

An Energy Budget for Intermittent Turbulence in the Free Atmosphere¹

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ABSTRACT

In order to study the physical aspects of intermittent turbulence in the free atmosphere, it is suggested that terms of an "instantaneous" turbulent energy budget be computed from data collected by aircraft. This can be accomplished through the application of numerical filters to specify mean and turbulence parameters as a function of distance along the aircraft track.

The proposed analysis is illustrated with data from a highly intermittent turbulence record derived from a flight through a rotor. Results are compatible with the known characteristics of the rotor, thus suggesting the feasibility of the technique. Some difficulties are encountered in the satisfaction of the Reynolds averaging rules because of the requirement for a gap in the velocity spectrum and the existence of a finite cutoff interval for a practical numerical filter.

1. Introduction

One of the best known, yet least studied characteristics of clear air turbulence (CAT) records is patchiness or intermittency.³ Only during the last few years, with the development of aircraft instrumentation capable of making detailed measurement of CAT, have the causes and effects of intermittency received much attention. Yet the property of intermittency of CAT appears to be an important one which should be taken into account in both the analysis and interpretation of the data (Dutton and Lane, 1969).

The effects of intermittency on the statistics of CAT records have been examined (Dutton *et al.*, 1969; Dutton, 1970), but there have been few attempts to deduce the detailed physical aspects. Dutton (1970) and Lester (1970) have derived statistical models of patchy turbulence based on hypothetical records composed of Gaussian bursts. While these models predict some of the gross statistical characteristics of intermittent CAT, they give little insight into the physics of the intermittency.

Other, more common methods of analyzing CAT data gathered by aircraft have frequently suppressed intermittency effects, or have neglected them entirely. For example, spectrum analysis studies (e.g., see Pinus *et al.*, 1967) often assume homogeneity and stationarity of the data in order to apply the spectral theories of

Kolmogorov (1941) and others. Such assumptions do not allow intermittency in the data. If the nature of CAT is to be determined, analytical techniques must be employed that do not suppress its intermittent character.

One possibility lies in the computation of the turbulent energy budget. Myrup (1969) and Lenschow (1970) have done this for flights in the planetary boundary layer, but in that portion of the atmosphere horizontal inhomogeneities appear to be less important, especially under unstable conditions, than they are at higher elevations. Turbulent intermittency shows a marked increase with height above the surface (Vinnichenko, 1969) and lateral inhomogeneities begin to become an important source of turbulent energy. It follows that simplifications in the turbulent energy budget that can be made in the boundary layer are not permissible when treating turbulence at higher levels. Averages along the aircraft path may eliminate important intermittent effects and so must be computed with great care.

This paper suggests an analytic technique for the isolation of the characteristics of individual turbulence bursts. The turbulence data are treated, *a priori*, as *intermittent*, and terms of the turbulent energy budget are considered as a function of distance along the aircraft track. In order to illustrate the application of the technique, computations are presented for some data obtained during a flight through a rotor under lee wave conditions.

2. The energy budget

The turbulent energy equation may be expressed in tensor notation (see Lumley and Panofsky, 1964)

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³ An intermittent record is one in which a large fraction of the total variance comes from a relatively small fraction of the total record (Dutton and Lane, 1969).

as

$$\frac{\partial \bar{E}}{\partial t} = -\bar{u}_j \frac{\partial \bar{E}}{\partial x_j} - \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u_i' u_j'} \frac{\partial u_i'}{\partial x_j} - \frac{1}{\bar{\rho}} \frac{\partial \bar{p}'}{\partial x_i} + \frac{g}{\bar{T}} \overline{T' u_3'} + \nu \overline{u_i' \frac{\partial^2 u_i'}{\partial x_j^2}}, \quad (1)$$

where $E = u_i'^2/2$, \bar{u}_i is the mean component of the wind, u_i' the turbulent component ($i=1,2,3$), ν the kinematic viscosity, and all other terms have their usual meaning. In the computation of the turbulent energy budget, it is generally assumed that the mean of each quantity, as, for example, the velocity component \bar{u} , may be defined as

$$\bar{u} = \frac{1}{T} \int_{-T/2}^{T/2} u(t) dt, \quad (2)$$

where T is the averaging interval. This method of separating the mean and turbulent flows simplifies the derivation of the turbulent energy equation (1) because of the satisfaction of the so-called Reynolds axioms (Charnock, 1957).

