The freezing of droplets by this mechanism would be accompanied by the ejection of ice splinters.

Regular ice crystals and frozen droplets can also be formed by secondary processes. Regular ice crystals can grow on small ice splinters and droplets can be frozen by the contact of supercooled droplets with ice crystals or ice splinters.

From the above review it can be seen that several mechanisms are likely to be active and perhaps more than one could be responsible for the observed ice particles. Our experiments do not indicate a mechanism, but do indicate a phenomenon which may be of meteorological significance.

Shock waves produced by a lightning stroke can induce freezing in a column, the diameter of which depends on the energy released by the stroke.

Freezing could also be triggered by supersonic flight through supercooled clouds, and in cloud-seeding procedures which employ explosive devices.

Acknowledgment. This research was supported by the National Science Foundation under Grant GA-3956.

REFERENCES

Abbas, M., and J. Latham, 1969: The electrofreezing of supercooled water droplets. *J. Meteor. Soc. Japan*, **47**, 65–74.

Alkezweeny, A., 1969: Freezing of supercooled water droplets due to collision. *J. Appl. Meteor.*, **8**, 994–995.

Edwards, G., L. F. Evans and S. D. Hamann, 1969: Nucleation of ice by mechanical shock. *Nature*, **223**, 390–391.

Engels, O., 1958: Fragmentation of waterdrops in the zone behind an air shock. *J. Res. Natl. Bur. Stds.*, **60**, 245–280.

Gitlin, S., and S. S. Lin, 1969: Shock waves and freezing. *J. Appl. Meteor.*, **9**, 716–717.

Fall Velocity of Snowflakes[1]

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28 June 1971 and 26 August 1971

1. Introduction

While the fall velocity of individual type crystals of symmetric form is reasonably well known (Magono, 1954; Bashkirova and Pershina, 1964; Holroyd, 1971), that of snowflakes is somewhat less certain. Most snowflakes fall at about 1–1.5 m sec^{-1}, as anyone with a stop watch and some patience can readily verify. However, one would like to understand better the governing variables and to have a reliable analytical expression for snowflake fall velocities.

Owing to the random collision process involved in snowflake aggregation, and the variety of shapes and densities that can result, it would be naive to expect too much from any fall velocity equation. One customarily assumes a spherical cluster, which appears adequate, although it will be observed that many flakes become aerodynamically flattened or are sometimes tapered to a bottom conical shape during fall. Nevertheless, the narrow range of average fall velocities mentioned suggests that shape, drag and density variables are not overly critical. Snowflakes that have accreted substantial supercooled cloud droplets will fall considerably faster and possess greater variability.

An attempt was made to develop simple but adequate fall velocity expressions for dry snowflakes as a function of diameter, the most readily measured parameter in the field; to test these expressions with measured data; and to resolve some seeming inconsistencies in prior reported work.

[1] This research was supported by the U. S. Department of Commerce, NOAA (Atmospheric Physics and Chemistry Laboratory), under Grant E22-91-71(G), and by the Atmospheric Science Research Center of the State University of New York.

2. Analytical expressions

Magono and Nakamura (1965) have done extensive work in this area, arriving at the empirical fall velocity equation

$$v_f = 377(\rho_f - \rho_a)^{1/2}, \tag{1}$$

where v_f is fall velocity, ρ_f the density of the flake, and ρ_a the density of the air (cgs units). They also experimentally determined a relationship between flake size (radius r_f) and density of the form

$$(\rho_f - \rho_a) r_f^2 = 0.005. \tag{2}$$

Their data in both cases included dry and wet (melting) snowflakes. Combining the above equations, the fall velocity can be expressed in terms of snowflake size as

$$v_f = 100/(r_f^{1/2}). \tag{3}$$

The obvious implication that small snowflakes fall faster than large ones is attributed by the authors to a significant flake density decrease with increasing size. Such an inverse velocity-size trend is in contrast to measurements of Langleben (1954). O'Brien (1970) also has arrived at empirical velocity expressions that indicate a directly proportional dependence on size, i.e.,

$$v_f = AD^n[\log_e(v_f D) - B], \tag{4}$$

where D is the maximum snowflake dimension, and A, B and n are empirical constants that were chosen, somewhat subjectively, to fit the observed data of Nakaya (1954) and Magono (1951, 1953, 1954) for 11 inferred types of snowfall.

Langleben's measurements were all made in terms of the melted snowflake size; his empirical velocity expression was

$$v_f = k d_m^{0.31} = 1.24 k r_m^{0.31} \approx 1.24 k r_m^{0.3}, \tag{5}$$

where d_m and r_m are melted snowflake diameter and radius, respectively, and k is a coefficient that varies with the type of crystals comprising the snowflake (i.e., $k=160$ for dendrites and 234 for a mixture of plates and columns).

