On the Relationship Between Horizontal Moisture Convergence and Convective Cloud Formation

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ABSTRACT

A technique suggested by H. L. Kuo for parameterizing the net effect of the latent heat released by cumulus convection on large-scale flow is evaluated to determine its applicability to regions of convection in middle latitude severe storm situations. The ratio of the vertically integrated horizontal moisture divergence to the moisture required for the development of a model cloud multiplied by a given arbitrary time interval is interpreted as the amount of model cloud produced in that time interval. The model cloud is characterized by pseudo-adiabatic temperature and moisture profiles above the lifting condensation level. The technique uses computer methods and regularly available rawinsonde data.

Results suggest that well-defined axes of horizontal moisture convergence generally accompany development of strong cumulus convection and that the use of a pseudo-adiabatic cloud to model the moisture requirements generally gives reasonable predicted cloud amounts. The patterns of predicted cloud formation often correlate well with observed radar patterns of strong cumulus convection. Generally, the correlation, which is based on radar data at 3-hr intervals, is good at the initial rawinsonde observation time, a little better 3 hr later, and decreases at greater time intervals between rawinsonde and radar observation time.

1. Introduction

Kuo (1965) developed a model cloud involving a theoretical parameterization of convective scale motions that permits an estimate of the percentage area covered by convective cloud elements. This study applies the scheme, which was developed for tropical convection, to middle latitude severe storm situations and evaluates its use in determining regions of convective activity in these cases. Krishnamurti (1968) similarly applied the theory to the study of an easterly wave positioned below a cold low in the tropics.

2. Theory

According to Kuo the moisture \( Q \) that must be added to a vertical column of air of unit horizontal area possessing a given temperature and moisture profile to produce the model cloud whose temperature and moisture profiles are pseudo-adiabatic can be considered to be composed of two parts: 1) \( Q_1 \), the amount required to warm the column from the environmental temperature to that of the appropriate pseudo-adiabat by the release of latent heat, and 2) \( Q_2 \), the amount required to saturate the column at the pseudo-adiabatic temperature.

The first part is obtained from the first law of thermodynamics. We consider a constant-pressure process in which the amount of water vapor that must be condensed per gram of dry air in order to raise the temperature from \( T_e \), the environmental temperature, to \( T_s \), the saturation temperature at the same level, is

\[
\frac{dq}{T_e - T_s} = \frac{c_p}{L} (T_s - T),
\]

where \( q \) is the mixing ratio, \( c_p \) the specific heat at constant pressure, and \( L \) the latent heat of condensation.

The amount of water vapor per unit area in the column from the bottom \( P_B \) to the top \( P_T \) is

\[
Q_1 = -\frac{c_p}{gL} \int_{P_B}^{P_T} (T_s - T) dP,
\]

where \( g \) is the acceleration of gravity and \( P \) pressure.

The evaluation of (2) consists of determining the area on a thermodynamic diagram between the environmental temperature profile and the pseudo-adiabat of the cloud profile in the layer from \( P_B \) to \( P_T \), and is thus a representation of the stability of the sounding.

The water vapor needed to saturate the column is

\[
Q_2 = -\frac{1}{g} \int_{P_B}^{P_T} (q_s - q) dP,
\]

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1 An abridged version of Tech. Memo. ERL.TM-NSSL-45, National Severe Storms Laboratory.
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where $q_s$ is the saturation mixing ratio of the pseudo-adiabat of the model cloud.

Using Phillips' (1957) expression for divergence on a polar stereographic map projection, the horizontal moisture divergence on this projection is

$$\nabla_H \cdot (\rho q V) = \sigma \left[ \frac{\partial (\rho q u/\sigma)}{\partial x} + \frac{\partial (\rho q v/\sigma)}{\partial y} \right].$$  \hspace{1cm} (4)

In (4), $\rho$ is the density of air, $V$ the horizontal wind velocity, $u$ and $v$ are the components of $V$ along the $x$ and $y$ axes of a rectangular grid on the image plane, and $\sigma$ the image scale factor.

