Radar Attenuation by Wet Ice Spheres[1]

LOUIS J. BATTAN

Institute of Atmospheric Physics, The University of Arizona, Tucson

(Manuscript received 29 September 1970)

ABSTRACT

Calculations have been made of the radar reflectivity and attenuation produced by exponential distributions of dry and wet ice spheres. Appropriate data are presented in the form of tables and graphs. It is shown that attenuation by wet spheres is substantially larger than that by dry spheres. If the ice spheres are coated with a layer of water 0.05 cm thick and extend in diameters to ~2 cm, they would produce two-way attenuations of about 7, 5 and 1 db km^{-1} at wavelengths of 3.21, 5.5 and 10.0 cm, respectively. Procedures for the radar detection of hail must take into account the attenuation caused by the hail itself.

1. Introduction

The attenuation of microwaves by rain has been discussed by various investigators (e.g., Gunn and East, 1954; Wexler and Atlas, 1963), but to the author's knowledge, calculations of the attenuation by showers of wet hail have not been reported. The lack of such information has made it difficult to evaluate various quantitative schemes for detecting hail by means of radar.

In order to obtain an estimate of hail attenuation, calculations were made of the attenuation cross sections of dry and wet ice spheres and of the attenuation coefficients k_h (db km^{-1}) for an assumed hail-size distribution.

2. Hail-size distribution

The hail-size distribution used in this study is one derived empirically by Douglas (1964). It is shown in Fig. 1 and is given by

$$N = N_0 e^{-3.09D}, \qquad (1)$$

where N is the number of hailstones of diameter D per cubic meter in the size interval ΔD taken to be 0.32 cm.

TABLE 1. Definition of hail-size distributions based on the equation $N = 31\, e^{-3.09D}$, where N is the number per cubic meter of ice spheres in the diameter interval ΔD (0.32 cm).

Size distribution	$D_{\max}$ (cm)
I	0.97
II	1.93
III	2.89
IV	4.17

The constant N_0 is equal to 31 cm^{-1} m^{-3}. Eq. (1) is normalized to a liquid water content of 1 gm m^{-3}. For any other liquid water content M the size distribution is given as NM. In the calculations to be described in this paper, it was assumed that the liquid water content was 1 gm m^{-3}.

Having Eq. (1), it still is necessary to specify the maximum ice-sphere diameter. Calculations were made for the four distributions shown in Table 1, where in every case it was assumed that there existed ice spheres within the diameter interval 0–0.32 cm.

In all calculations, the ice particles were taken to be spheres, and they were assumed to be coated with a layer of water whose thickness ranged from zero (dry ice) to over 0.1 cm. It is recognized that it is unlikely to have a water thickness as large as this on a falling, solid ice sphere, but it is possible that a solid ice sphere

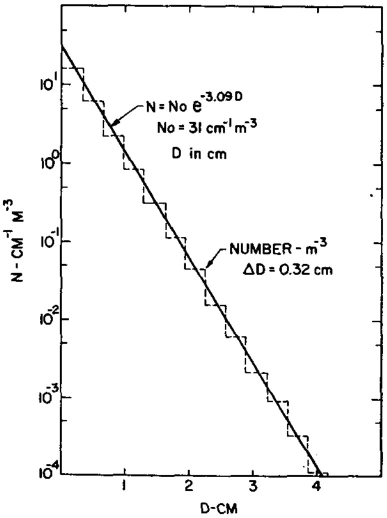

FIG. 1. Hail-size distribution for a liquid-water content of 1 gm m^{-3}.

---

[1] Research supported by the Atmospheric Science Section, National Science Foundation, under Grants GA-1431 and GA-24134.

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC

TABLE 2. Assumed complex indices of refraction in the form $N = m(1 - ik)$. The values are based on data taken from Gunn and East (1954) and are for water and ice at 0C.

Substance	Wavelength (cm)	m	k
Ice	1.87–10	1.78	0.00135
Water	1.87	5.72	0.556
Water	3.21	7.14	0.405
Water	4.67	7.95	0.277
Water	5.5	8.25	0.236
Water	10.0	8.99	0.164

coated with a layer of spongy ice having a water content equivalent to a water thickness equal to 0.1 cm, for example, may attenuate as if it were an ice sphere coated with such a layer of water. A corresponding effect was found to be the case when the backscattering from wet ice spheres was measured (Joss and List, 1963; Atlas *et al.*, 1964).

