Chapter 8

Multiscale Temporal Mean Features of Perturbation Kinetic Energy and Its Budget in the Tropics: Review and Computation

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ABSTRACT

The authors examined the maintenance mechanisms of perturbation kinetic energy (PKE) in the tropical regions for multiple time scales by computing and analyzing its budget equation. The emphasis has been placed on the mean features of synoptic and subseasonal variabilities using a 33-yr dataset. From analysis of the contributions from $u$-wind and $v$-wind components, the PKE maximum in the Indian Ocean is attributed less to synoptic variability and more to intraseasonal variability in which the Madden–Julian oscillation (MJO) dominates; however, there is strong evidence of seasonal variability affiliated with the Asian monsoon systems. The ones in the eastern Pacific and Atlantic Oceans are closely related to both intraseasonal and synoptic variability that result from the strong MJO and the relatively large amplitude of equatorial waves.

The maintenance of the PKE budget mainly depends on the structure of time mean horizontal flows, the location of convection, and the transport of PKE from the extratropics. In the regions with strong convective activities, such as the eastern Indian Ocean to the western Pacific, the production of PKE occurs between 700 and 200 hPa at the expense of perturbation available potential energy (PAPE), which is generated by convective heating. This gain in PKE is largely offset by divergence of the geopotential component of vertical energy flux; that is, it is redistributed to the upper- and lower-atmospheric layers by the pressure field. Strong PKE generation through the horizontal convergence of the extratropical energy flux takes place in the upper troposphere over the eastern Pacific and Atlantic Ocean, and is largely balanced by a PKE loss due to barotropic conversion, which is determined solely by the sign of longitudinal stretching deformation. However, over the Indian Ocean, there is a net PKE loss due to divergence of energy flux, which is compensated by PKE gain through the shear generation.

1. Introduction

The perturbation kinetic energy (PKE) is extensively used to measure and study transient wave activity in the tropics (e.g., Webster and Chang 1988; Arkin and Webster 1985; Chen and Yanai 2000; Yanai et al. 2000). Murakami and Unninayar (1977) showed that the region of maximum PKE along the equator in January and February of 1971 was located in the vicinity of the equatorial westerlies. Using National Meteorological Center (NMC) operational tropical objective analyses from 1968 to 1979, Arkin and Webster (1985) indicated that the maximum PKE at 200 hPa in the tropics is collocated with a zone of equatorial westerlies where...
convective activity is minimum. Furthermore, Webster and Yang (1989) extended the analysis of Arkin and Webster by considering the vertical distribution of the mean PKE and its relationship with the structure of mean zonal wind field. Their study and others (e.g., Liebmann 1987; Webster and Chang 1988) showed quite similar results: the tropics can be divided into two regimes conditioned on the direction of the time-mean upper-level zonal wind at the equator, and that PKE maximum is located within the westerlies.

It needs to be pointed out that the PKE utilized in aforementioned studies was defined as the time mean kinetic energy of the motions on having time scales of less than one month. In consequence, the intraseasonal variability containing the Madden–Julian oscillation [MJO; Madden and Julian (1971), (1972), (1994); Zhang (2005); see also chapter 4 (Maloney and Zhang 2016) and chapter 5 (Krishnamurti et al. 2016) in this monograph], which is among the most prominent large-scale motions in the tropical atmosphere, had been underestimated. Yanai et al. (2000) examined the PKE in the intraseasonal time scale and its budget for the TOGA COARE intensive observation period (IOP; hereafter COARE IOP) by integrating the power spectra and cospectra over the period range of 30–60 days. They showed that, in addition to two PKE maxima over the equatorial eastern Pacific and Atlantic Oceans where the westerlies were predominant in the upper troposphere, a large PKE center associated with strong convective activity over the warm pool region was located above the 200-hPa level. It was found that the interaction between convection and large-scale circulation, via production and conversion of perturbation available potential energy (PAPE) in the middle troposphere, plays a major role in the maintenance and growth of the MJO over the Indian Ocean–western Pacific warm pool, where the wave energy flux is clearly radiating upward and downward from the convective source region. In the central-eastern Pacific, where deep cumulus convection is suppressed, there are strong equatorward fluxes of wave energy from the subtropics of both hemispheres, causing horizontal convergence of wave energy flux in the equatorial upper troposphere. The results obtained from the climatological data indicate that the conclusions derived from the COARE IOP cases by Yanai et al. (2000) can be generalized to a great degree (Chen and Yanai 2000).

The most conspicuous feature of the tropical PKE is the collocation of PKE maximum with the equatorial westerlies in the upper troposphere. Various theories and hypotheses have been proposed to explain this feature. Charney (1969) studied the meridional propagation of large-scale wave disturbances into the tropics and suggested that disturbances propagating into the tropics will tend to be confined in the upper troposphere and lower stratosphere where zonal winds are weak easterly or westerly. Bennett and Young (1971) discussed in detail the effects of horizontal shear on the meridional propagation of disturbances. They pointed out that 1) disturbances with large eastward phase speeds cannot propagate into the tropics, 2) disturbances whose phase speeds coincide with the mean flow somewhere are absorbed at the critical latitude, and 3) disturbances with greater westward phase speeds than the mean flow may freely propagate into the tropics. Several studies (e.g., Webster and Holton 1982; Magaña and Yanai 1991; Zhang and Webster 1992) further emphasized the interaction of tropical and extratropical activity, and indicated that the regional maximum in PKE is a result of the equatorward propagation of disturbances into the tropics through the westerly duct (i.e., the convergence of equatorward wave energy flux). On the other hand, Webster and Chang (1988) and Chang and Webster (1990, 1995) have offered another explanation, that is, the PKE maximum may be associated with the accumulation of wave energy flux emanating from the tropical source region through nonuniform zonal flow. In addition, Wang and Xie (1996) suggested that a westerly vertical shear provides a favorable condition for the trapping of Rossby and Yanai waves in the upper-troposphere waves and may be partially responsible for the PKE maximum in the upper-tropospheric westerly duct.

The PKE budget analysis has further been used to diagnose the energetics of the intraseasonal variability and MJO in model simulations. Mu and Zhang (2006) examined the PKE budget in the modified NCAR CAM3, and pointed out that different mechanisms are responsible for the PKE production at different locations. Deng and Wu (2011) computed the PKE budget to delineate the physical processes that led to improved MJO simulations by a general circulation model. These studies demonstrated that the PKE budget could be a powerful tool to evaluate model performance and to investigate the physical processes resulting in the development and maintenance of the simulated tropical disturbances.

