The Response of Constant-Density Balloons to Sinusoidal Variations of Vertical Wind Speeds

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Constant-density balloons have been used for years as tracers of air motion in the atmosphere. A balloon is filled with a lighter-than-air gas until the skin is rigid and a superpressure is attained that will cause the balloon to float at a desired height in the atmosphere. When the balloon is displaced from this equilibrium height by a vertical wind gust, buoyant forces will attempt to return it to the equilibrium height. Angell *et al.* (1968) have flown balloons in the boundary layer and detected vertical air motions having periods on the order of 30 min. Constant-level balloons were used by Booker and Cooper (1965) and Vergeiner and Lilly (1970) to study atmospheric waves that form to the lee of mountain ranges. Vertical wave motions having magnitudes of several meters per second and periods greater than a few minutes were discovered. Booker and Cooper used a digital computer to simulate the vertical motion of a balloon responding to sinusoidal vertical air motions. The response to only one such sine wave was published. For a vertical sine wave of magnitude 1000 ft and period 15 min, the amplitude of the balloon trajectory was 15% low and the balloon led the air motion by $\sim 20°$. Vergeiner and Lilly also analyzed the balloon's response by solving an equation of motion for the balloon. In these two analyses, the dynamic buoyancy and acceleration drag forces that appear on the right-hand side of Eq. (1) below were neglected.

Angell *et al.* and others have recently begun to analyze the vertical fluctuations of balloon position, in order to derive the statistics of air motions from balloon motions. However, the balloon does not react perfectly to vertical air motions. The equation of motion for the vertical speed of several types of balloons was solved for vertical sinusoidal wind motions having a range of magnitudes between 0.1 and 2.0 m sec^{-1}, and a range of periods between 200 and 2600 sec. The response is analyzed in terms of the relative magnitude

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of the balloon and air speeds, and the phase lead angle between the sinusoidal balloon and air motions; these are shown here to be functions of a single dimensionless number for periods of motion that are greater than the Brunt-Väisälä period.

The vertical equation of motion for the balloon has the form

$$M_b\frac{\partial w_b}{\partial t}=M_a\frac{\partial w_a}{\partial t}+\frac{1}{2}M_a\left(\frac{\partial w_a}{\partial t}-\frac{\partial w_b}{\partial t}\right)-M_b g\left(\frac{\rho_b-\rho_a}{\rho_b}\right)-\rho_a A\frac{C_D}{2}(w_b-w_a)|w_b-w_a|. \qquad (1)$$

The symbols are defined as follows:

M_b(gm)	mass of the balloon system (the balloon, the gas inside it, string and other payload)
M_a(gm)	mass of the air displaced by the balloon system
w_b(cm sec^{-1})	vertical speed of the balloon
w_a(cm sec^{-1})	vertical speed of the air
g(cm sec^{-2})	acceleration due to gravity
ρ_b(gm cm^{-3})	density of balloon system
ρ_a(gm cm^{-3})	density of air
A(cm^2)	cross-sectional area presented to the vertical air stream by the balloon
C_D	drag coefficient

The term on the left-hand side of Eq. (1) is the net force on the balloon. The terms on the right-hand side are the dynamic buoyancy force due to the acceleration of the air, the acceleration drag due to the difference between the air's and the balloon's accelerations, the static buoyancy force, and the drag force. Prandtl (1952) describes the nature of the dynamic buoyancy and acceleration drag forces.

Eq. (1) is simplified by making the Boussinesq assumption that fluctuations in density are unimportant except in terms where they occur in conjunction with the gravitational acceleration. The hydrostatic stability s is defined by

$$s=-g\frac{\partial}{\partial z}(\rho_a/\rho_b). \qquad (2)$$

The ratio $s^{\frac{1}{2}}/(2\pi)$ is the Brunt-Väisälä or "bouncing" frequency, which is the frequency with which the balloon oscillates when displaced vertically from its equilibrium level in a stable atmosphere. It is assumed that the ratio ρ_a/ρ_b is equal to unity at the equilibrium height ($z=0$) of the balloon. Furthermore, if the radius of the balloon is R, it is assumed that the ratio $\rho_a A/M_b=3/(4R)$, which is strictly true for a spherical balloon whose density equals the air density. With these assumptions and dividing by the mass M_b, Eq. (1) becomes

$$\frac{3}{2}\frac{\partial}{\partial t}(w_b-w_a)=-g(1-e^{-sz/g})-\frac{3C_D}{8R}(w_b-w_a)|w_b-w_a|. \qquad (3)$$