3. Radar cross sections

By means of the Mie scattering equations and the procedure reported by Herman and Battan (1961 a,b), calculations were made of the principal scattering cross sections of dry and wet ice spheres. A range of wavelengths between 1.87 and 10.0 cm was assumed and complex indices of refraction at 0C were employed as shown in Table 2. A range of sphere diameters up to 8.0 cm and water shell thickness as large as 0.5 cm were considered. Note that in those cases where the assumed water thickness equalled the sphere radius, the particle was taken to be an all-water drop. The results of these calculations have been reported by Battan *et al.* (1970a, b).

Having the scattering cross sections for particular sphere diameters, water-shell thickness and wavelength, and assuming the size distribution given as Eq. (1), it is a straightforward matter to calculate such quantities as the effective radar reflectivity Z_e and the attenuation coefficient k_h.

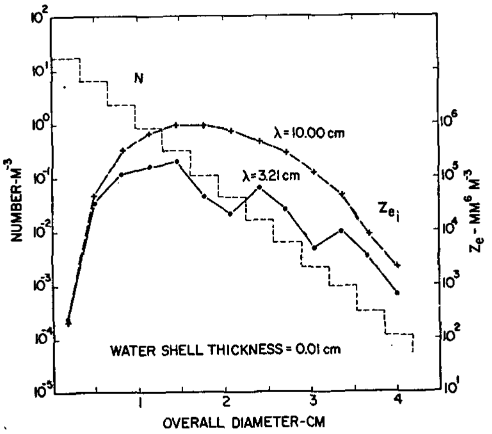

FIG. 2. Contributions to Z_e by wet spheres of different sizes.

TABLE 3. Calculated values of Z_e (mm⁶ m⁻³) for various ice sphere size distributions at $\lambda = 3.21$ cm.

Size distribution	$D_{\max}$ (cm)	Water shell thickness (cm) 0*	0.001	0.01	0.05
I	0.97	1.1×10^5	1.1×10^5	1.5×10^5	8.8×10^5
II	1.93	3.3×10^5	2.7×10^5	5.8×10^5	1.9×10^6
III	2.89	4.1×10^5	3.5×10^5	6.9×10^5	2.1×10^6
IV	4.17	4.5×10^5	3.8×10^5	7.1×10^5	2.1×10^6

* Dry ice sphere.

Atlas and Ludlam (1961) define the quantity Z_e (mm⁶ m⁻³) as

$$Z_e=3.52\times10^3\lambda^4\eta, \tag{2}$$

where the reflectivity η is equal to the summation of the backscattering cross sections over a unit scattering volume. If σ_B is the calculated normalized backscattering cross section,

$$Z_e=3.52\times10^3\lambda^4\sum_0^{D_i} N_i\frac{\pi D_i^2}{4}\sigma_B, \tag{3}$$

where N_i is the number per cubic meter of spheres of overall (ice plus water) radius D_i in the size range ΔD_i. In this calculation D_i is in centimeters and $\Delta D_i=0.32$ cm.

Assuming various size distributions and water thicknesses, calculations were made of Z_e at wavelengths of 1.87, 3.21, 4.67, 5.5 and 10 cm. Some of the results are shown in Tables 3 and 4.

Douglas (1964) also calculated Z_e for dry and wet spheres, using the values of Z_e as a function of D published by Atlas and Ludlam (1961). They had used the backscattering cross sections of ice spheres coated with a layer of water 0.01 cm thick. The values of Z_e in Table 3 are in good agreement with those reported by Douglas. This is not surprising considering the fact that essentially the same size distribution and scattering equations were employed.

The data in Table 3 show that the radar reflectivity of an exponential distribution of ice spheres, having mass equivalents to those in an average hailstone size distribution, is determined principally by the more numerous smaller particles. It is seen that the ice spheres having diameters $\gtrsim 1.9$ cm contribute little to the radar reflectivity.

TABLE 4. Calculated values of Z_e (mm⁶ m⁻³) for the size distribution IV.