In Dutton's (1969) study of a layer of stratospheric CAT, deviations were determined by the removal of linear trends from gust data which were first subjected to a high-pass filter (scales $\gtrsim 2$ km were eliminated). Covariances were computed in the usual manner, that is, an overbar in (1) was defined as in (2). It was assumed, with T equal to the time it takes the sampling aircraft to traverse a long stretch of CAT, that lateral homogeneity existed and $\bar{u}_3 = 0$.

The energetics of *intermittent* CAT cannot be studied by this method because radical changes in turbulent energy often occur in short distances along the aircraft track. An apparently simple way to account for the energy of bursts of turbulence might be to divide the length of the turbulent record into a number of smaller records of equal length and to compute individual budgets over each of the subsections. In the limit, this is equivalent to computing equally weighted running averages, over intervals of width T , i.e.,

$$\bar{u}(t) = \frac{1}{T} \int_{-T/2}^{T/2} u(t + \tau) d\tau, \quad (3)$$

so that the mean becomes a function of space (sampling time). However, it is well known (see, e.g., Holloway, 1958) that such an averaging process produces a damping of fluctuation amplitudes. Also, the Reynolds averaging rules hold for (3) only if u is a constant or a linear function of time. Such is not usually the case in regions of CAT. So the simple approach suggested by (3) is not valid.

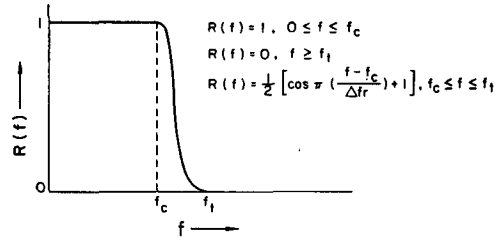


FIG. 1. Schematic diagram of the response function, $R(f)$, of the Martin-Graham low-pass filter. General response characteristics are given on the right-hand side of the diagram. See text for further explanation.

Isakson,⁴ in a classical work, has devised a scheme for the separation of scales for which the Reynolds axioms hold (see, also, Charnock, 1957). Representing the total motion as the sum of a mean and a deviation, the mean is given by

$$\bar{u}(t) \equiv \int_{-\infty}^{\infty} w(\tau) u(t + \tau) d\tau, \quad (4)$$

where $w(\tau)$ is a weighting function. By recognizing that the response function, $R(\omega)$, is the Fourier transform of the weighting function and by expressing $u(t)$ in terms of a Fourier integral, (4) may be written as

$$\bar{u}(t) = \int_0^{\infty} R(\omega) [A(\omega) \cos \omega t + B(\omega) \sin \omega t] d\omega. \quad (5)$$

It can be shown that the rule that $\bar{u}(t) = \bar{\bar{u}}(t)$ holds only if $R(\omega)$ is either 0 or 1. Defining

$$\left. \begin{aligned} R(\omega) &\equiv 1, & 0 < \omega < \Omega_c \\ R(\omega) &\equiv 0, & \omega > \Omega_c \end{aligned} \right\}, \quad (6)$$

where Ω_c is the cutoff frequency, the mean flow is defined as

$$\bar{u}(t) = \int_0^{\Omega_c} [A(\omega) \cos \omega t + B(\omega) \sin \omega t] d\omega, \quad (7)$$

and the deviations are obtained by subtracting the mean from the total record. Note from (6) and (7) that $\bar{u}' = 0$. Isakson has shown that $\bar{u} u' = 0$ only if a gap of width $(3\Omega_c - \Omega_c)/2$, centered on Ω_c , exists in the spectrum of u ; also, in order for $\bar{u}^2 = \bar{u'^2}$, energy in the mean flow components should be absent at frequencies $> \Omega_c/2$ (see also, Charnock, 1957).