This chapter reviews and expands on the seminal work by Yanai et al. (2000). The primary objective is to examine the maintenance mechanisms of PKE in various synoptic and subseasonal time scales (i.e., 2–15, 17–24, and 30–60 days) by evaluating the PKE budget equation using a long period dataset, and, for the sake of model evaluation, to provide a climatological mean distribution of the PKE budget. It is also intended to serve as a reference for the PKE and PAPE budget computations, specifically the computation of energy transformation and generation functions using cross-spectrum analysis. The formula of the generation function calls for the
apparent heat source \((Q; \text{Yanai et al. 1973})\), which is the budget residual of the thermodynamic equation; therefore, we describe its computation to demonstrate the principle of computing budget equations in the spherical-pressure coordinates.

The data and analysis procedures are briefly described in section 2. Section 3 examines spatial distribution of the tropical PKE in different time scales. Sections 4, 5, and 6 present results from the terms of shear generation, conversion of PAPE, and energy flux in the PKE budget equation, respectively. In addition, an analysis for the PKE budget residual is also included in section 6. The summary and discussions are given in section 7. Finally, methods for computing cross-spectra and budget equation residuals are described in the appendixes.

### 2. Data and analysis procedures

The primary dataset used in this study is the European Centre for Medium-Range Weather Forecasts (ECMWF) interim reanalysis (ERA-Interim) which is the latest global atmospheric reanalysis produced by the ECMWF to replace ERA-40. The ERA-Interim was generated by a frozen global data assimilation system along with an observational database and overcame several difficult data assimilation problems that were mostly related to the use of satellite data (Dee et al. 2011). The ERA-Interim provided an improved representation of the hydrological cycle, a more realistic stratospheric circulation, and better temporal consistency on a range of time scales. An advantage of using reanalysis is that the dataset is not subject to changes in the operational analyses. Furthermore, in the tropics, especially in the areas with sparse observations, both operational analysis and reanalyses depend greatly on the first guess supplied by the forecast model, and consequently depend critically on the diabatic heating distribution produced by the physical parameterization of the model. Comparison between the precipitation from the ERA-Interim, and independent surface- and satellite-based observations indicate that the ERA-Interim has substantial improvement in the precipitation due to improvements in the cloud and convection schemes and that the analyzed circulation is probably near reality.

The ERA-Interim dataset used in the study covers a 33-yr period from 1979 to 2011 and was averaged into daily mean data with a grid mesh of 1.5° × 1.5° at 28 pressure levels. The domain of study is 50°S–50°N and 0°–360°.

Following Yanai et al. (2000), the PKE is defined as

\[
\frac{\partial \overline{K}}{\partial t} = -\nabla \cdot \mathbf{v} - \nabla \cdot \mathbf{v} - \nabla \omega \cdot \frac{\partial \mathbf{v}}{\partial p} - \alpha \omega^2
\]

\[
- \mathbf{v} \cdot \overline{F_h} - \frac{\partial F_p}{\partial p} + \overline{\nabla \cdot \mathbf{f}},
\]

In Eq. (8-1), \(\mathbf{F_h} = \overline{\nabla \mathbf{v}} + \overline{\mathbf{v}} \mathbf{v} + \mathbf{v}^2 \mathbf{v}/2\), and \(F_p = \overline{\mathbf{v}} \mathbf{v} + \overline{\mathbf{v}} \mathbf{v} + \mathbf{v}^2 \mathbf{v}/2\), where \(\mathbf{v}\) is the horizontal velocity, \(\omega = dp/dt\) (the vertical \(p\) velocity), \(\rho\) is the pressure, \(\alpha\) is the specific volume, \(\mathbf{v}\) is the isobaric gradient operator, \(\phi\) is the geopotential, and \(\mathbf{f}\) is the frictional force per unit mass.

The right-hand side terms of Eq. (8-1) represent, respectively, the production of PKE through barotropic conversion process (the shear generation terms), the conversion from the perturbation available potential energy (more details in section 5) through \((\alpha, -\omega)\) correlation, the horizontal and vertical convergence of wave energy flux \((\mathbf{F_h}, F_p)\), and the work done by frictional force. Furthermore, the energy flux \((\mathbf{F_h}, F_p)\) is separated into two components: \((\mathbf{F_h}, F_p) = (\mathbf{F_h}, F_p)_{gh} + (\mathbf{F_h}, F_p)_{PKE}\), where \((\mathbf{F_h}, F_p)_{gh} = (\overline{\nabla \mathbf{v}}, \overline{\mathbf{v}} \mathbf{v})\) is the geopotential component that contributes to redistribution of energy by pressure, and \((\mathbf{F_h}, F_p)_{PKE} = (\overline{\mathbf{K}} \mathbf{v} + \mathbf{v}^2 \mathbf{v}/2, \overline{\mathbf{K}} \mathbf{v} + \mathbf{v}^2 \mathbf{v}/2)\) mainly represents PKE flux.

The terms mentioned above (i.e., various energy transformation functions) involve covariance between two quantities. By utilizing cross-spectrum analysis technique, the contribution from variability at a certain frequency to the total covariance (i.e., cospectrum) can be obtained. Therefore, Eq. (8-1) can be calculated at different frequencies. In this study the lag-correlation method is used in the spectral analysis to conserve variance and covariance (see details in appendix A).

We used a moving block bootstrap method to estimate the covariance terms. Since we are interested in synoptic to intraseasonal time scales, the data were grouped into 189 successive overlapping (by 63 days) segments with a length of 183 days. The calculation was carried out for each segment and the final result was obtained by averaging over all available segments of the 33-yr record. The mean of each segment was first removed. In the lag-correlation method the maximum lag number \((M)\) is set to 60 days, then the variability can be resolved at periods of \(2M/k, (k = 0, 1, 2, \ldots, M)\) (i.e., the mean, 120 days, 60 days, 40 days, 30 days, etc.). The variance and covariance were grouped into the contributions from three period bands: 30–60 days (intraseasonal time scale), 17–24 days (interim time scale), and 2–15 days (synoptic time scale).

