A Simple Graphical Approach to Penman's Method for Evaporation Estimates[1]

J. Y. WANG AND S. C. WANG[2]

University of Wisconsin, Madison, Wis.

3 November 1961 and 22 January 1962

1. Introduction

Water vaporized from bare soil or water surfaces is called evaporation; from plants it is known as transpiration. The combination of evaporation from the surface of soil, water, or plants is termed evapotranspiration.

Evapotranspiration is affected by such factors as radiation, available soil moisture, vertical gradient of water vapor, temperature of evaporating surfaces (particularly, thermal diffusivity of the soil and air), wind velocity and the effectiveness of turbulent mixing in the lower air layer. Therefore, it is a topic of interest to scientists in many fields (Wang and Barger, 1962).

Three phases of evaporation are commonly studied: i.e., measurement, estimation, and application. Estimation, which is here defined as the graphical determination of evaporation without direct measurement, is the major concern of this paper.

2. Methods and material

Over the years, many relations for the estimation of evaporation or evapotranspiration from the other known factors have been developed and tested (e.g., Dalton, 1802; Thornthwaite, 1948; Penman, 1948; Sanderson, 1950; Albrecht, 1951; Ivanov, 1954; Davis, 1956). Many of these are applicable only to certain areas and seasons.

Penman's method has been widely used because it has some theoretical foundation, and has more general applicability than other methods. Several authors (e.g., Gilbert and van Bavel, 1954; King, 1956; Guerrini, 1957) have reported the successful use of Penman's approach. Therefore, we have chosen this method as the basis for our study.

Penman's equation is a simplified energy budget equation. It is:

$$E_0=\frac{H\Delta+\gamma E_a}{\Delta+\gamma}$$

where E_0 = evaporation from open water surfaces in mm day^{-1}

H = net radiation, expressed in equivalent mm day^{-1} of evaporation

$$H=R(1-A)(0.18+0.55\,n/N) \\ -\sigma T_a^4(0.56-0.092\sqrt{re_s})(0.10+0.90\,n/N)$$

R = mean monthly extraterrestrial radiation, expressed in equivalent mm day^{-1} of evaporation

A = reflection coefficient, i.e., $A=0.05$ for water

n/N = per cent of possible sunshine; it is the ratio of the actual duration of sunshine to the maximum possible sunshine

[1] This paper was prepared under the U. S. Weather Bureau Contract Cwb 10221, and the Industrial Grant from the International Division, Libby, McNeill & Libby, Chicago, Ill., Project Number 1132 with the Departments of Soils and Meteorology, University of Wisconsin.

[2] Assistant Professor of the Departments of Soils and Meteorology, and Research Assistant of the Department of Meteorology.