Wavelength (cm)	Water shell thickness (cm) 0*	0.001	0.01	0.05
1.87	1.9×10^5	1.3×10^5	1.5×10^5	2.5×10^5
3.21	4.5×10^5	3.8×10^5	7.1×10^5	2.1×10^6
4.67	1.1×10^6	1.0×10^6	1.4×10^6	7.0×10^6
5.5	1.5×10^6	1.5×10^6	1.7×10^6	1.0×10^7
10.0	3.5×10^6	3.5×10^6	4.6×10^6	1.1×10^7

* Dry ice sphere.

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC

TABLE 5. Normalized attenuation cross sections of ice spheres.

Wave-length (cm)	Sphere diameter (cm)														
	0.5			1.0			1.5			2.0			3.0		
	Water-shell thickness (cm)														
	Dry	0.01	0.05	Dry	0.01	0.05	Dry	0.01	0.05	Dry	0.01	0.05	Dry	0.01	0.05
1.87	0.264	1.67	2.27	3.14	3.28	2.89	4.61	3.73	2.67	3.64	3.03	2.38	2.11	2.10	2.34
3.21	0.0308	0.492	1.13	0.472	1.97	2.61	2.01	3.18	2.73	3.26	3.84	2.65	4.61	3.50	2.67
4.67	0.00760	0.175	0.360	0.109	0.639	2.28	0.528	1.79	2.82	1.41	3.19	2.90	3.32	4.26	2.70
5.5	0.00436	0.115	0.188	0.0566	0.379	1.83	0.285	1.00	2.77	0.811	2.32	3.06	3.22	3.67	3.30
10.0	0.000904	0.0343	0.0279	0.00600	0.0830	0.146	0.0266	0.156	0.479	0.0827	0.284	1.37	0.410	0.849	3.67

The relative contribution to the backscattering of wet ice spheres (water shell 0.01 cm thick) by various size spheres is illustrated in Fig. 2. The points along the curve labeled Z_{e_i} show the contribution to the total Z_e by spheres having diameters indicated by each indicated point and having the number concentration given by the N curve. It is clear that at the two wavelengths shown most of the backscattering occurs from spheres having diameters between about 0.7 and 1.7 cm. Observations of hailstone spectra are often cut off below the diameter of 0.5 cm. Fig. 2 shows that the inclusion of small particles has an insignificant effect on the calculated backscattering cross section of an exponentially distributed hailstone sample.

It should be noted that on other grounds Atlas *et al.* (1964) concluded that quantitative observations of hailstorms could best be explained if most of the power being backscattered were by hailstones about 1 cm in diameter.

The data in Tables 3 and 4 show that a very thin layer of water ($\leqslant 0.001$ cm) has little influence on the reflectivity of hail, but the effects of the water become appreciable when the water layer exceeds 0.01 cm. For the exponential size distribution of ice spheres, at all the wavelengths considered, a water shell thickness of 0.05 cm caused Z_e to be substantially higher than would have been the case had the spheres been dry.

4. Attenuation by ice spheres

On the basis of calculations by Herman and Battan (1961b), Atlas and Ludlam (1961) published curves indicating the attenuation by dry hailstones, uniform in diameter and in such concentrations as to represent a water content of 1 gm m^{-3}. The curves indicate maximum one-way attenuation of about 1.3, 1.0 and 0.5 db km^{-1} at wavelengths of 3.3, 4.67 and 10 cm, respectively. The new calculations by Battan *et al.* (1970a, b) show that when hailstones are wet the attenuation coefficients may be very much higher than when the ice spheres are dry. This point is illustrated by the selected data in Table 5. Note for any sphere size that the attenuation coefficient, for a concentration of uniform ice spheres, is directly proportional to σ_T, the *normalized* attenuation cross section of the ice spheres, where

$$\sigma_T = q_T/(\pi D^2/4), \tag{4}$$

and q_T is the attenuation cross section. If there are N_i ice spheres per cubic meter having attenuation cross section q_{T_i} (cm^2) the *one-way* attenuation coefficient k_h (db km^{-1}) is given by

$$k_h = 0.4343 \sum N_i q_{T_i}, \tag{5}$$

where only single scattering is assumed (Gunn and East, 1954). Herman (1965) concluded, in extensive showers composed of large hailstones, that the assumption of single scattering may lead to significant errors, but there is no way from his analysis of estimating the effects of multiple scattering in the case of ice spheres having the size distribution given by Eq. (1).