The selection of these time scales is ultimately subjective. The multiscale tropical convection–coupled disturbances are known to have broadband time spectra and often scale-invariant characteristics below the
intraseasonal time scale (e.g., Tung and Yanai 2002a; Tung et al. 2004). As exemplified by the high-resolution spatial–temporal modes of tropical convection signals extracted from satellite infrared brightness temperature throughout 1984–2006 in Tung et al. (2014), ~30–60-day spectral maxima are prevailing features in the time spectra across various modes, accompanied with a power-law decaying range down to below 10 days until the diurnal cycle (e.g., Fig. 5 in their paper). The 2–15-day scale was chosen for known equatorial waves active in this range (e.g., review by Kiladis et al. 2009) and its significance in short- to extended-range synoptic weather forecasting. The interim 17–24-day range is thus named accordingly.

3. Distribution of PKE

Figures 8-1a–d show the PKE averaged between 15°S and 15°N as functions of longitude and height (pressure) (hereafter x–p section) for the total and the three period bands. In Fig. 8-1a, the maximum centers in total PKE are located above the tropical Indian, the eastern Pacific, and Atlantic Oceans; they occupy the mid- to upper troposphere between 400 and 100 hPa. The two maxima in the Western Hemisphere are stronger than the one in the Eastern Hemisphere and reside in a lower altitude. The spatial distribution of PKE in the three period bands bears a resemblance to that of total PKE, but naturally with smaller magnitudes. It is noted that, in the intraseasonal time scale, the maximum in the Eastern Hemisphere shows nearly equal intensity to those in the Western Hemisphere (Fig. 8-1b). However, in the synoptic time scale, the former appears weaker than the latter (Fig. 8-1d). Moreover, the PKE maxima in the intraseasonal time scale are relatively higher (~150–200 hPa) than those in the other two time scales (~200 hPa). In this time scale, the maximum center around 120°W is more eastward displaced than other period bands. Compared with the PKE in the other two period bands, that in the interim 17–24-day band has very small contribution to the total PKE.

To interpret the PKE in the total PKE (Fig. 8-1a), one must consider the mean PKE (zero frequency) and a portion of the seasonal PKE (120-day period) in addition to the three period bands (see section 2 and appendix A). In general, the mean PKE accounts for 10% of the total PKE, and the 120-day PKE accounts for 15% (neither is shown here). Particularly, the intensity of seasonal PKE is evidently quite strong in the Indian Ocean, with the 120-day PKE reaching up to 25%–30% of the total PKE maximum center. This large seasonal PKE is considered to be associated with the Asian monsoon system, which migrates annually between the Northern and Southern Hemispheres in accordance with the apparent movement of the sun (e.g., Webster et al. 1998; Chao and Chen 2001). It may also be related to the seasonality of the

![Fig. 8-1. Longitude–height cross sections of PKE in the (a) total period range, (b) 30–60-day band, (c) 17–24-day band, and (d) 2–15-day band (J kg⁻¹)].
tropical intraseasonal variability (e.g., reviews by Madden and Julian 1994; Lau and Waliser 2012).

Figures 8-2 and 8-3 show \( u_0^2/2 \) and \( v_0^2/2 \), the respective contributions of zonal and meridional wind components to the PKE in Fig. 8-1. As seen in Figs. 8-2a and 8-3a, and compared with Fig. 8-1a, the contribution from the \( u_0^2/2 \) to the total PKE is larger than that from the \( v_0^2/2 \). In particular, much greater contribution from \( u_0^2/2 \) to the

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**Fig. 8-2.** As in Fig. 8-1, but for \( u_0^2/2 \).

**Fig. 8-3.** As in Fig. 8-1, but for \( v_0^2/2 \).
PKE maximum over the Indian Ocean is observed. The contribution to total $u^2/2$ by the intraseasonal band is overall comparable to that by the synoptic band (Figs. 8-2b and 8-2d); however, much larger contribution from seasonal (not shown) and intraseasonal variability (Fig. 8-2a) can be seen over the Indian Ocean. From Figs. 8-3b and 8-3c, contribution to the total $v^2/2$ from variability of the 2–15-day band is much more significant in the eastern Pacific and Atlantic Oceans, indicating the presence of relatively large-amplitudes Rossby waves that are considered to be associated with an equatorward propagation of wave energy from higher latitudes by many authors (e.g., Kiladis 1998; Straub and Kiladis 2003).

Moreover, comparing Fig. 8-2b with Fig. 8-3b, in the 30–60-day band $u^2/2$ is much stronger than $v^2/2$; comparing Fig. 8-2b with Fig. 8-1b, about $\frac{1}{4}$ of PKE in 30–60-day band comes from the contribution of $u^2/2$, suggesting that the MJO plays a key role in the PKE distribution because of the large MJO amplitude in $u$-wind component especially at its “dry” phase across the Western Hemisphere (e.g., review by Madden and Julian 1994). This is further corroborated by Fig. 8-4, which shows the frequency–wavenumber power spectra of the zonal wind at 850 and 200 hPa. The spectra are overlaid with dispersion curves as in previous work by Hayashi (1982), Takayabu (1994), and Wheeler and Kiladis (1999); here, they were calculated by assuming a static base state, with the marked equivalent depths for each equatorially trapped shallow-water wave type (Matsuno 1966; Lindzen and Matsuno 1968). These spectra indicate that the eastward-moving MJO is the most dominant intraseasonal signal in both wind fields. By examining Fig. 8-2d and Fig. 8-3d together, it is shown that the contribution to PKE in the 2–15-day period band from $v^2/2$ is comparable to that from $u^2/2$ in terms of magnitude. The relatively large contribution of $v^2/2$ to PKE in the 2–15-day period might be related to strong wave activities in particular over the eastern Pacific and Atlantic Oceans. A pronounced feature in this region is the upper-tropospheric “westerly duct” in the northern fall through spring that allows extratropical Rossby waves to propagate through (Webster and Holton 1982; Tomas and Webster 1994; Yang and Hoskins 1996). In the 17–24-day period, both $u^2/2$ and $v^2/2$ are much smaller than those in the other two period bands (Figs. 8-1c, 8-2c, and 8-3c).

In summary, two PKE maxima in the eastern Pacific and Atlantic Oceans mostly comprise the variance of two time scales: the 30–60-day intraseasonal time scale...
and the 2–15-day synoptic time scale. The variance from both time scales, as well as from the 120-day seasonal variation, contribute to the PKE maxima in the Indian Ocean. On the intraseasonal time scale in all three regions, the dominant contributor to the PKE is the zonal wind perturbations. Moreover, the synoptic time-scale activities are stronger in the eastern Pacific and Atlantic Oceans than those of the Indo-Pacific warm-pool sector, and are marked with comparable contributions from both zonal and meridional wind perturbations.