The vertically integrated horizontal moisture divergence in a column bounded below by the surface of the earth at $Z_0$ and above by a level at $Z_1$ at a constant height above sea level is

$$\int_{Z_0}^{Z_1} \nabla_H \cdot (\rho q V) dZ = \int_{Z_0}^{Z_1} \rho q V dZ \cdot \nabla_H Z_1 - \nabla_H Z_0 \cdot (\rho q V)_{Z_0}. \hspace{1cm} (5)$$

The second term on the right is zero because $\nabla_H Z_1 = 0$. The third term on the right is neglected in this study.

Substitution for $\rho$ in (4) from the equation of state for moist air and then substitution into (5) yields

$$\int_{Z_0}^{Z_1} \nabla_H \cdot (\rho q V) dZ = $$

$$= \sigma \frac{\partial}{\partial x} \int_{Z_0}^{Z_1} \frac{\rho q u}{RT \sigma} dZ + \sigma \frac{\partial}{\partial y} \int_{Z_0}^{Z_1} \frac{\rho q v}{RT \sigma} dZ_1, \hspace{1cm} (6)$$

where $R$ is the gas constant for dry air and $T^*$ the virtual temperature.

The horizontal moisture convergence, multiplied by the number of hours $\Delta t$ over which it occurs and divided by the moisture requirement, can be interpreted as the fraction of convective cloud cover produced in the column during time $\Delta t$. For the processes being considered here, $\Delta t$ is of the order of a few hours. Calculations are made for a unit time interval of 1 hr for ease in considering time increments in multiples of 1 hr.

![Fig. 1. The area of analysis and the grid. United States rawinsonde stations are shown by black dots.](image)

### 3. Data

Nine periods with severe weather occurrences were chosen for study as shown in Table 1. These periods range in length from 1–4 days (2–8 rawinsonde observation times) and are represented by 30 observation times. The National Meteorological Center provided the 1967 data, and the National Severe Storms Forecast Center (NSSFC) provided the 1968 data. The radiosonde data were checked for consistency using the operational checking program at NSSFC (Inman, 1968).

### 4. Computations

The pseudo-adiabat that defines the model cloud is characterized by the wet bulb potential temperature $\theta_w$ of a parcel of air whose thermodynamic properties are representative of the lowest 100-mb surface layer. The average temperature and mixing ratio of this layer for evening soundings [1800 (all times CST)] were used as parcel properties, but allowance for daytime surface heating was made in the morning soundings (0600) by adding 2C to the observed temperature 100 mb above the surface and then assuming a dry adiabatic lapse rate from that point to the surface. The wet bulb potential temperature $\theta_w$ was calculated using an empirical relation reported by Prosser and Foster (1966).\(^8\)

A check was made for "overrunning," a situation with a layer of relatively cool, dry air at the surface and relatively warm, moist air above. If the lift occurs primarily in the warm air, calculations based on conditions in the lower layer do not represent the dominant physical processes. This situation was identified by computing wet bulb temperatures at each reported

\(^8\) Calculations discussed in the first two paragraphs of this section are described by Prosser and Foster.

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**Table 1. Periods of severe weather used in study.**

<table>
<thead>
<tr>
<th>First rawinsonde observation</th>
<th>Number of consecutive rawinsonde observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0600 CST 14 May 1967</td>
<td>2</td>
</tr>
<tr>
<td>0600 CST 30 May 1967</td>
<td>2</td>
</tr>
<tr>
<td>0600 CST 9 June 1967</td>
<td>4</td>
</tr>
<tr>
<td>0600 CST 3 April 1968</td>
<td>4</td>
</tr>
<tr>
<td>0600 CST 14 April 1968</td>
<td>2</td>
</tr>
<tr>
<td>0600 CST 16 April 1968</td>
<td>2</td>
</tr>
<tr>
<td>0600 CST 6 May 1968</td>
<td>4</td>
</tr>
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<td>0600 CST 14 May 1968</td>
<td>8</td>
</tr>
<tr>
<td>0600 CST 10 June 1968</td>
<td>2</td>
</tr>
</tbody>
</table>
pressure level between the top of the surface layer and 700 mb and comparing them with the temperatures of the parcel lifted along the pseudo-adiabat through $\theta_e$. If the wet bulb temperature exceeded the parcel temperature, the level at which this difference was the largest was taken to be the new starting level of the parcel.