When all the ice spheres in a unit volume are the same size, $k_h = 0.4343\, N q_T$, where N is the number per cubic meter and q_T is in square centimeters.

Table 5 shows that even a coating of water 0.01 cm thick, a value considered to be reasonable by Atlas *et al.* (1960), has a profound effect on the attenuation caused by ice spheres. The increase of attenuation as the spheres are wetted is particularly striking at the wavelengths most often used for hail detection, i.e., between 3 and 10 cm and at the smaller ice sphere diameters 0.5–1.5 cm.

TABLE 6. One-way attenuation coefficients (db km^{-1}) produced by uniform ice spheres constituting an ice mass of 1 gm m^{-3}. Attenuation would be proportional to the ice mass concentration.

Wavelength (cm)	Sphere diameter (cm)											
	0.5			1.0			1.5			2.0		
	Water-shell thickness (cm)											
	Dry	0.01	0.05	Dry	0.01	0.05	Dry	0.01	0.05	Dry	0.01	0.05
3.21	0.044	0.71	1.6	0.34	1.4	1.9	1.0	1.5	1.3	1.3	1.4	1.0
5.5	0.0063	0.17	0.27	0.041	0.27	1.32	0.14	0.48	1.3	0.29	0.84	1.1
10.0	0.0013	0.050	0.040	0.0043	0.060	0.11	0.013	0.075	0.23	0.030	0.10	0.50

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC

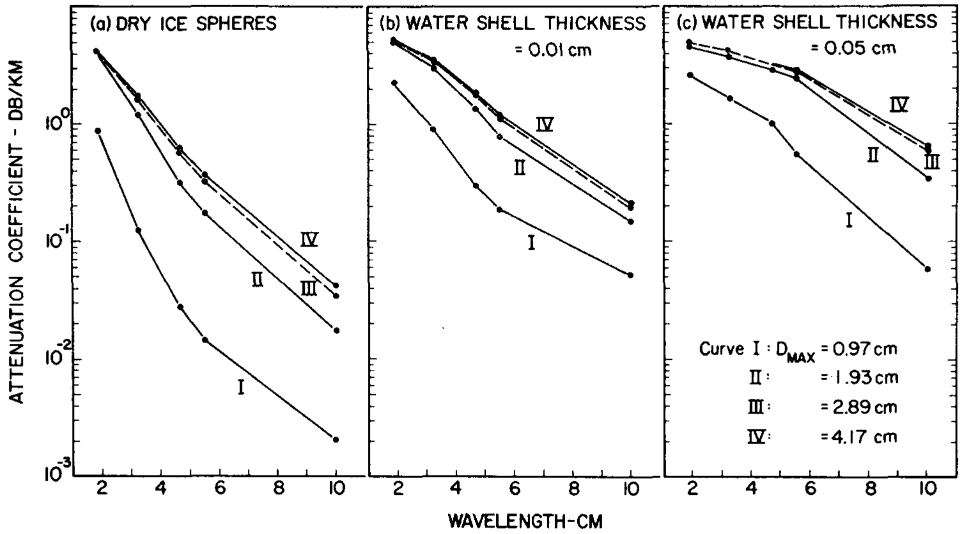

FIG. 3. One-way attenuation coefficient of exponentially distributed dry and wet ice spheres as a function of wavelength.

The attenuation produced by a wet 0.5-cm diameter ice sphere is more than 10 times greater at 3.21 cm and almost 40 times larger at 10 cm than the attenuation of a dry sphere of the same size.

At large sphere diameters, greater than about 1.5 cm, at wavelengths between 1.87 and 4.67 cm, the normalized attenuation cross section varies only little with either wavelength or sphere diameter. This was shown earlier by Battan *et al.* (1970b). It was also shown that very large ($D \geqslant 3$ cm), very wet (effective water thickness $\geqslant 0.05$ cm) ice spheres have normalized attenuation cross sections between 2 and 4 and do not change much over the wavelengths 1.87–10 cm.