Figure 8-5 shows the longitude–height section of zonal wind averaged between 15°S and 15°N. The total PKE maximum centers over the eastern Pacific and Atlantic Oceans in Fig. 8-1a are collocated with the equatorial westerlies in the upper troposphere, while large values of PKE and easterlies in the upper levels coincide in the regions from the Indian Ocean to the western Pacific where the most active convection exists. The collocation in the eastern Pacific and Atlantic Oceans was recognized a long time ago and well documented (e.g., Murakami and Unninayar 1977; Arkin and Webster 1985; Webster and Chang 1988), but recognition of the one in the Eastern Hemisphere is relatively new (Yanai et al. 2000).

Figure 8-6 is the sum of all terms on the right-hand side of Eq. (8-1) except friction forcing for all period ranges. Practically, the time tendency of PKE is zero for a long time mean, so the residual represents the negative friction forcing (i.e., $-\frac{\mathbf{v} \cdot \mathbf{f}}{C_1}$) plus the computation error. It can be seen that the $\frac{\mathbf{v} \cdot \mathbf{f}}{C_1}$ is negative everywhere and especially in the boundary layer, although areas with small positive values are found over the oceans around 700 hPa and on the upwind side of the Andes and the Ethiopian Highlands at around 200 hPa. These positive signals are nontrivial; they may well be associated with momentum transports of convection and convectively coupled waves [e.g., review by Sui and Yanai (1986); Tung and Yanai (2002a,b); and
fundamental theories in chapter 9 (Khoudier and Majda 2016) and chapter 10 (Majda and Stechmann 2016) as well as mountain-induced internal gravity waves (e.g., Durran 1990). Therefore, the residual indicates that our budget calculation is rather accurate and reliable as well.

In the subsequent sections, we will discuss processes that maintain the PKE distribution by calculating and analyzing the PKE budget equation. Since the interim band contributes minimally to the total PKE, we mainly focus on the discussions of total, intraseasonal, and synoptic period bands. The interim band is mentioned only when there are sufficiently significant findings.

4. Horizontal shear generation

To gain a better understanding of the role played by the horizontal shear generation or barotropic conversion in the maintenance of PKE, it is helpful to rewrite this term as

\[
\mathbf{\nabla} \mathbf{\nabla} \cdot \mathbf{V} \cdot \mathbf{\nabla} = - \frac{\partial \Pi}{\partial x} - \frac{\partial \Pi}{\partial y} - \frac{\partial \Pi}{\partial y} \left( \frac{\partial \Pi}{\partial x} + \frac{\partial \Pi}{\partial x} \right).
\]

That is, SG (horizontal shear generation of PKE) is divided into the PKE generation through the work done by stretching deformation of mean horizontal flows (SG1 and SG2), and by shearing deformation (SG3).

Figures 8-7a–c show the longitude–height sections of SG for total, intraseasonal, and synoptic period ranges. From Fig. 8-7a (total period range), three large positive generation values in the upper troposphere can be found around the eastern coast of Africa (40°–60°E), the western coast of America (130°–80°W), and the central Atlantic Ocean (30°W–10°E). Moreover, the negative values cover Africa, America, and a broad region from the central Indian Ocean to the eastern Pacific Ocean. In the intraseasonal and synoptic period ranges (Figs. 8-7b and 8-7c), both distributions of SG are quite similar to Fig. 8-7a but overall SG in the intraseasonal time scale is stronger than that in the synoptic time scale.

Terms SG1, SG2, and SG3 are shown in Figs. 8-8a–c, Figs. 8-9a–c, and Figs. 8-10a–c for the total, intraseasonal, and synoptic period range, respectively. Among the three terms, SG1 is the dominant one in all period ranges and determines net effect of the horizontal shear generation (SG). As a result, SG1 shows a very similar structure as SG. In all period ranges SG2 shows an opposite sign with SG1 and offsets a portion of the contribution of SG1 although the offsetting values are different in various period ranges. Finally, SG3 is rather small and negligible in comparison with SG1.

The sign of SG, which indicates generation or destruction of PKE, is mainly determined by that of SG1, which is, in turn, decided solely by the sign of longitudinal stretching deformation \( \frac{\partial \Pi}{\partial x} \) due to \( u' u' / \Pi > 0 \); that is, in the region of \( \frac{\partial \Pi}{\partial x} < 0 \) (\( \frac{\partial \Pi}{\partial x} > 0 \), PKE is generated (destroyed). The longitude–height section of \( \frac{\partial \Pi}{\partial x} \) is illustrated in Fig. 8-11. From Fig. 8-11 together with Figs. 8-5, 8-7, and 8-8, it can be seen that, through work done by longitudinal stretching of the mean flow, the generation of PKE occurs to the east of the equatorial westerly and to the west of the easterly where \( \frac{\partial \Pi}{\partial x} < 0 \), and the destruction occurs to the west of the westerly and to the east of the easterly where \( \frac{\partial \Pi}{\partial x} > 0 \). From the wave propagation view, the term SG1 is related to the accumulation of wave energy that accompanies wave action flux in a spatially varying mean flow. Webster and Chang (1988) and Chang and Webster (1990) theoretically explored the phenomenon of energy accumulation of transient equatorially
trapped modes, and pointed out that regions where \(\partial u/\partial x > 0\) (\(\partial u/\partial x < 0\)) are regions of wave action flux (i.e., wave energy density) divergence (convergence). Thus, the accumulation of wave energy occurs to the east of the maximum westerly wind that enhances the intensity of disturbances (i.e., the generation of PKE).

In the period ranges discussed, PKE generation from vertical shear of the mean flow [i.e., \(\frac{2}{\rho_0} \frac{\partial v}{\partial p}\)] (not shown) is very small and its contribution is negligible.