The temperature at the lifting condensation level (LCL) was determined with an approximate equation not requiring an iterative technique (Inman, 1969). The pressure at this level was obtained from the definition of potential temperature.

The total moisture requirement $Q$, the distance that the parcel must be lifted to reach saturation (called "lifted height" here), and the two integrals in the horizontal moisture divergence of (6) were obtained for each rawinsonde report; using an objective analysis scheme, the same values were then obtained for grid points at a spacing of 68.5 n mi for the eastern two-thirds of the United States (Fig. 1). Finally, the field of integrated horizontal moisture divergence was determined.

The objective analysis (Inman, 1970) [essentially that developed by Cressman (1959)] involves a successive approximation procedure but with the weight function modified in a manner similar to that devised
by Endlich and Mancuso (1968) so that observations along the wind direction contribute more to a grid point value than do crosswind observations.

5. Procedures

Station values of the moisture requirement are calculated for the layer from the LCL to 400 mb rather than to the level near the tropopause at which the parcel becomes colder than the environment, i.e., the top of the cloud according to parcel theory. This level is generally above 400 mb when moderate or strong convection is likely to occur. Computations were not made to a higher level primarily because the loss of thermodynamic sounding data above 400 mb often becomes significant enough to adversely affect the objective analysis. Consequently, the moisture requirement $Q$ is low when the temperature of the parcel is warmer than that of the environment at 400 mb. When the parcel is colder than the environment at 400 mb, $Q$ is generally
negative and larger in magnitude than $Q_2$. Thus, the total moisture requirement is negative, which for the purposes of this study is all that is required.

Horizontal moisture divergence was calculated in the layer from the surface to 10,000 ft (MSL) because most of the water vapor is in this layer and because loss of wind data becomes significant above this level. The divergence was also calculated for the layer between the surface and 6000 ft (MSL). Patterns were similar to those of the deeper layer, and the values were lower.

The fractional coverage, the ratio of the integrated horizontal moisture divergence multiplied by 1 hr to the moisture requirement, expressed in percent, was calculated only at grid points where moisture convergence was occurring and where the moisture requirement was positive, since cumulus convection can occur in this model only under these conditions.

Since the coverage occasionally exceeded 100%, it should then, perhaps, be considered in terms of the production of cloud air. This condition often occurred when the horizontal moisture convergence and the moisture requirement were both small; one example of this frequently occurs in the southwestern United States. There the atmosphere is generally dry, and the LCL is high. This reduces the layer of integration and the moisture requirement. At the same time the horizontal moisture convergence is small because the low level air is dry. The calculated percent coverage under these conditions can become unrealistically large even though appreciable cumulus convection is unlikely. Thus, grid point percentages are set to zero when the parcel must be lifted more than 7000 ft (a little more than 200 mb in the lower layers), and the moisture requirement is less than 0.7 gm cm$^{-2}$, values chosen by a qualitative examination of several test cases. These criteria proved to be not very restrictive. It is a physically realistic modification, however. When the coverage exceeds 100% and the above criteria are not met, the coverage is set equal to 100%.

6. Example

It is virtually impossible to obtain detailed quantitative information on the distribution of strong cumulus convection over the region of the grid; thus, the computed results are evaluated only qualitatively.

The 30 data sets available for the 8 periods shown in
Table 1 were computer processed, and the fields of horizontal moisture divergence, moisture requirements, and predicted percent coverage were compared with radar echo patterns and with severe weather reports. U. S. Weather Bureau radar summary charts, available at 3-hr intervals for the times and areas of interest, provided radar echo patterns. Severe weather reports were obtained from U. S. Department of Commerce (1967).