It is important to emphasize at this point that the bar and prime notation have taken on a more general meaning. The application of Eq. (7) allows one to define a *continuous* mean (or deviation). However, there are practical problems which must be considered before applying the procedure to CAT records.

⁴ A summary of Isakson's work is given by J. M. Burgers in *Das Turbulenzproblem*, by C. W. Oseen (1930).

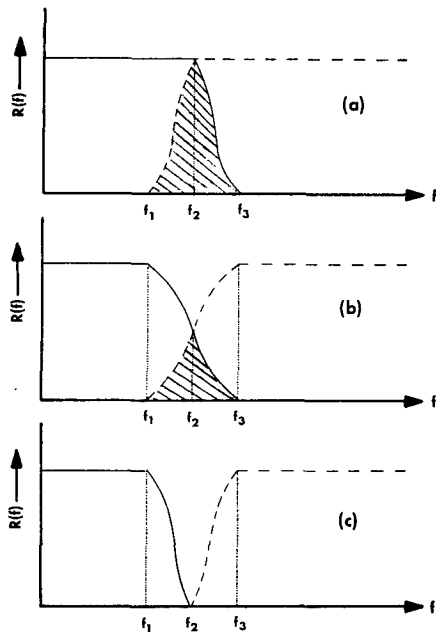


FIG. 2. Schematic representation of three possible arrangements of response functions for high-pass (dashed line) and low-pass (solid line) filters. Regions of filter overlap are shaded. See text for discussion.

Isakson's filter is based on infinite integrals, but most CAT data consist of discrete points and all CAT records are finite. The fact that $w(\tau)$ and $R(\omega)$ are Fourier transforms of each other has facilitated the design of numerical filters for application to finite records (Holloway, 1958). One of these is the Martin-Graham filter (Martin, 1962; Graham, 1963), which is characterized by its excellent response and flexibility. For these reasons and because it has been used extensively in CAT investigations (Crooks *et al.*, 1968), this filter was applied to the data in the present study of CAT energetics.

The Martin-Graham low-pass filter has a response function, shown schematically in Fig. 1, where f_c is the cutoff frequency (Hz), f_t the termination frequency, and $\Delta f_r = (f_t - f_c)$ is the "roll-off" interval.

The associated high-pass filter is given by the difference between the low-pass filter and the all-pass filter (central weight, 1; all other weights, 0). The filter is completely determined by the specification of the cutoff frequency f_c , the termination frequency f_t , and the number of weights NW .

Another problem arises because the cutoff frequency for the ideal filter is replaced by a finite interval for the numerical filter, that is, a region where the response function decreases from 1 to 0. This is illustrated in Fig. 1 and causes difficulties in the satisfaction of the Reynolds averaging rules. For example, consider the three schematic diagrams in Fig. 2. The response function for the mean (low-pass filter) is given by the solid line while the dashed line represents the response func-

tion for the deviation (high-pass filter). In 2a, low-pass filtering of the raw data admits all frequencies $< f_2$ with negligible amplitude dampening. The high-pass filter performs similarly on the raw data for all frequencies $> f_2$. However, the two filters share a common frequency band (shaded area) because of the finite interval over which the cutoff is accomplished. The result is that $u \neq \bar{u} + u'$ and $\overline{u'} \neq 0$; therefore, the advantage gained by insuring that frequencies above and below f_2 may be passed with undamped amplitudes by one filter or the other is offset by the fact that two important Reynolds axioms are violated. The arrangement in 2b insures that $u = \bar{u} + u'$ because the high-pass filter is the complement of the low-pass filter. However, the region of overlap again does not allow $\overline{u'} = 0$. Although the arrangement in 2c overcomes the latter problem by not permitting the filters to overlap, again $u \neq \bar{u} + u'$ because of the frequency gap created.