5. Conversion of perturbation available potential energy

Figures 8-12a–c illustrate the longitude–height cross sections of the total covariance of specific volume (\(\alpha\)) and vertical motion (\(-\omega\)) and the partial covariance in the 30–60-day, and 2–15-day period ranges, respectively. The term \(-\alpha \omega > 0\) represents conversion from PAPE to PKE through rising (sinking) motion of warm (cold) air parcels (baroclinic conversion). In the four period ranges, the large conversion from PAPE to PKE occurs almost exclusively in the deep tropospheric layer between 500 and 200 hPa. In the total period range (Fig. 8-12a), the generation of PKE takes place in almost all the tropical domain shown, with the largest values in a region from the Indian Ocean across the western Pacific warm pool beyond the date line and in the continents of South America and Africa where the most active cumulus convection is observed. In the intraseasonal time scale, the most significant conversion from PAPE to PKE is confined to the Indo-Pacific warm-pool region, which is in agreement with a well-known and well-documented fact that the most active convection associated with the MJO exists only over a region from the central Indian Ocean to the central Pacific Ocean (see review by Zhang 2005). In the synoptic time scale, in addition to the large and broad generation of PKE existing between the Indian and Pacific Oceans, two centers are found in the continents of Africa and South America suggesting a strong convection activity in the synoptic time scale.

**Fig. 8-8.** As in Fig. 8-7, but for SG1.

**Fig. 8-9.** As in Fig. 8-7, but for SG2.
Following Yanai et al. (2000), the PAPE is defined as
\[ \tau = \frac{\alpha^2}{2S}, \]
where \( S = -\frac{\alpha \bar{h}}{\bar{\rho}} \) is the static stability factor and \( \theta \) is the potential temperature; the approximate equation for the time change of PAPE is
\[ \frac{\partial \tau}{\partial t} = -\frac{1}{S} \bar{\nabla} \bar{v} \cdot \nabla \bar{\omega} - \bar{v} \cdot \bar{E}_h - \frac{\partial \bar{E}_p}{\partial \bar{p}} + \frac{R}{c_p S \bar{\omega}} \frac{\alpha}{\bar{\rho}} Q_1, \]
(8-2)
with \( \bar{E}_h = \bar{\varepsilon} + \bar{v} \cdot \bar{\omega} \frac{\alpha^2}{2S} \) and \( \bar{E}_p = \bar{\varepsilon} + \bar{v} \cdot \bar{\omega} \frac{\alpha^2}{2S} \), \( R \) is the gas constant of dry air, \( c_p \) is the specific heat capacity at constant pressure, and \( Q_1 \) is the diabatic heating. The right-hand side of Eq. (8-2) represent, respectively, the conversion of mean available potential energy through horizontal heat flux due to perturbation, the conversion from PAPE into PKE through the \((\alpha, \bar{\omega})\) correlation [cf. Eq. (8-1)], the horizontal and vertical convergence of fluxes of PAPE fluxes \((\bar{E}_h, \bar{E}_p)\), and production of PAPE through the \((\alpha, Q_1)\) correlation. Here, \( Q_1 \) can be evaluated as the residual of the large-scale thermodynamic equation budget (e.g., Yanai et al. 1973; Yanai and Johnson 1993; see appendix B for computation). Figures 8-13a–c show the term \((R/c_p S \alpha Q_1)\) for the four period ranges. Compared with Figs. 8-12a–c, the terms \((R/c_p S \alpha Q_1)\) and \(-\frac{\alpha}{\bar{\rho}} \bar{\omega} \) bear a remarkable resemblance to each other with regard to the distributions and the magnitudes, indicating that the latent-heating release and transport of heat in deep convection is a primary source of PAPE, and is almost solely converted into PKE through the correlation of \((\alpha, \bar{\omega})\). This particular PKE generation and conversion were first noted in the Marshall Island area by Nitta (1970, 1972), and confirmed by Wallace (1971) and Kung and Merritt (1974).

Figures 8-12 and 8-13 indicate that the maximum heating and the conversion from PAPE to PKE take place above midtroposphere between 400 and 300 hPa for all frequency bands. In the tropics, apart from deep convection, a great amount of stratiform cloud exists, including mesoscale anvils, as well as those not...
associated with convective systems (e.g., Houze 1982; Schumacher and Houze 2003). Lin et al. (2004) attributed the top heaviness of the heating profile of the MJO to stratiform precipitation. For other time scales, further investigation is warranted.

6. Energy flux and its convergence

a. Energy flux

Figures 8-14a–c show \((F_x, F_p)_{gh}\) (as in the geopotential component of wave energy flux) along the equatorial band between 15°S and 15°N for the total and two period ranges (see appendix C for the procedure to produce the plot). The most outstanding feature recognized in Fig. 8-14a is that the wave energy flux emanates upward and downward from a region (250–400 hPa) of wave energy flux over the warm pool from the eastern Indian Ocean to the date line. In addition, two regions between 100 and 400 hPa are identified off the coast of South America (≈90°W) and East Africa (≈30°E) where a very strong westward horizontal transport exists besides the vertical emanations. Compared with Fig. 8-12, it can be found that the regions from where wave energy fluxes emanate away in Fig. 8-14a are collocated with the areas where the large production of PKE from PAPE occurs. In the 30–60-day band (Fig. 8-14b), \((F_x, F_p)_{gh}\) shows noticeable upward and downward emanations in the Indo-Pacific sector where the deep convective activity associated with the MJO resides, and dominant westward components between 100 and 400 hPa from South America. Moreover, \((F_x, F_p)_{gh}\) in the 2–15-day band (Fig. 8-14c) possesses the largest contribution to the total flux and shows a similar structure to those of the total period and the 30–60-day band.

The PKE components of wave energy flux along the 15°S–15°N latitude band [i.e., \((R/c_p S_p)\alpha Q_1\)] are illustrated in Figs. 8-15a–c for the total and two period ranges. It is striking that, for all bands, the regions of large eastward
FIG. 8-14. As in Fig. 8-7, but for \((F_x, F_p)_{gh}\) (J m s\(^{-1}\) kg\(^{-1}\)).
Fig. 8-15. As in Fig. 8-14, but for \((F_x, -F_p)_{PE}\).
horizontal flux between 400 and 100 hPa are from the date line to the American west coast, as well as from the Atlantic Ocean to the west coast of Africa with the largest convergence around South America. Vertical overturning features are also seen in these two regions, in particular in the 2–15-day period. From the total wave energy flux \( [i.e., (F_x, F_y)_{gh} + (F_x, F_y)_{PKE}] \) (Figs. 8-16a–c), the potential and PKE components of wave energy flux are partially canceled out over the Western Hemisphere. The potential component dominates the total wave energy flux and largely determines the distribution of the energy wave flux for all but the 2–15-day band in which the overturning features seen in the PKE component over the eastern Pacific and Atlantic Ocean are carried into the total wave energy flux.