The major difference in the above results apparently is due to the fact that the Magono-Nakamura (1965) data include both dry and partially melted snowflakes. In many cold snowstorms, the latter are not present. When Magono and Nakamura grouped their data into fixed density categories (influenced by the degree of melting), they also observed a weak increase of fall velocity with size. The excellent snowflake density-diameter data of these authors (their Fig. 3) were re-analyzed with all the wet snowflake data eliminated. A least-squares fit (Holroyd, 1971) then resulted in the following expression for all types of dry snowflakes:

$$\rho_f r_f = 0.0085. \tag{6}$$

While an inverse relation between crystal density and size still exists, it is not so strong as that of (2).

One may relate the sizes of purposely melted (Langleben approach) and dry snowflakes using mass equalities. Considering first dendritic flakes, we have

$$\left.\begin{array}{c}\underbrace{\text{Mass (melted flake)}} = \underbrace{\text{Mass (dry flake)}} \\ (\tfrac{4}{3})\pi r_m^3 \rho_l = \sum_{i=1}^{j} n_{ci} m_{ci}\end{array}\right\}, \tag{7}$$

where ρ_l is the density of liquid water, and n_c and m_c are the concentration and mass of individual crystals in the flakes. Assuming uniform-size crystals in the flake and the Nakaya-Terada (1934) mass-size relation for dendrites (i.e., $m_c = 15.2 \times 10^{-4} r$), the combination of (5) and (7) then yields

$$v_f = 90 n_c^{0.1} r^{0.2}. \tag{8}$$

Note that r is the individual crystal size and we would prefer to convert this to the more easily measured snowflake size, r_f. Making the customary assumption of spherical flakes and using an approach in which the mass of the snowflake is set equal to the total mass of the individual crystals in the flake, we have

$$r^{0.2} = 2.21 \rho_f^{0.1} r_f^{0.3} / n_c^{0.1}. \tag{9}$$

Substituting this latter expression into (8) and applying the snowflake density expression (6), the terminal fall velocity of dendritic snowflakes is reduced to just one variable (size), as follows:

$$v_f = 123 r_f^{0.2}. \tag{10}$$

One may go through an identical exercise to obtain the fall velocity of dense snowflakes consisting of plates and columns; in this case we have

$$v_f = 178 r_f^{0.2}. \tag{11}$$

For snowflakes containing both dendritic and compact crystals, or if doubt exists as to crystal composition, the use of an average of (10) and (11) appears legitimate, i.e.,

$$v_f = 150 r_f^{0.2}. \tag{12}$$

Observations in New York State indicate that dendritic type snowflakes are the most common, a mixture of dendrites and thin plates the next most prevalent, with thick plate and column snowflake aggregates being quite rare. The latter are indicative of an inactive cloud near ice saturation, where the high crystal concentrations necessary for significant collisions and aggregation (Jiusto, 1971) apparently are not commonly present. They have been observed (Holroyd and Jiusto, 1971) to fall from supercooled clouds that were heavily seeded (Weickmann *et al.*, 1970).

The various velocity functions are plotted in Fig. 1. The Magono-Nakamura function, for small snowflakes, reflects melting flakes that steadily approach the liquid phase with decreasing size. The functions (10)–(12) yield velocity values that tend to cluster in the 1–1.5 m sec^{-1} range; they also show a slight velocity increase with size, being more pronounced at the smaller end of the size spectrum. O'Brien's curve for dendritic ag-

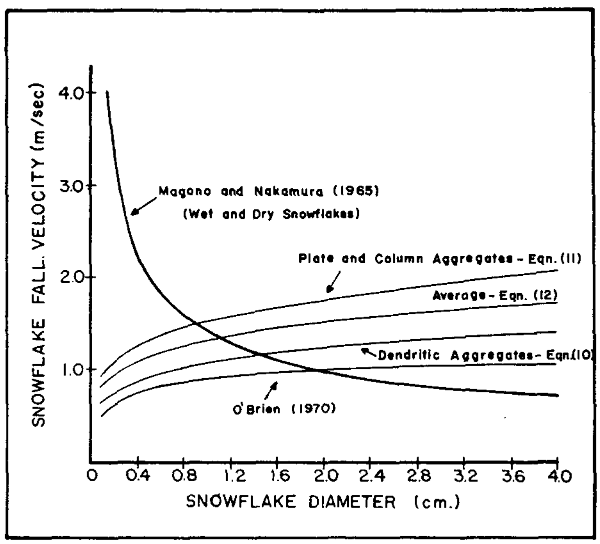

FIG. 1. Analytical functions for the terminal fall velocity of snowflakes.

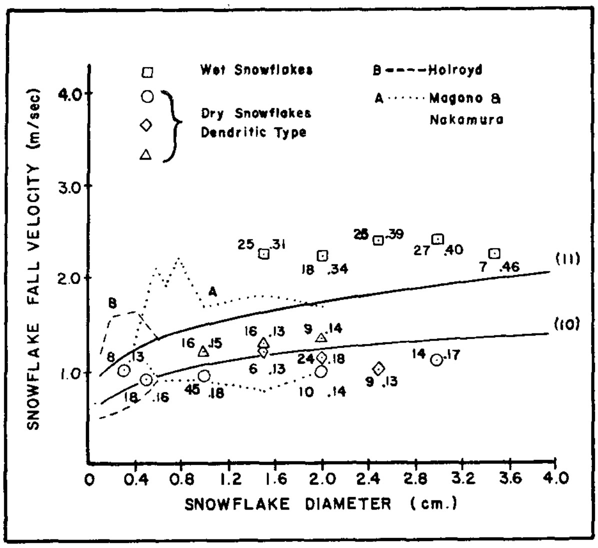

FIG. 2. Snowflake fall velocity data. The number of measurements are indicated to the left of a plotted point, the standard deviation to the right.

gregates is in reasonably good agreement with expression (10).