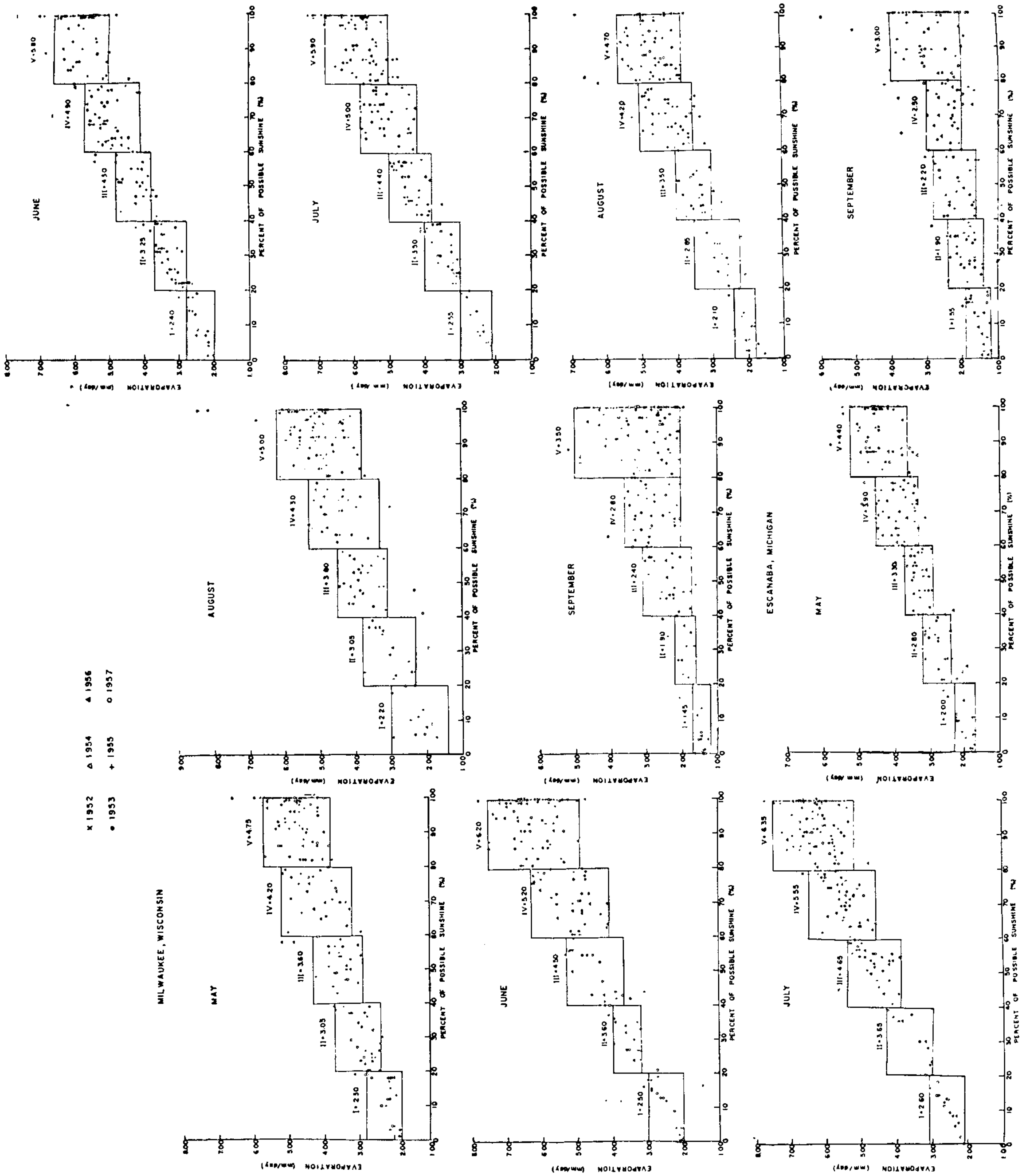

FIG. 1. Scatter diagrams of evaporation versus the per cent of possible sunshine, 1952–1957.