The effects of the various arrangements of filters on the Reynolds axioms were tested on real data. On the basis of the tests it was decided to use the arrangement shown in Fig. 2c so that $\overline{u'} = 0$, but in so doing, to keep the interval $(f_3 - f_1)$ as small as practical; thus, the fraction of total energy lost from the mean and from the deviation was kept to a minimum.

A third problem arises because a gap of width $(3f_c - f_c)/2$ centered at f_c must be present in the energy spectrum in order for $u'\bar{u} = 0$. A spectral gap implies that the atmospheric flow is clearly separable into mean and turbulent components that are uncoupled in the sense of suppressed interactions. If the spectral gap does not exist, then interactions become large and $u'\bar{u}$, which is a measure of such interactions, may be of the same order of magnitude as $\overline{u'^2}/2$.

Ideally, the proposed analysis of the turbulent energy budget is carried out in the following manner: the spectra of the data are first examined for a possible gap in the mesoscale; based on the location of the gap, the data are separated (filtered) into means and deviations, both of which are functions of time (or space). Finally, energy budget terms are computed with the realization that the overbar in (1) henceforth implies low-pass filtering of the particular quantity in question and the prime represents high-pass filtering. For example, $\overline{u'^2}$ would be calculated by first high-pass filtering to determine u' , squaring to yield u'^2 , and finally, low-pass filtering to determine $\overline{u'^2}$.

In practice, the placement of the cutoff frequency in a gap is not always possible. Bretherton (1969) has discussed a number of the problems involved in the specification of the gap from atmospheric data. In the present study it was found that the position of the depression of the spectral energy (when it existed) was ill-defined and did not extend over the desired range. Furthermore, the apparent gap often shifted its position in frequency space as a function of the size of the sample

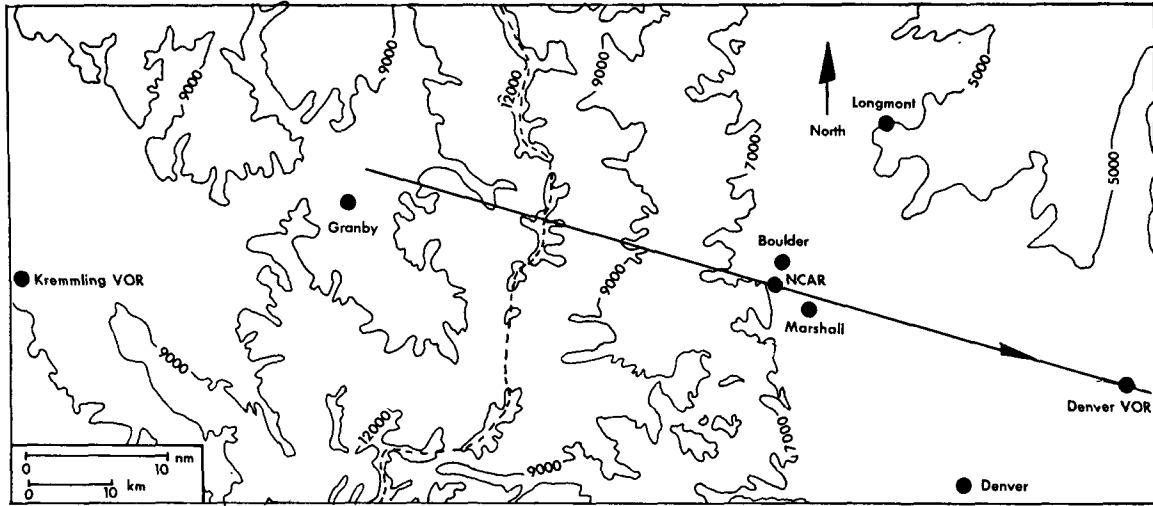


FIG. 3. Topographical chart of geographical region in which the Rocky Mountain Lee Wave Experiment was conducted. Contours are labeled in feet (MSL). Heavy solid line denotes flight path of aircraft. The continental divide is indicated with the dashed line.

analyzed and the spatial position of the sample with respect to the turbulent burst or bursts.