3. Measured data

A wooden snow shelter was constructed with an opening in the roof through which snow flakes could descend. Their fall times over a distance of 3.35 m were clocked with a precision stopwatch. The diameter of a flake was measured after it settled on black velvet, recognizing that some enlargement error was occasionally introduced. If nonspherical flakes were involved, the average of the long and short axes of the snowflake was taken.

Mean velocity data, as shown in Fig. 2, were obtained on four days during which 287 snowflakes were timed and sized. On three days, the snowflakes consisted of dry, primarily dendritic forms, while melting flakes occurred once. The number of observations per plotted point is indicated to the left of each symbol and the velocity standard deviation (σ) to the right. Snowflakes equal to or greater than 1.5 cm in diameter always consisted of more than 40 individual crystals, while smaller flakes rarely comprised fewer than 10 crystals.

The mean data points in the dry snowflake cases tend to straddle the analytical curve for dendritic aggregates. A slight increase in fall velocity with size was observed on two of the three days. Evidently, as indicated by Magono, the reverse can take place with certain types of snowflakes. The consistent σ values of dry snowflakes appear compatible with random shape and density variations and measurement errors (estimated 7% probable error). The wet snowflakes fell substantially faster than their dry counterparts, with a similarly weak dependence on size suggested. As might be expected, the velocity standard deviations were considerably larger than for dry snowflakes.

The Magono-Nakamura measured data on low density ($\leqslant 0.04$ gm cm^{-3}, i.e. presumably dry) snowflakes are outlined by area A (approximately 370 measurements), while Holroyd (1971) data for smaller dry snowflakes are indicated by area B (approximately 80 measurements). The data are compatible with our analytical curves.

4. Conclusions

In summary, simple fall velocity equations were obtained for dry snowflakes (dendritic and columnar forms) as a function only of flake size by combining Langleben's velocity expression, an adjusted Magono-Nakamura snowflake size-density expression, and the Nakaya-Terada crystal mass-size relations. The results show a slight dependence of fall velocity on snowflake size. Measured velocity data agreed well with these approximation equations.

Snowflakes consisting of dendrites or a mixture of dendritic and plate forms are dominant in New York State snowstorms, if not in most convective storms in the Northeast. Dense snowflakes comprised of thick plate and columnar crystals appear rare in storms, but have been observed in heavily seeded clouds.

REFERENCES

Bashkirova, G. M., and T. A. Pershina, 1964: On the mass of snow crystals and their fall velocity. *Tr. Gl. Geofiz. Observ.*, **165**, 90–100.

Holroyd, E. W., 1971: The meso- and microscale structure of Great Lakes snowstorm bands. Ph.D. dissertation, Dept. of Atmospheric Science, State University of New York, Albany.

——, and J. E. Jiusto, 1971: Snowfall from a heavily seeded cloud. *J. Appl. Meteor.*, **10**, 266–269.

Jiusto, J. E., 1971: Crystal development and glaciation of a supercooled cloud. *J. Rech. Atmos.*, **5**, 69–85.

Langleben, M. P., 1954: The terminal velocity of snowflakes. *Quart. J. Roy. Meteor. Soc.*, **80**, 174.

Magono, C., 1951: On the fall velocity of snowflakes. *J. Meteor.*, **8**, 199–200.

——, 1953: On the growth of snowflake and graupel. *Sci. Repts. Yokohama Natl. Univ., Sect. 1*, No. 2, 18–40.

——, 1954: On the falling velocity of solid precipitation elements. *Sci. Repts. Yokohama Natl. Univ., Sect. 1*, No. 3, 33–40.

——, and T. Nakamura, 1965: Aerodynamic studies of falling snowflakes. *J. Meteor. Soc. Japan, Ser. 2*, **43**, 139–147.

Nakaya, U., 1954: *Snow Crystals, Natural and Artificial.* Harvard University Press, 510 pp.

——, and T. Terada, 1934: Simultaneous observations of the mass, falling velocity and form of individual snow crystals. *J. Faculty Sci., Hokkaido Univ., Ser. II*, **1**, 191.

O'Brien, H. W., 1970: Visibility and light attenuation in falling snow. *J. Appl. Meteor.*, **9**, 671–683.

Weickmann, H. K., J. E. Jiusto, G. E. McVehil, R. J. Pilie and J. Warburton, 1970: The Great Lakes project. *Preprints of Papers, 2nd Natl. Conf. Weather Modification*, Santa Barbara, Calif., Amer. Meteor. Soc., 34–40.