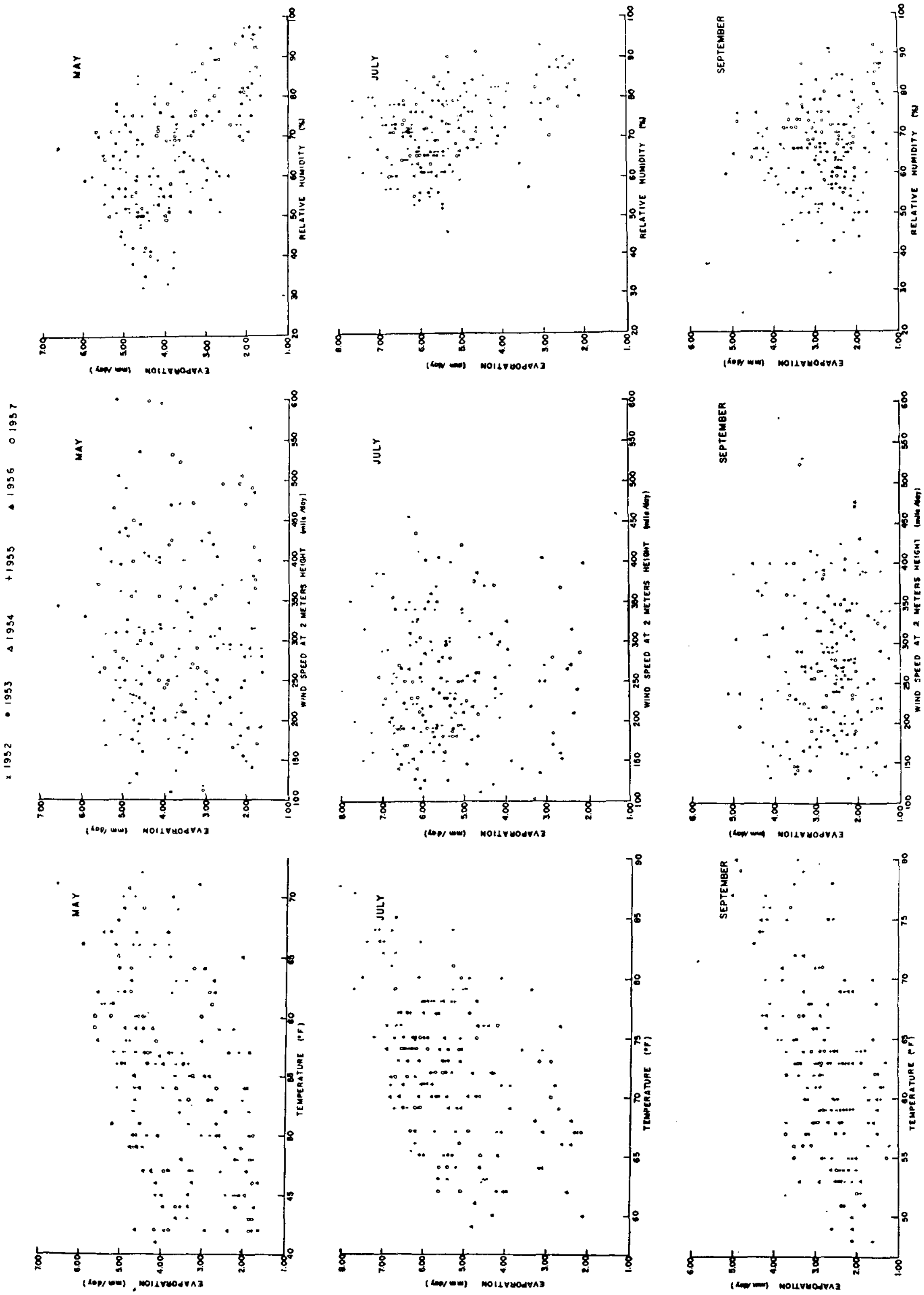

FIG. 2. Scatter diagrams of evaporation versus the various climatic factors at Milwaukee, Wis., 1952–1957.

σ = Stefan-Boltzmann constant = 2.01×10^{-9} mm (deg K)$^{-4}$ day^{-1}
T_a = air temperature in deg K
r = relative humidity in per cent
e_s = saturated vapor pressure of air in mm Hg
Δ = slope of saturated vapor pressure curve at the air temperature
γ = the psychrometer constant ($\gamma=0.27$ is used in this calculation)
E_a = a product of wind function and vapor pressure deficit, i.e.,

$$E_a=f(u)(1-r)e_s \text{ in mm day}^{-1}$$

where

$$f(u)=0.262\left(0.5+1.2\frac{U_2}{100}\right)$$

and where

U_2 = wind speed at 2-m height in mi day^{-1}.

According to Penman's earlier experiments, the wind function term is $f(u)=0.35(1+9.8\times10^{-3}\,U_2)$. The one presently used in this paper is adopted from the experimental results of the Lake Hefner studies (U. S. Geological Survey, 1954).

Penman's equation is too lengthy for hand computation. For this reason, several investigators have established relationships between evaporation, as computed by Penman's formula, and observable meteorological quantities (e.g., Rijkoort, 1954; Kohler *et al.*, 1955; Mistry, 1958; Pruvis, 1961; and Lamoreux, 1962). Their methods are complicated by involving either too many factors or lengthy procedures. We have attempted to fill in the need with a simple, graphical approximation of the Penman estimate. Our methods have been tested by data collected from nine Wisconsin and neighboring stations as listed in Table 1. Most of the climatic data for the present computation can be obtained from the Local Climatological Data of the U. S. Weather Bureau for the respective stations in Table 1. At a few stations where data were not available, these factors were estimated from nearby stations. The Rainbow Reservoir station used the data for the per cent of possible sunshine from Escanaba, and Fennimore station used the data for per cent of possible sunshine and wind from Madison.