The importance of the quantities in Table 5 become more evident, if as was done by Atlas and Ludlam (1961), calculations are made of the attenuation caused by uniform ice spheres having concentrations such that the sum of the masses of all spheres in a cubic meter would amount to 1 gm m^{-3}. Such values can be obtained from the expression

$$k_h = 0.724\frac{\sigma_T}{D}, \tag{6}$$

which follows directly from (5) by writing it as $k_h = 0.4343\ Nq_T$, and substituting for the quantity N obtained from the expression $M = (\pi D^3/6)\rho N$. As already noted M is taken to be 1 gm m^{-3} and the ice density ρ is assumed to be 0.9 gm cm^{-3}. Table 6 gives some representative values of the one-way attenuation coefficients. At a wavelength of 10 cm, the attenuation coefficients of wet spheres may be appreciable, particularly for sphere diameters >1 cm. At shorter wavelengths the attenuation by ice spheres, particularly wet ones, can be quite important. At $\lambda = 3.21$ cm, the two-way attenuation of wet spheres 1 cm in diameter is ~3 db km^{-1}, substantially larger than the attenuation which would have been caused by dry spheres.

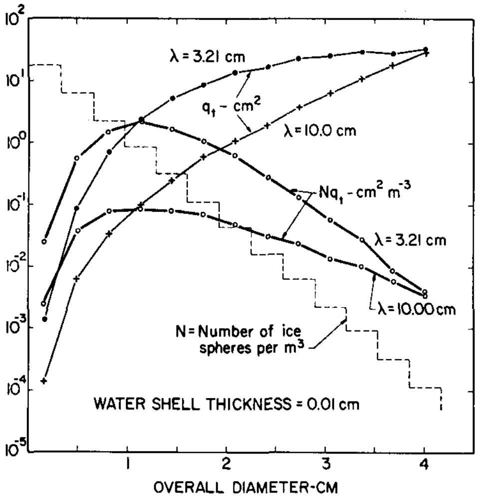

FIG. 4. Relative contributions to the one-way attenuation by wet spheres of different sizes. Attenuation is given by $0.4343\,\Sigma\, Nq_t$.

The one-way attenuation by a *spherical* hail-size distribution given by Eq. (1) is shown in Figs. 3 a–c. In each diagram k_h is plotted as a function of wavelength for hail spectra having the maximum diameters shown on the diagram. Fig. 3a is for dry ice spheres, while 3b and 3c are for ice spheres having water shells 0.01 and 0.05 cm thick, respectively. As would be expected, the attenuation decreases markedly as wavelength increases. Also, the diagrams show that wet ice spheres generally produce substantially more attenuation than dry spheres.

The relative contribution of various wet sphere sizes to the attenuation at 3.21 and 10.0 cm is illustrated in Fig. 4, which presents, as a function of sphere diameter, the number density N, the attenuation cross section q_t, and the product Nq_t. This latter quantity is directly proportional to the attenuation coefficient. It is clear