The existence of a spectral gap, although desirable, is not an absolute necessity in the proposed analysis. However, in its absence, interactions between the mean and turbulent components of the flow become important and must not be neglected.

3. A case study

A few terms of the turbulent energy budget were computed using data obtained during a downwind flight through a rotor in order to demonstrate the proposed

method. As opposed to other types of turbulence-producing mechanisms in the free atmosphere, mesol scale aspects of the rotor have been observed in detail for many years (see, e.g., Kuettner, 1939a, 1939b, 1959; Holmboe and Klieforth, 1954; Harrison, 1957). The phenomenon, which is associated with mountain lee-wave flow, may be described as extremely turbulent, quasi-stationary, roughly symmetric with respect to an axis oriented parallel to, and downstream of, the mountain ridge which is disturbing the flow, and with dimensions of a few kilometers in radius and many kilometers in length. The rotor data were purposely

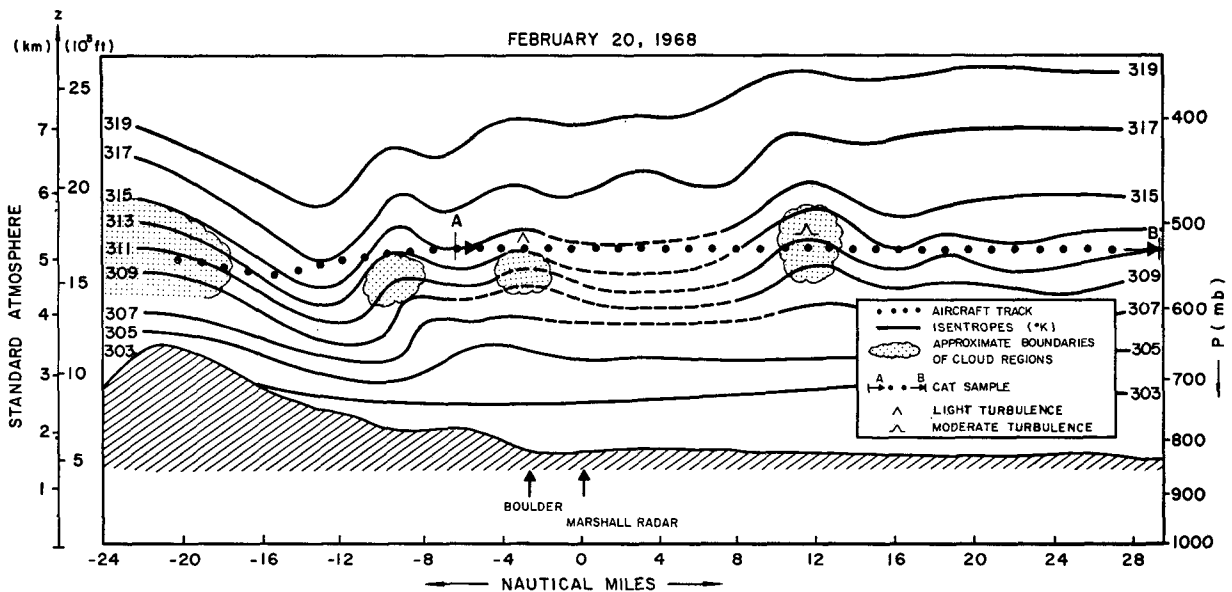


FIG. 4. Vertical cross section oriented along the aircraft track. Isentropes are dashed where data did not show vertical consistency or were not available. The mean terrain profile is shaded (original analysis by D. K. Lilly of NCAR).

selected in the present study because of the extreme intermittency of the record and the known characteristics of the rotor phenomenon.