3. Result and Discussion

The scatter diagrams of computed evaporation values as related to each of the four factors—per cent of possible sunshine, temperature, wind speed at 2-m height, and relative humidity—have been plotted for these nine stations. Parts of the plotted diagrams are shown in Figs. 1 and 2. From these scatter diagrams, is is easily seen that a trend exists in the scatter diagram of evaporation versus per cent of possible sunshine. For the sake of simplicity, the per cent of possible sunshine is divided into five class intervals: 0–20 per cent, 21–40 per cent, 41–60 per cent, 61–80 per cent and 81–100 per cent. A mean value of evaporation for every class interval can be obtained by taking the central approximate value of the points of each box (Fig. 1).

Fig. 3 shows that a large rate of evaporation occurred in June and July; a small rate of evaporation in September; and obviously, that the higher the per cent of possible sunshine, the larger the evaporation value.

The standard error of the monthly mean evaporation value may be represented by $\sigma/\sqrt{n}$, where σ is the standard deviation and n is the total number of stations. The standard error of all stations for the month of May to the month of September is listed in Table 2.

TABLE 2. Standard error of all stations.

Month	Per cent of possible sunshine 0–20	21–40	41–60	61–80	81–100
May	0.095	0.139	0.116	0.136	0.125
June	0.084	0.086	0.086	0.114	0.123
July	0.079	0.083	0.097	0.123	0.164
August	0.069	0.082	0.101	0.123	0.173
September	0.045	0.038	0.066	0.087	0.109

TABLE 1. Station index.

Station	County	Lat.	Long.	Elev. (ft)	Years of record
Duluth	St. Louis, Minn.	46 50	92 11	1409	1952–57, May–Sept.
Escanaba	Delta, Mich.	45 45	87 03	594	1952–57, May–Sept.
Fennimore	Grant, Wis.	43 00	90 39	1180	1954–57, May–Sept.
Green Bay	Brown, Wis.	44 29	88 08	690	1952–57, May–Sept.
La Crosse	La Crosse, Wis.	43 52	91 15	650	1952–57, May–Sept.
Madison	Dane, Wis.	43 08	89 20	860	1952–57, May–Sept.
Michigamme	Iron, Mich.	45 57	88 12	600	1952–57, May–Sept.
Milwaukee	Milwaukee, Wis.	42 57	87 54	670	1952–57, May–Sept.
Rainbow Reservoir	Oneida, Wis.	45 50	89 33	1600	1952–57, May–Sept.