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC

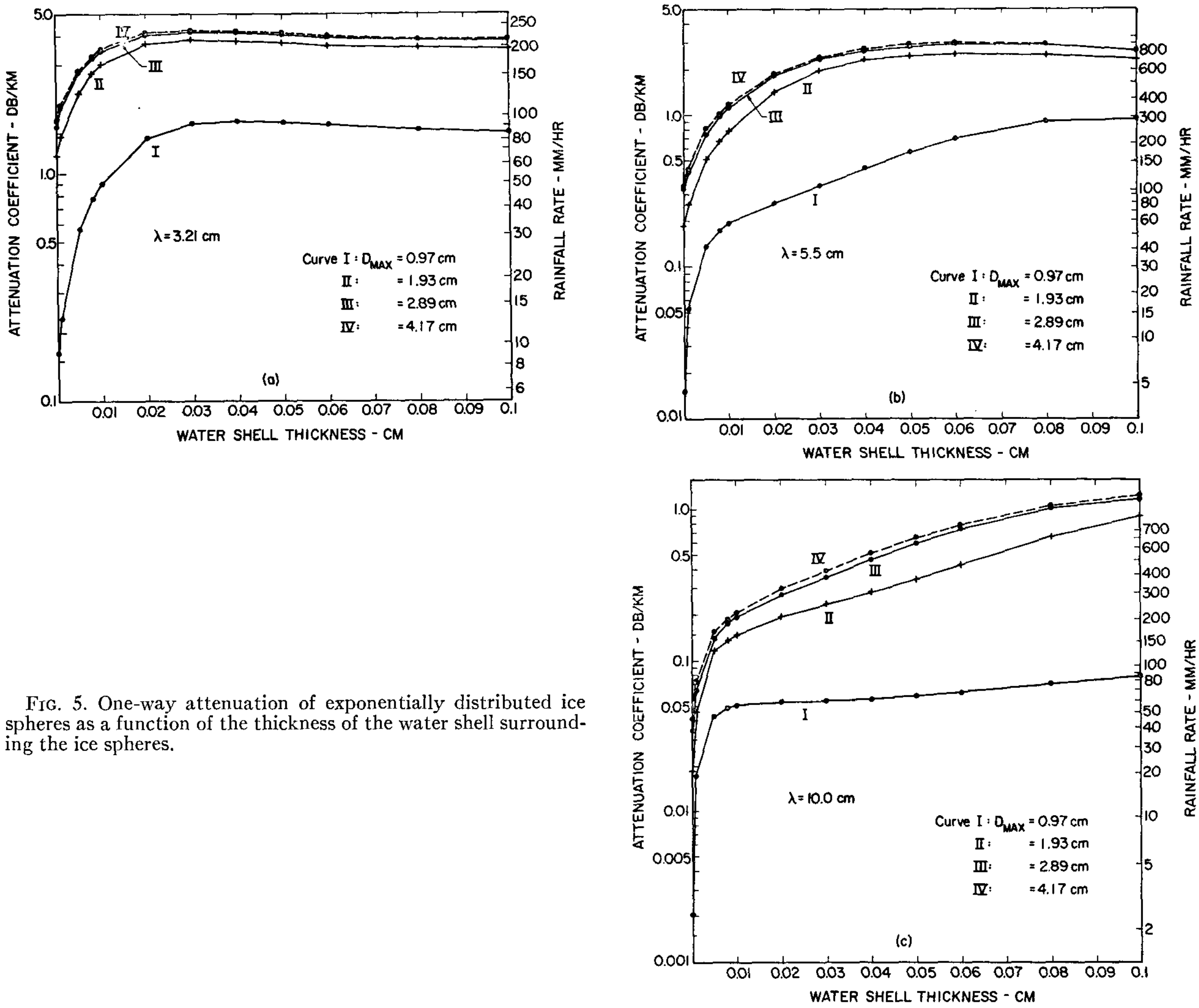

Fig. 5. One-way attenuation of exponentially distributed ice spheres as a function of the thickness of the water shell surrounding the ice spheres.

that most of the attenuation by an exponential distribution of ice spheres is produced by those having diameters between about 0.5 and 2 cm. Very small and very large ice spheres produce relatively little attenuation, the former because of their small attenuation cross sections and the latter because of their low concentrations. Fig. 3 shows that at all the wavelengths considered the larger spheres made very small contributions to the attenuation.

These results indicate that hailstorms may produce appreciable attenuation, particularly of X- and C-band radar signals when there are hailstones as large as 2 cm in diameter and the hailstones are wet, that is, they are composed of ice spheres coated with a layer of water or of spongy ice. Fig. 3b shows that if spherical hailstones as large as 1.93 cm in diameter were coated with 0.01 cm of water, the two-way attenuation would amount to 6 db km^{-1}, a very large quantity indeed.

Table 7. One-way attenuation (db km^{-1}) of various distributions of spherical hail where $N = 31e^{-3.09D}$ and $D_{\max}$ is as shown. The distributions are normalized to a liquid water content of 1 gm m^{-3}.