On 20 February 1968, an instrumented Queen Air A80 encountered light to moderate turbulence at a flight level of 5.2 km in the lee of the Continental Divide during the period 0949 to 0957 MST. The flight path and topography are shown in Fig. 3. The flight was one of a number which were made during the 1968 phase of the Rocky Mountain Lee Wave Experiment (Kuettner and Lilly, 1968). Data were obtained from the National Center for Atmospheric Research (NCAR).⁵ The aircraft instrumentation has been described by Lilly and Tutenhoofd (1969) and Lenschow (1970).

The vertical cross section (Fig. 4) indicates that the path of the aircraft crossed over two roll clouds and intersected a third. Light turbulence was reported near the second rotor and moderate turbulence in the third. Some difficulty was encountered in the analysis of the region indicated by dashed lines in Fig. 4. Discussions with the aircraft observer and reference to the flight log suggest that patterns changed during the time it took to complete the previous flight leg and return over the same track at a lower level.

Although most of the terms of (1) could not be computed in the present case because certain parameters were not measured, it is sufficient for illustrative purposes to consider the behavior of a few terms of the energy equation along the aircraft track. These include

$$-\frac{\bar{u}}{2} \frac{\partial \overline{u'^2}}{\partial x}, \quad -\frac{\partial \bar{u}}{\partial x} \overline{u'^2}, \quad -\frac{1}{2} \frac{\partial \overline{u' u'^2}}{\partial x}. \quad (8)$$

The cross product, $\bar{u} u'$ has also been computed to illustrate the importance of interactions between the mean flow and the turbulence. Since the turbulence data were collected along a flight track which was approximately parallel to the mean wind vector, the positive x axis is taken in the direction of the mean wind vector, and u is the longitudinal wind component.

Because the "turbulence" considered in the present study is defined in terms of aircraft response, the cutoff frequency was selected to correspond with a scale of about 1 km. The mean (low-pass filtered) u component along the aircraft path was derived from the Doppler navigation system while the turbulent (high-pass filtered) u component was based on the true air speed (TAS). This step was necessary because at low frequencies (scales of about 1–5 km for the Queen Air) TAS fluctuations include the phugoid oscillation of the aircraft. Also, at low frequencies, ground speed fluctuations become of the same order of magnitude or larger than TAS fluctuations. The Doppler system, however, is capable of resolving the lower frequencies (e.g., Pinus, 1963; Fujita, 1966). The compatibility of the

Doppler u component and TAS over the intermediate scales of 1–5 km was checked by analyses of the spectra of both variables. Slopes and magnitudes of the spectra showed excellent agreement over the range in question.

Prior to acquisition for use in the present study, data had been averaged arithmetically in groups of four points, thus yielding information at intervals of 0.125 sec (original sampling rate 32 sec⁻¹). The effects of this preliminary averaging caused a slight loss of energy at frequencies $\gtrsim 0.5$ Hz (scales < 200 m).

The filtered mean and turbulent portions of the wind component along the aircraft track are shown in Fig. 5. Martin-Graham filter characteristics are given in the legend of the figure. The segment of the flight selected for analysis lies between points A and B in Fig. 4. The undulating nature of the mean component (\bar{u}) verifies the existence of mesoscale lee waves. The turbulent record (u') shows that the magnitudes of the maximum gusts are about 5 m sec⁻¹ in the rotor. The well-defined, sinusoidal character of the record of u' near 0956 MST indicates that some sort of wave motion may be present.

The mean kinetic energy of the longitudinal gust component ($\overline{u'^2}/2$) is shown in Fig. 6. The energy at the central peak (in the rotor) exceeds that of the rest of CAT record by an order of magnitude or more. Minor energy peaks occur near 0949.75 and 0956.

The advection of kinetic energy of the longitudinal gust component by the mean wind, $(-\bar{u}/2)(\partial \overline{u'^2}/\partial x)$, has a large sinusoidal fluctuation in the vicinity of the maximum energy peak and smaller oscillations near the less intense bursts. The loss of energy due to the advection of less turbulent air into the rotor exceeds the energy gain due to the advection out of the rotor, on the downstream side. The reason for the asymmetry can be seen from the plots of \bar{u} (Fig. 5) and of $(\overline{u'^2})/2$ (Fig. 6) immediately downstream of the energy peak. The mean wind reaches a minimum in that region, and the magnitude of the energy decrease in the same area is less than the magnitude of the increase on the upwind side of the burst.