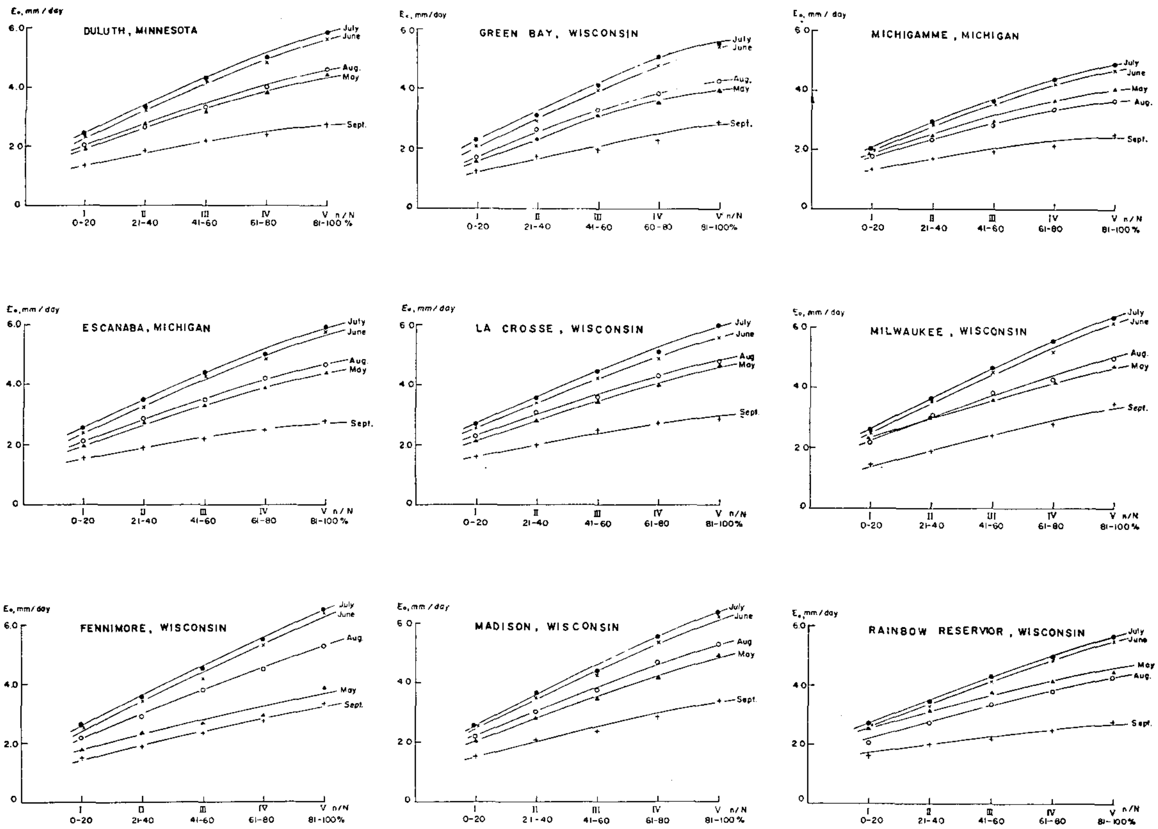

FIG. 3. Classification of evaporation rate by month at nine stations in Wisconsin and vicinities.

Although the daily data for evaporation do not show any correlation with temperature, wind, and relative humidity, the monthly mean evaporation correlates well with temperature at some stations. The longer the duration of sunshine, the higher the temperature would be, which implies that solar intensity is generally a controlling factor of the daily temperature. However, when cold advection occurs, the reverse condition may be noted. The correlation coefficients of monthly evaporation for the four factors for two stations are shown in Fig. 4.

The estimate of monthly evaporation for five stations has been made by using the monthly mean temperature (abscissa), and per cent of possible sunshine (ordinate) as shown in Figs. 5 and 6.[3] According to Tanner and Pelton (1960), the measured evapotranspiration exceeds Penman's estimates. The major error in the evaporation estimates is due to the E_a term in which the wind function term employed does not account for the surface roughness. A different wind function term is needed because both the wind and vapor pressure, and temperature profiles are affected by the surface roughness, so that when a vapor pressure deficit is measured over the rough surface, it must be combined with a wind function appropriate for that rough surface in Penman's equation. However, the main purpose of this paper is to find a simple graphical approach to Penman's evaporation estimates for a free water surface and not for a rough surface.

4. Application of results

A large area coverage in evaporation estimates may be possible either by establishing net radiation stations or by evaluating the per cent of possible sunshine from a combination of cloudiness records and daily synoptic maps. The former was established in 1961 in Wisconsin and neighboring states (Wang, 1961). A collection of data for future years will be necessary for evaporation estimates. The latter method for which the raw data are available should be used in future studies.

[3] The daily variation of sunshine and temperature during the month of July is much greater than that of September in Wisconsin and its vicinity, therefore, each distribution for each 0.2 mm per day of evaporation differs from Fig. 5 to Fig. 6. In Fig. 5, a linear spacing is shown, whereas in Fig. 6, there is wider spacing for 1.9, 2.9, and 3.1 isolines of evaporation. This latter case shows that there is little change of evaporation as related to temperature and sunshine when evaporation value is low. In the month of July, the hours of sunshine and temperature have more effect than in the month of September. In July, the rate of evaporation is over twice as great as that in September.