Horizontal maps showing the total horizontal wave energy flux \([i.e., (F_x, F_y)]\) in a vertical layer between 200 and 225 hPa are displayed in Figs. 8-17a–d for the total, intraseasonal, interim, and synoptic period ranges. It is noted that as time scales reduce from Figs. 8-17a to 8-17d, the prevailing energy flow patterns morph from being largely rotational in Fig. 8-17a to being convergent in Fig. 8-17d, likely reflecting the distinction between the geostrophic (Rossby) mode and the inertia–gravity mode of atmospheric motions. This gradual transition is more obvious in the Eastern Hemisphere than the Western Hemisphere where background westerlies permit extratropical Rossby waves to propagate through (cf. Fig. 8-5). In the total and 30–60-day period ranges (Figs. 8-17a and 8-17b), two rotational centers straddle the equator over the Indian Ocean. From the Pacific to the Atlantic regions, the energy flux alternates between northward and southward cross-equatorial transports. The vast area over the western Pacific between 10°S and 10°N is occupied by a northward flux, while the eastern Pacific has a southward flux. There is a southward flux from the extratropics to the Maritime Continent at around 120°E, adjacent to a rotational center between 120° and 150°E. Along the equator, the energy flux is clearly eastward over the Indian Ocean; two likely centers of energy flux convergence are around 120°E and −150°W.

In the shorter period ranges (Figs. 8-17c,d), the energy flux over the equatorial Indian Ocean is westward, the opposite of that in Figs. 8-17a,b. Here, the energy flux converges over Africa and South America on the equator. Another center of convergence is evident over the Maritime Continent at ~120°E in Fig. 8-17c. Near the date line, the center of divergence is seen in both Fig. 8-17c (~150°W) and Fig. 8-17d (~180°). The eastern Pacific exhibits the most pronounced cross-equatorial southward energy fluxes delivered from the Northern Hemispheric extratropics.

In all time scales, but most significantly in the 2–15-day range, the northward flux from the southern extratropics and the southward flux from the northern extratropics meet near the equatorial regions from the central to eastern Pacific Ocean and the equatorial Atlantic Ocean, where the mean westerlies prevail showing the propagation of wave energy into the tropics from higher latitudes. From horizontal maps of \((F_x, F_y)_{gh}\) and \((F_x, F_y)_{PKE}\) (not shown), generally, the two west–east components \([i.e., (F_x)_{gh}\) and \((F_x)_{PKE}\) mostly offset each other for all four period ranges with relatively larger values in the geopotential part. But, the two north–south components \([i.e., (F_y)_{gh}\) and \((F_y)_{PKE}\) have the same signs in most regions where the PKE part is dominant, leading to an enhancement in the north–south components of total wave energy flux, particularly, the northward flux from the southern extratropics and the southward flux from the northern extratropics in the sector from the central to eastern Pacific Ocean and Atlantic Ocean.

It should be pointed out that there is an apparent inconsistency between the zonal component of the total fluxes at ~200 hPa in Fig. 8-16a and those around the equator in Fig. 8-17a, especially between 0°–120°E and 120°W–0°. As shown in Fig. 8-17a, the total fluxes in these areas have a very strong east–west component beyond 10°S and 10°N in the upper troposphere. They end up dominating the local sign of the averaged \(F_x\) in Fig. 8-16a. The averaged fluxes in Figs. 8-16b and 8-16c are not as much affected by the moderately strong zonal component in the higher latitudes, however.

b. Convergence of energy flux

The convergence of the geopotential and the PKE components of the energy fluxes is further divided into horizontal and vertical parts \([i.e., \nabla \cdot (F_x)_{gh}, \nabla \cdot (F_x)_{PKE}, -\partial (F_y)_{gh}/\partial p, and -\partial (F_y)_{PKE}/\partial p]\). Figures 8-18a–c illustrate the longitude–height cross sections of \(-\nabla \cdot (F_x)_{gh}\) averaged between 15°S and 15°N. In Fig. 8-18a, (total period band) wave energy flux convergence \([-\nabla \cdot (F_x)_{gh} > 0]\) exists in most areas from the western Indian Ocean across the Pacific Ocean to the eastern Pacific in the upper troposphere above 600 hPa. In addition, two fairly narrow but strong convergence centers cover most of Africa and South America. Two strong negative extrema are located at both coasts of Africa (around 5° and 45°E), and a relatively weak negative center is close to the western coast of South America (i.e., around 90°W). In the 30–60-day band (Fig. 8-18b), the patterns are quite similar to those in the total period band, but with a much stronger negative center off the west coast of South America (~90°W). In the 17–24-day band (not shown), strong convergence is found in
As in Fig. 8-14, but for \((F_x, -F_p)\).
the sector from the central Pacific to the middle Atlantic Oceans. In the 2–15-day band (Fig. 8-18c), except for a weak negative center in the eastern coast of Africa, strong convergence \[ \frac{2}{C_1} (F_h)_{gh} \] almost covers the entire globe in the middle to high troposphere.

The longitude–height cross sections of \[ \frac{2}{C_1} (F_h)_{PKE} \] are shown in Figs. 8-19a–c for the total and two period ranges. Compared with the cross sections for \[ \frac{2}{C_1} (F_h)_{gh} \] (i.e., Figs. 8-18a–c), in the regions between the Indian Ocean to the western Pacific, \[ \frac{2}{C_1} (F_h)_{PKE} \] is nearly compensated by that of geopotential component for all period bands. On the other hand, fairly strong convergence [i.e., \( \frac{2}{C_1} (F_h)_{PKE} > 0 \)] takes place in the regions from the central to eastern Pacific, particularly, with the largest values in the 2–15-day band, which considerably enhances the convergence of the total wave energy flux there. In the other regions, \[ \frac{2}{C_1} (F_h)_{PKE} \] has an opposite sign to those of the geopotential component with relatively smaller values, indicating that \( \frac{2}{C_1} (F_h)_{gh} \) mainly determines the distributions of \( \frac{2}{C_1} F_h \) in these regions.