In Fig. 6, $(-\overline{u'^2})(\partial \bar{u}/\partial x)$ represents energy changes due to energy redistribution by convergence of the mean wind. The relatively large positive maximum is due to the large turbulent kinetic energy in a region where the mean velocity along the aircraft track is decreasing downstream (convergence), while the minimum occurs where the mean flow is diverging; the magnitudes of the divergence of \bar{u} are $\sim 5 \times 10^{-3}$ sec⁻¹.

In the immediate vicinity of the rotor there are narrow zones of flux convergence and divergence ($\pm \frac{1}{2} \partial \overline{u' u'^2}/\partial x$). The maximum value attained by this term is one-half or less the maxima of the two terms discussed above.

The terms presented in Fig. 6 are frequently neglected in the usual method of computing the turbulent kinetic energy budget (e.g., see Lumley and Panofsky, 1964;

⁵ Sponsored by the National Science Foundation.

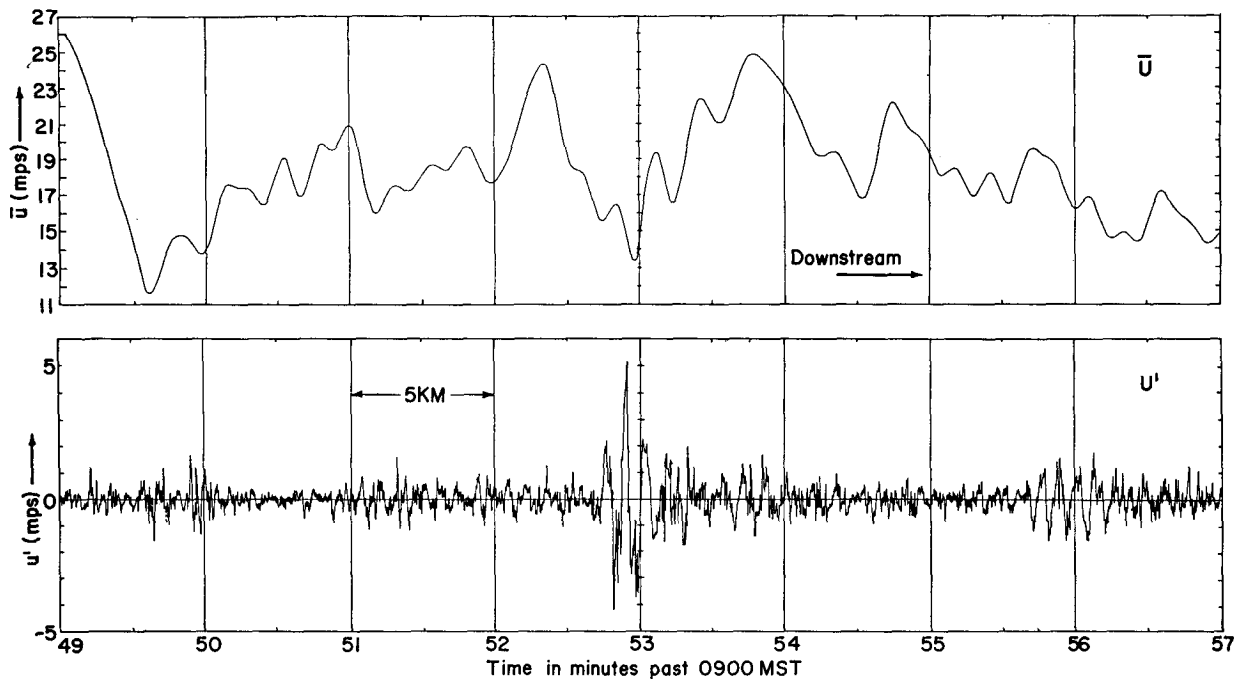


FIG. 5. Top: Velocity component along the aircraft track after low-pass filtering ($f_c=0.070$ Hz, $f_t=0.085$ Hz, $NW=1001$). Bottom: Velocity component along the aircraft track after high-pass filtering ($f_c=0.085$ Hz, $f_t=0.100$ Hz, $NW=1001$). Time axes are identical on both diagrams.