Morning (0600 CST) and evening (1800 CST) rawinsonde data for the severe storm case of 14 May 1967 illustrate the technique. Surface and 500-mb analyses are shown in Figs. 2a–d for 0600 and 1800. An axis of horizontal moisture convergence (Fig. 3a) lay from eastern Texas through Oklahoma to northern Kentucky. Areas of large moisture divergence were centered in northern Texas and Mississippi. Moisture requirements (Fig. 3b) are high over most of the southeastern United States with an axis of maximum values extending from southern Texas to Tennessee. A minimum in the lifted height field (Fig. 3c) occurred in western Oklahoma where it appeared that low-level velocity divergence and dry advection occurred. The minimum in the lifted height field in western Illinois occurred near the region of maximum horizontal moisture convergence and probably defined the area of maximum low-level moisture.

Radar echo patterns were better organized at 0845 CST (Fig. 3f) and 1145 (Fig. 3g) than at 0545 (Fig. 3e). The radar echo patterns at 0845 agreed well with the percent coverage field (Fig. 3d). By 1145 the thunderstorm activity had begun to move southeastward from
the percent coverage field pattern. At 0545 rain and thundershowers over the lower Mississippi Valley occurred in an area of zero percent coverage. By 0845 this area had dissipated, implying the predictive nature of the model. The small areas of rain and thundershowers north of the primary convective area weakened during the morning and occurred in an area of zero percent coverage.

Hail was reported in south central Oklahoma at about 0600 just outside of the non-zero percent area. Heavy rains and three funnels occurred in Kentucky during the morning.

By 1800 the axis of horizontal moisture convergence (Fig. 4a) extended from the Louisiana coast to western Tennessee. The area of divergence behind the front was still present. Moisture requirements (Fig. 4b) decreased over most of the region that had positive values at 0600. The axis of minimum lifted height (Fig. 4c) agreed quite well with the radar echo pattern at 1800.

At 1800 the cloud cover percentage patterns (Fig. 4d) were not quite as accurate as those 12 hr earlier, but most of the principal features, especially the squall line extending from Kentucky southwestward to the Gulf Coast, was within the non-zero area. However, the area of thundershowers over Indiana and northern Kentucky at 1800 did not show up because of the moisture divergence in the region. No convection was predicted over most of the area of light rain north of the front (Figs. 4e–g). Light rain and some thundershowers were occurring in the northeastern United States where the percentages were very large. Generally, these large
percentages resulted from small values of convergence and moisture requirements.

Severe weather reports between 1800 and 2400 included a hailstorm near Shreveport, and funnels, strong winds and heavy rains in east central Alabama and northern Georgia. All of these occurrences were within non-zero areas.

7. Summary and conclusions

The amount of moisture per unit area required to produce a model cloud whose temperature and moisture profiles are pseudo-adiabatic in a layer representative of moderate cumulus convection was calculated. The base of the layer is the lifting condensation level, the level of saturation of moist air raised with dry adiabatic expansion, and the top is 400 mb (~24,000 ft). The amount of model cloud produced by horizontal moisture convergence in the layer bounded by the surface of the earth and 10,000 ft MSL (a layer sufficiently thick to contain most of the atmospheric water vapor) was determined. The amount of model cloud produced was compared with the convective activity indicated by the U. S. Weather Bureau radar network and by severe weather reports.

The examination of the 30 data sets suggests that the technique works best in strong, well-defined synoptic situations. Generally, the correlation between the patterns of predicted cloud formation and the radar echo patterns is good at rawinsonde observation time, a little better 3 hr later, and decreases at greater time intervals between rawinsonde and radar observation times. Thus, the model is predictive on a short time scale rather than purely descriptive.

The orientation of the areas of non-zero percentages and, in particular, the relative maxima within non-zero areas agrees well with the orientation of convective cloud patterns. The model generally predicts zero percentage in areas of stratiform rain, a type of precipitation not adequately described by the model. Convection is often predicted over most of the warm sector of an extratropical cyclone. However, the percentages are usually less than 5% in portions of the warm sector that do not have radar echoes. Minima in the lifted height field are usually associated with considerable low-level moisture, especially in the warm sector of cyclones.

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REFERENCES