Figures 8-20a–c illustrate the longitude–height cross sections of \( \frac{2}{C_1} F_h \) along the equator. In Fig. 8-20a (total period band) wave energy flux convergence \( \frac{2}{C_1} (F_h > 0) \) exists in the areas from the western Indian Ocean to the eastern Pacific Ocean in the upper troposphere above 600 hPa; and the maximum of convergence is located at the central to eastern Pacific Ocean around 160°W. In addition, two other convergence centers can be identified off the eastern coast of America (\(-40°W\)) and over Africa. Strong divergence of wave energy flux \( \frac{2}{C_1} (F_h < 0) \) occurs around the Arabic Sea and the western coast of Africa, and a weak divergence center is also seen in the eastern Pacific. In the 30–60-day band (Fig. 8-20b) the largest convergence of energy flux takes place in the two areas from the central to eastern Pacific Ocean and from the eastern coast of America to the central Atlantic Ocean, and the divergence is sporadically distributed with three centers in the central Atlantic Ocean, the eastern coast of Africa, and the eastern Pacific Ocean.

In 2–15-day band (Fig. 8-20c), the convergence of energy flux exists in most of the domain except around
the central Indian Ocean where there is small divergence. The largest convergence center is located at the eastern Pacific Ocean, and it extends westward to the western Pacific Ocean and eastward across the Atlantic Ocean to the eastern coast of Africa with two centers at 30°W and 20°E, respectively. The two convergence centers are found in the eastern Pacific Ocean and the central Atlantic Ocean in the 17–24-day band (not shown), and the small divergence occurs in other regions. Apart from the maximum of convergence in the regions from the central to eastern Pacific Ocean around 160°W in the upper troposphere above 600 hPa, which is strengthened by the contributions from both geopotential and PKE components, the distributions of $-\nabla \cdot \mathbf{F}$ is mainly decided by the divergence and convergence of geopotential component of energy wave flux for all period bands.

Figures 8-21a–c show the longitude–height cross sections of $-\nabla \cdot (\mathbf{F}_{gh})$ along the equator for the total and two period bands. The most outstanding features are that the predominant divergence occurs in the middle troposphere of 400–200 hPa for the whole longitudinal domain, and that the maximum divergence is collocated with the most intensive convection. In the 30–60-day period band (Fig. 8-21b), the strongest divergence exists in the sector between the Indian and Pacific Oceans where the eastward propagation of convection associated with the MJO is found. On the other hand, in the 2–15-day band (Fig. 8-21c) the strong divergence covers all tropical longitudes in the middle troposphere and is sandwiched by two positive areas in the lower and upper troposphere, although local variation is evident. The longitude–height cross sections of $-\partial \mathbf{F}_{gh}/\partial p$ {i.e., $-\left[\partial (\mathbf{F}_{gh})/\partial p + \partial (\mathbf{F}_{PKE})/\partial p\right]$} are displayed in Figs. 8-22a–c. Compared with Figs. 8-21a–c {i.e., $-\partial \mathbf{F}_{PKE}/\partial p$}, the contribution of geopotential component largely determines the distribution of total vertical divergence and convergence of wave energy flux, and $-\partial (\mathbf{F}_{PKE})/\partial p$ (not shown) is very small and its contribution is negligible.
7. Summary and discussion

In this chapter we reviewed and demonstrated the diagnostic utility of PKE in the tropical regions for various time scales. We calculated all terms of the PKE budget equation except the one related to frictional force by applying cross spectrum analysis techniques to the computation of variance and covariance. This approach is designed in such a way that the budget of PKE can be analyzed in a desired frequency or period domain. Our emphasis has been placed on the time mean features of subseasonal variability based on a 33-yr dataset. The computational methods are described in appendixes A and B for the readers’ reference.

In the course of our demonstration, three large PKE centers were found in the upper troposphere over the Indian Ocean, the eastern Pacific, and Atlantic Oceans; these are consistent with previous findings. From the analysis of the contributions from zonal and meridional perturbation wind components, the PKE maximum center in the Indian Ocean was shown to be dominated by the zonal component and is associated with synoptic variability and intraseasonal variability in which the MJO dominates. There was also evidence of strong seasonal variability, likely associated with the Asian monsoon systems and the seasonality of the tropical intraseasonal variability. The PKE maxima in the eastern Pacific and Atlantic Oceans appeared closely related to both intraseasonal and synoptic variability that result from the strong MJO and relatively large amplitude of tropical waves. These synoptic waves have comparable contributions from both zonal and meridional wind perturbations. In addition, the collocation of PKE maximum with the equatorial westerlies in the upper troposphere is evident over the eastern Pacific and Atlantic Oceans.

From an energetics point of view, to make the PKE budget balance, gain of PKE through certain processes must be compensated by the loss of the energy through
other processes. This energy balance is achieved, from a dynamical point of view, by adjusting the atmosphere to a new structure to be compatible with the energetics constraints through the dynamics and thermodynamics operating within the atmosphere. Our subsequent analysis revealed that the maintenance of the PKE budget mainly depends on the distribution of time mean horizontal flows, the location of convection, and the transport of PKE from the extratropics.

The most important conclusions can be summarized as follows for all period bands discussed. In the regions with strong convective activity, such as the Indian Ocean to the western Pacific, the gain of PKE occurs between 700 and 200 hPa through a positive correlation of (a, −ω) at the expense of PAPE generated by convective heating through positive (α, Q1) correlation. Such a gain is largely offset by the divergence of the geopotential component of vertical energy flux; that is, it is redistributed to the upper and lower atmospheric layers by the pressure field. Strong PKE generation through the horizontal convergence of the extratropical energy flux takes place in the upper troposphere over the eastern Pacific and Atlantic Ocean, and is largely balanced by PKE loss due to the work done by horizontal shear, which is decided solely by the sign of longitudinal stretching deformation ∂ω/∂x. However, over the Indian Ocean, there is a net PKE loss due to divergence of energy flux, which is compensated by PKE gain through the shear generation.

Finally, our calculation is performed by integrating the variance or covariance over a predefined period range. Notable differences can be found for the variability for different periods or frequencies. However, the results presented here can only serve as a general and static picture for a particular variability such as the MJO, because the detailed features of spectra are somewhat obscured by strong noise in the spectrum (e.g., weather noise and that resulted from budget calculations). To obtain the details of the PKE budget associated with a particular tropical system, proper prefiltering or significant tests must be performed. Moreover, for this chapter to be a first primer of PKE analysis, our focus is only placed on the time mean feature. More detailed statistical features, such as those conditioned on the phase of interannual variability, are of great interest for future research.