		Wavelength (cm)											
		3.21				5.5				10.0			
		Water-shell thickness (cm)											
Size distribution	$D_{\max}$ (cm)	Dry	0.01	0.05	0.10	Dry	0.01	0.05	0.10	Dry	0.01	0.05	0.10
I	0.97	0.12	0.91	1.68	1.50	0.015	0.19	0.56	0.94	0.0020	0.051	0.058	0.080
II	1.93	1.21	3.01	3.72	3.49	0.18	0.79	2.48	2.30	0.017	0.15	0.34	0.89
III	2.89	1.66	3.46	4.03	3.79	0.33	1.12	2.82	2.60	0.034	0.19	0.60	1.18

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC

The change of attenuation as a function of water shell thickness at wavelengths of 3.21, 5.5 and 10 cm is shown in Figs. 5 a-c. Some quantities from the illustrations are given in Table 7. It is evident that the development of a thin layer of water greatly increases the attenuation caused by exponentially distributed spherical hailstones. At a wavelength of 3.21 cm, attenuation increases markedly as the water film thickness increases to a depth of about 0.02–0.03 cm. At greater water thicknesses, attenuation remains essentially constant. When maximum hailstone diameters exceed 2 cm in diameter the two-way attenuation amounts to $\sim$8 db km^{-1}.

At a wavelength of 5.5 cm, maximum two-way attenuations are $\sim$6 db km^{-1} and would be experienced when spherical hailstones are exponentially distributed, have maximum diameters $\geqslant$3 cm, and have a water coating $\sim$0.05 cm thick.

At a wavelength of 10 cm, an exponential distribution of wet, spherical hailstones extending to diameters of $\sim$3 cm may give two-way attenuations amounting to $\sim$2 db km^{-1}.

The values of attenuation given in Figs. 3 and 5 and Table 7 are for a spherical hailstone distribution representing a liquid water content of 1 gm m^{-3}. At other values of M, in grams per cubic meter, all these figures would have to be multiplied by a factor equal to M.

When considering the backscattering (Z_e) or attenuation (k_h) of microwaves by wet ice spheres there are many degrees of freedom. One must consider the wavelength, the size distribution, and the thickness of the water layer surrounding the ice sphere. In this analysis certain conditions were selected to give what are considered to be significant, representative values of backscattering and attenuation. The calculations of scattering cross sections of Battan *et al.* (1970b) allow the ready determination of Z_e or k_h for other sets of conditions.

Acknowledgments. Sincere thanks are extended to S. R. Browning for his assistance in carrying out the calculations discussed in this report.

REFERENCES

Atlas, D., and F. H. Ludlam, 1961: Multi-wavelength radar reflectivity of hailstorms. *Quart. J. Roy. Meteor. Soc.*, **87**, 523–534.

——, K. R. Hardy and J. Joss, 1964: Radar reflectivities of storms containing spongy hail. *J. Geophys. Res.*, **69**, 1955–1961.

——, W. G. Harper, F. H. Ludlam and W. C. Macklin, 1960: Radar scatter by large hail. *Quart. J. Roy Meteor. Soc.*, **86**, 468–482.

Battan, L. J., S. R. Browning and B. M. Herman, 1970a: Tables of the radar cross sections of dry and wet ice spheres. Tech. Rept. 21, Inst. Atmos. Phys., The University of Arizona, 11 pp.

——, ——, and ——, 1970b: Attenuation of microwaves by wet ice spheres. *J. Appl. Meteor.*, **9**, 832–834.

Douglas, R. H., 1964: Hail size distribution. *Proc. Eleventh Wea. Radar Conf.*, Boulder, Colo., Amer. Meteor. Soc., 146–149.

Gunn, K. L. S., and T. W. R. East, 1954: The microwave properties of precipitation particles. *Quart. J. Roy. Meteor. Soc.*, **80**, 522–545.

Herman, B. M., 1965: Multiple scatter effects on the radar return from large hail. *J. Geophys. Res.*, **70**, 1215–1225.

——, and L. J. Battan, 1961a: Calculations of Mie backscattering from melting ice spheres. *J. Meteor.*, **18**, 468–478.

——, and ——, 1961b: Calculations of the total attenuation and angular scatter of ice spheres. *Proc. Ninth Wea. Radar Conf.*, Kansas City, Mo., Amer. Meteor. Soc., 259–265.

Joss, J., and R. List, 1963: Backscattering cross sections of mixture of ice and water. *Z. Angew. Math. Phys.*, **14**, 376–380.

Wexler, R., and D. Atlas, 1963: Radar reflectivity and attenuation of rain. *J. App.. Meteor.*, **2**, 276–280.

Unauthenticated | Downloaded 03/31/23 05:57 AM UTC