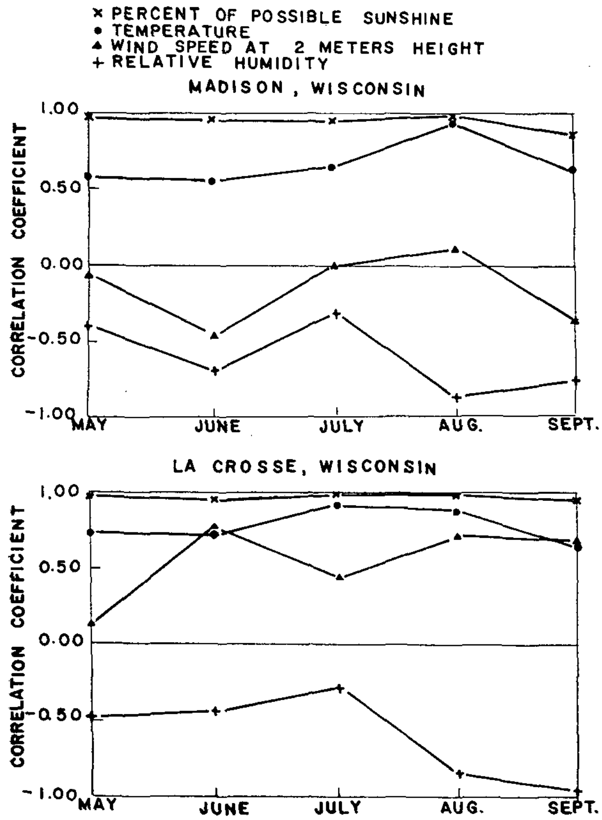

FIG. 4. The correlation coefficient between evaporation and the various climatic factors, 1952–1957.

The estimation of evaporation in the wintertime is necessary and possible. When freezing is continuous, the difference of the water equivalents of snow between two successive intervals of time (e.g., a 5-day period) gives a good estimate of evaporation of snow. Therefore, the estimation of winter evaporation should also be emphasized.

Acknowledgments. The authors wish to express their gratitude for critical reviews and suggestions for improvement to Professors Robert H. Shaw, of the Iowa State University; Champ B. Tanner, Verner E. Suomi, and Heinz H. Lettau, of the University of Wisconsin; and Dr. J. H. Chang, of the Experimental Station of the Hawaiian Sugar Planters' Association. To the U. S. Weather Bureau and the International Division of Libby, McNeill & Libby, the authors are obliged for their financial support. Also, thanks to Mr. S. M. Robinson of the University of Wisconsin for the preparation of the IBM 650 program and Mr. P. D. Mistry for his assistance in the computation.

REFERENCES

Albrecht, F., 1951: Die Methoden zur Bestimmung der Verdunstang der natürlichen Erdoberfläche. (Methods for determination of evaporation from the natural land surface.) *Arch. Meteor., Geophys. u. Bioklimatol.*, Ser. B., 2, 1–38.

Dalton, J., 1802: Experiments and observations made to determine whether the quantity of rain and dew is equal to the quantity of water carried off by rivers and raised by evaporation, with an inquiry into the origin of springs. *Mem. Lit. Phil. Soc.*, 5, 346–372.

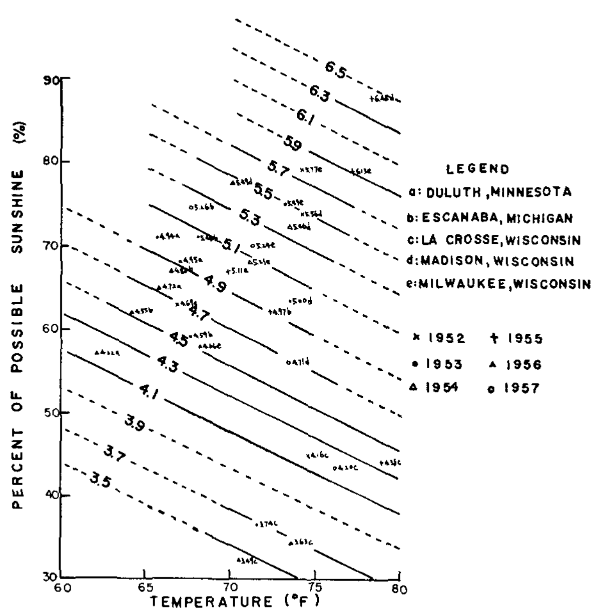

FIG. 5. Nomogram for the estimation of evaporation in Wisconsin and vicinities for the month of July.