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APPENDIX A

An Application of Cross-Spectrum Analysis to the Calculation of the Energy Transformation Functions

The terms in the PKE budget in Eq. (8-1) (i.e., various energy transformation functions) involve covariance between two quantities. Following the classic cross-spectrum analysis of equally spaced finite records (e.g., in early studies of tropical waves by Maruyama 1967, 1968 and Yanai et al. 1968), we consider two time series...
$x_i(i = 1, 2, \ldots, N)$ and $y_i(i = 1, 2, \ldots, N)$, in which time means [i.e., $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ and $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$] are removed. We first estimate the cross-covariance function by $C_{xy}(l) = \frac{1}{(N-l)}\sum_{i=1}^{N-l} x_i y_{i+l}$ where $l = 0, 1, 2, \ldots, M(\leq N/2)$ and $M$ is the maximum of lag number that determines the frequency resolution (or interval) by $\Delta f = 1/(2M\Delta t) = f_M/M$ ($\Delta t$ is the time interval and $f_M$ is the Nyquist frequency). Then we construct the symmetric (or even) and the antisymmetric (or odd) parts {i.e., $C_+(l) = (1/2)[C_{xy}(l) + C_{yx}(l)]$ and $C_-(l) = (1/2)[C_{xy}(l) - C_{yx}(l)]$}. The cospectrum is obtained by the cosine transform of the symmetric component, that is, $K_x(k) = 4\Delta t\sum_{l=0}^{M} C_+(l) \cos(k\pi l/M)\delta_l$ for $k = 0, 1, 2, \ldots, M$; for $l = 0, M, \delta_l = 1/2$, otherwise $\delta_l = 1$. The quadrature spectrum is the sine transform of the antisymmetric component {i.e., $Q_y(k) = 4\Delta t\sum_{l=0}^{M} C_-(l) \sin(k\pi l/M)\delta_l$}, $k = 0, 1, 2, \ldots, M$. It can be shown that the covariance is $(1/N)\sum_{i=1}^{N} x_i y_i = \sum_{k=0}^{M}[K_x(k) + Q_y(k)],$ and the variance is $(1/N)\sum_{i=1}^{N} y_i^2 = \sum_{k=0}^{M}[K_x(k)].$ Therefore, the terms $K_x(k) + Q_y(k)$ and $K_x(k)$ can be identified as the contributions of perturbations with frequency of $k/(2M\Delta t)$ to the total covariance and variance, respectively; and the contribution from a particular frequency band can be obtained by aggregating those components within that frequency band. Using this technique we can calculate the covariance and variance terms, such as $u^2$, $v^2$, $\omega^2T$, etc., for a certain period band, and analyze the PKE budget for that particular period band.

**APPENDIX B**

**Example of Budget Computation Using Finite-Difference Approximation in the Spherical-Pressure Coordinates: $Q_1$**

In research we frequently perform budget evaluations given state variables and equations of conservation laws. When the computation is executed with observational data, the results are always tainted with measurement and computational errors. To suppress the latter type of errors, the finite-difference approximation of these equations cannot be arbitrary. As a tribute to late Prof. Yanai, we use the apparent heat source, $Q_1$, which is the budget residual of the thermodynamic equation, as an example to show the appropriate finite-difference approximation in the spherical-pressure coordinates. The readers can code the approximation for the PKE in Eq. (8-1) and obtain the time-mean budget residual (i.e., $\overline{\psi \cdot F}$) (Fig. 8-6) with the information provided in appendixes A and B.

Following Yanai et al. (1973) and Yanai and Johnson (1993), the apparent heat source, $Q_1$, is the budget residual of the thermodynamic equation:

$$
Q_1 = c_p \left( \frac{p}{p_0} \right)^{\kappa} \left( \frac{\partial \bar{\theta}}{\partial t} + \bar{v} \cdot \nabla \bar{\theta} + \omega \frac{\partial \bar{\theta}}{\partial \phi} \right)
$$

$$
= Q_R + L(\bar{v} - \bar{\theta}) - \nabla \cdot \bar{s} \nabla - \frac{\partial \bar{s} \omega}{\partial \phi},
$$

where $p_0 = 1000$ hPa, $\kappa = R/c_p$. $Q_R$ is the radiative heating rate, $s$ is the dry static energy, and $c$ and $e$ are the rates of condensation and evaporation (of cloud water) per unit mass of air, respectively. We have assumed that the Reynolds conditions and their consequences hold with sufficient accuracy. Note that, unlike in the main text or appendix A, which deal with time mean and its perturbations, the overbar here denotes the running horizontal average with respect to a large-scale area and the prime denotes the deviation from the average. That is, the scale separation takes place in space. The variables $u, v, \omega$, and $\theta$ resolved at the grid points of the reanalysis are regarded as “large scale” and are marked with overbars in the first equation. They are used as the direct input to calculate the residual $Q_1$, which is then interpreted according the second equation as the total effect of radiative heating, latent heat released by net condensation, and the convergence of fluxes of sensible heat due to subgrid-scale eddies such as cumulus convection and turbulence.

The finite-difference scheme of the thermodynamic equation is casted in a vertically staggered grid so that $Q_1$ is computed between two standard isobaric levels of the reanalysis data. Furthermore, its advective form is consistent with its flux form. In the discrete spherical-pressure coordinate system ($\lambda$, $\phi$, $p$, and $t$), where $\lambda$ is the longitude and $\phi$ is the latitude, let $i$ be the longitude index, $j$ be the latitude index, $k$ be the pressure index, and $n$ be the time index. The distance between two longitudinal grid points depends on their latitudinal location $j$, that is, $\Delta \lambda = a \cos \phi_i \Delta \lambda$, with $a$ the earth’s radius. The distance between two latitudinal grid points is a constant, that is, $\Delta \phi = a \Delta \phi$.

Following is a pseudocode for the finite-difference approximations for the terms in the first equation, at an arbitrary grid point with indices ($i$, $j$, $k$, $n$):

$$
\left( \frac{\partial \bar{\theta}}{\partial t} \right)_{i,j,k,n} \approx \frac{1}{2} \left( \bar{\theta}_{i,j,k,n+1} - \bar{\theta}_{i,j,k,n-1} \right) \frac{1}{2\Delta t},
$$

$$
\left( \frac{\partial \bar{\theta}}{\partial \phi} \right)_{i,j,k,n} \approx \frac{1}{2} \left( \bar{\theta}_{i,j,k,n+1} - \bar{\theta}_{i,j,k,n-1} \right) \frac{1}{2\Delta \phi}.
$$
For more aspects of computing heat and moisture budget

Therefore, the (up, wp) should be used to make the plot with (uρ, wρ) = (u, Rxw/Rz) or (uρ, wρ) = (Rzw/Rx, w).

REFERENCES


