boulder, Colo., from 1503–1602 MDT, a., and from 1602–1714 MDT, b.

frequency for typical drop size distributions encountered in rainfall. This results from the fact that for small drops the absorption coefficient decreases with increasing temperature; for drops of $\sim$1 mm in radius the absorption is essentially independent of temperature, and for larger drops the temperature dependence is reversed.

The analysis presented here contains another implicit assumption. In using the absorption cross section, we
assume that the emission does indeed originate from liquid rather than ice. Since thunderstorms are generally mixtures of ice and water particles it is difficult to say which is dominant. To aid in judging in which regions of the storm the assumption may be valid, we have indicated the 0°C isotherm (freezing level) on the contour diagrams and note that no hail reports from the 18 July storm were known to us. It seems likely that quantitative radar reflectivity data would be very useful in determining the validity of this assumption.

5. Conclusions

The technique described here shows promise as a tool for studying liquid water in clouds, although some further experimental work is needed to delineate its limitations. In addition to an estimate of integrated liquid water per unit area encountered by a radio ray, it provides a partial picture of how the liquid is distributed and some idea of the flux of liquid water in a particular volume. This information could be useful in predicting the amount of water available for precipitation and in observing the changes produced by storm modification experiments. In such a study, the method would, of course, be used with other techniques, especially radar. Measurement of attenuation in the storm is also an important use of the technique for the communication engineer, although this has not been elucidated here in detail.

Acknowledgments. We express our gratitude to J. M. Pepio for his valuable contributions, both to the experiments and to the data reduction procedure.

REFERENCES