This equation was solved using a digital computer for several values of the parameters s, C_D, R, assuming that the vertical air speed w_a varied sinusoidally as

$$w_a=W_a\sin(2\pi t/T), \qquad (4)$$

with magnitude W_a and period T. All combinations of the following parameters were used:

Vertical air speed magnitude W_a	0.1, 0.2, 0.3, ..., 2.0 m sec^{-1}
Vertical air speed period T	200, 400, 600, ..., 2600 sec
Stability parameter s	0.57×10^{-3} sec^{-2} and 1.15×10^{-3} sec^{-2} (isothermal)
Drag coefficient C_D	0.8
Radius of balloon R	0.13 m and 0.65 m

In addition, a few results were obtained for drag coefficients C_D equal to 0.45 and 1.20. After a few time periods T had elapsed during each run, when the vertical balloon speed w_b was independent of its initial value, the phase angle and magnitude response were estimated. The phase angle ϕ is defined as the angle by which the balloon motion leads the air motion, and the magnitude response is the ratio W_b/W_a of the maximum vertical balloon speed to the amplitude of the vertical air motion.

For air speed periods $T\gtrsim500$ sec, the numerical results showed that the phase lead angle ϕ was a monotonically increasing function only of the dimensionless number $(RTs)/(C_D W_a)$, as plotted in Fig. 1.

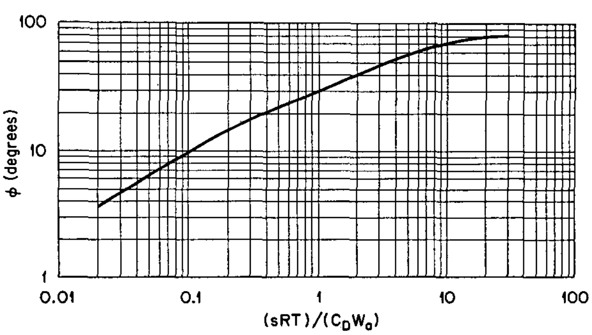

FIG. 1. The angle by which the balloon motion leads the air motion as a function of the dimensionless number $(sRT)/(C_D W_a)$, where s is a stability parameter, R the balloon radius, C_D the balloon drag coefficient, and W_a and T the speed amplitude and period of the vertical air motions. This relation is valid for periods $T\gtrsim500$ sec, i.e., about two times the Brunt-Väisälä period.

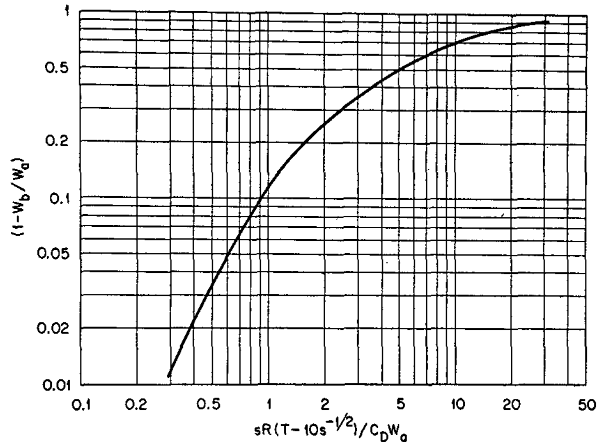

FIG. 2. The decrease in amplitude response $(1-W_b/W_a)$ as a function of the dimensionless number $(sR/C_DW_a)\,(T-10s^{-\frac{1}{2}})$. This relation is valid for $T>10s^{-\frac{1}{2}}$, i.e., $T\gtrsim 500$ sec for the stabilities considered here.