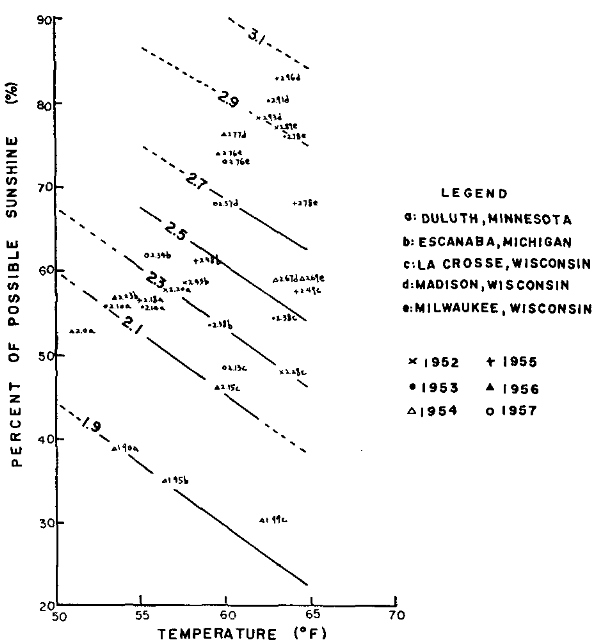

FIG. 6. Nomogram for the estimation of evaporation in Wisconsin and vicinities for the month of September.

Davis, J. R., 1956. Evaporation and evapotranspiration research in the United States. Lafayette, Purdue University, 87 pp.

Gilbert, M. H., and H. M. van Bavel, 1954: A simple field installation for measuring maximum evapotranspiration. *Trans. Amer. geophys. Union*, **35**, 937–943.

Guerrini, V. H., 1957: An analysis of evapotranspiration observations at Valentia Observatory, August 1952–July 1956. Erie Meteor. Serv., Tech. Note 25, 6 pp.

Ivanov, N. N., 1954: Ob opredelenii velichiny ispariaemosti. (The determination of the rate of evaporation.) *Izvest. Vsesosiuz, Geograf. Obshchestvo*, **86**, 189–196.

King, K. M., 1956: Pasture irrigation control according to soil and meteorological measurements. Ph.D. Thesis, Madison, University of Wisconsin, 70 pp.

Kohler, M. A., T. J. Nordenson and W. E. Fox, 1955: Evaporation from pans and lakes. Research Paper No. 38, U. S. Weather Bureau, 21 pp.

Lamoreux, W. W., 1962: Modern evaporation formulae adapted to computer use. *Mon. Wea. Rev.*, **90**, 26–28.

Mistry, P. D., 1958: Simplified approach to Penman estimates of evapotranspiration. M. S. Thesis, Madison, University of Wisconsin, 26 pp.

Penman, H. L., 1948: Natural evaporation from open water, bare soil and grass. *Proc. Roy. Soc.*, Ser. A., **193**, 120–145.

Purvis, J. C., 1961: Graphical solution of the Penman equation for potential evapotranspiration. *Mon. Wea. Rev.*, **89**, 192.

Rijkoort, P. J., 1954: A nomogram for the determination of the pan evaporation adapted from the Penman method. Roy. Netherlands Meteor. Inst. Rept. III, 143 pp.

Sanderson, M., 1950: Measuring potential evapotranspiration at Norman Wells, 1949. *Geograph. Rev.*, **40**, 636–645.

Tanner, C. B., and W. L. Pelton, 1960: Potential evapotranspiration estimates by the approximate energy balance method of Penman. *J. geophys. Res.*, **65**, 33403–33407.

Thornthwaite, C. W., 1948: An approach toward a rational classification of climate. *Geograph. Rev.*, **38**, 55–94.

U. S. Geological Survey, 1954: Water loss investigations, Lake Hefner studies. *Geological Survey Professional Paper*, 1(269).

Wang, J. Y., 1961: A brief description of agrometeorological pilot station network at the University of Wisconsin. Dept. of Meteor., University of Wisconsin, Madison, 12 pp.

Wang, J. Y., and G. L. Barger, 1962: Bibliography of agricultural meteorology. Madison, University of Wisconsin Press, 688 pp.