This result can be derived analytically from Eqs. (2) and (3) by making the assumptions that the balloon speed also varies sinusoidally, leads the balloon motion by the angle ϕ, and that the magnitude response is unity. The resulting quadratic equation in $\cos\phi$ can easily be solved, yielding

$$\cos\phi=-\left(0.150\frac{RTs}{C_DW_a}-\frac{8.85R}{C_DW_aT}\right)+\left[\left(0.150\frac{RTs}{C_DW_a}-\frac{8.85R}{C_DW_aT}\right)^2+1-\frac{17.70R}{C_DW_aT}\right]^{\frac{1}{2}}. \quad (5)$$

The magnitude of the first term, which contains the dimensionless ratio used in Fig. (1), is more than five times the magnitude of the second term for periods $T>17.2s^{-\frac{1}{2}}$, or about 500 sec for the stabilities considered here. For periods $T\lesssim 500$ sec, the period of the balloon motion is close to the Brunt-Väisälä period $2\pi s^{-\frac{1}{2}}$, and resonance with this "natural" period occurs.

Similarly, the magnitude response W_b/W_a calculated from the numerical experiments is a function of a single dimensionless number $(Rs/C_DW_a)(T-10\,s^{-\frac{1}{2}})$ for periods $T>10\,s^{-\frac{1}{2}}$. This relationship is plotted in Fig. 2. The time $10\,s^{-\frac{1}{2}}$, which is simply an empirical number that best fits the data, is close to the Brunt-Väisälä period. For periods $T<10\,s^{-\frac{1}{2}}$, the magnitude response is often greater than unity, due to resonance with the natural Brunt-Väisälä oscillations.

Using Figs. (1) and (2) it is possible to estimate the phase lead angle ϕ between balloon and air motions and the magnitude response W_b/W_a, based on a knowledge of the balloon's radius R and drag coefficient C_D, the hydrostatic stability s, and the amplitude W_b and period T of the vertical air motion. It is important to remember to account for variations of balloon density with height when calculating the stability s.

As an example of the use of these figures, consider the typical "tetroon" used by the Air Resources Laboratories of NOAA:

$$R=0.65 \text{ m},$$
$$C_D=0.8.$$

Assume that the stability $s=0.57\times10^{-3}$ sec^{-2}, corresponding to an isothermal atmosphere in which the vertical derivative of the density of the balloon is one-half of the vertical derivative of the density of the air. For W_a and T equal to 0.5 m sec^{-1} and 1000 sec, respectively, the phase lead angle ϕ is $\sim30°$ and the magnitude response $W_b/W_a\sim0.95$. For W_a and T equal to 0.25 m sec^{-1} and 2500 sec, respectively, $\phi\approx65°$ and $W_b/W_a\approx0.50$. Sinusoidal air motions with periods and magnitudes nearly equal to these values were reported by Angell *et al.*, Vergeiner and Lilly, and Booker and Cooper. Clearly these observed balloon motions need to be interpreted with great care. From Figs. (1) and (2) it is also evident that better response is obtained from small balloons with large drag coefficients.

Energy spectra calculated from observed vertical fluctuations of a balloon do not necessarily correspond to the energy spectra of vertical air fluctuations. The error is likely to be greater for smaller frequencies and for smaller total turbulent energies. Furthermore, phase lead angle differences strongly affect cross spectra between the vertical speed component and the horizontal speed component or temperature. The horizontal balloon motions at these speeds and periods are in phase with the air motions while the vertical balloon motions are out of phase by the angles given in Fig. 1. As a result, cross spectra calculated from observations of balloons are likely to differ quite markedly from true cross spectra.

Nothing has been said about the reaction of balloons to eddies whose diameters are on the order of or less than the diameter of the balloon. In that case the air on one side of the balloon has a different velocity from the air on the other side and additional terms are necessary in the equation of motion for the balloon. Also, sinusoidal motions with periods of the same order of magnitude as the Brunt-Väisälä period are not discussed here. Nevertheless, the above analysis applies to much of the range of vertical eddy sizes that balloons are likely to encounter in the boundary layer at heights above $\sim$100 m.

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