Dutton, 1969). On the scale of the entire CAT sample, the assumption of longitudinal homogeneity would probably be verified for the present case; that is, integration of the three terms over the entire patch of CAT

would yield very small average values. However, the instantaneous values of these terms near the major burst may be quite important.

The relative importance of the three terms would

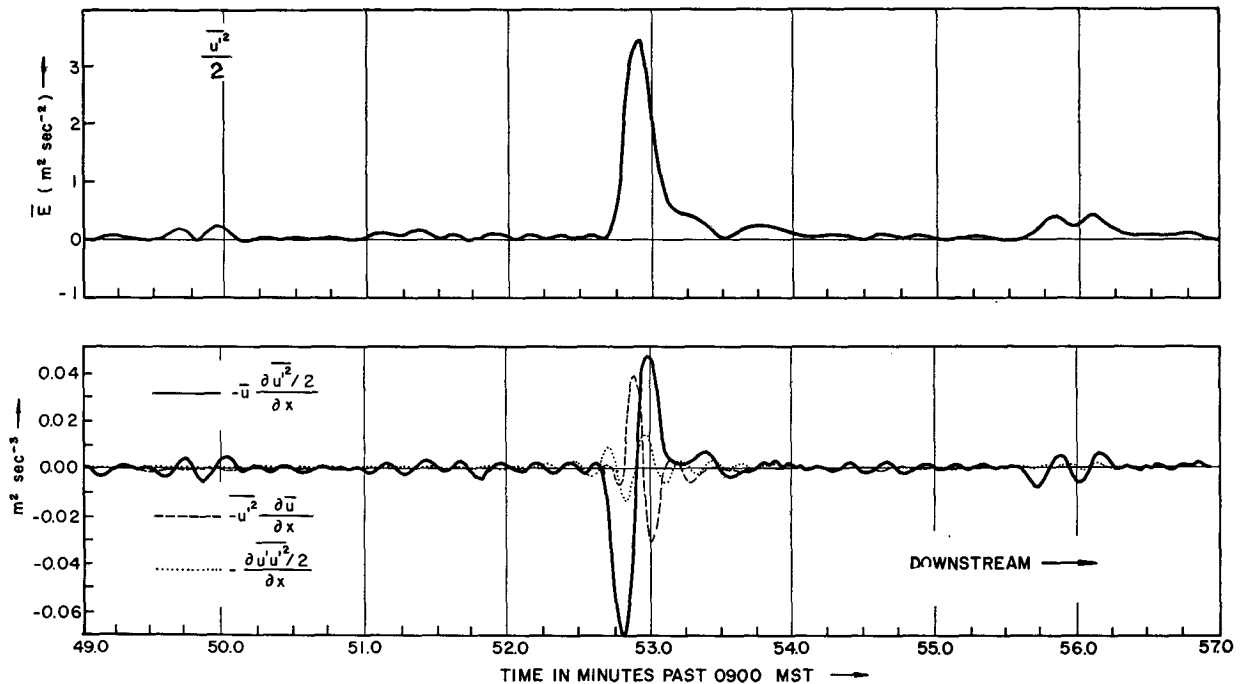


FIG. 6. Top: Distribution of the kinetic energy of the longitudinal gust component along the aircraft track. Bottom: Distribution of selected energy budget terms along aircraft track. See text for elaboration. Time axes are identical on both diagrams.

normally be determined by comparing them to the other, more familiar, terms of the energy budget. This procedure is not possible in the present case since terms such as the energy feeding due to mean vertical wind shear ($-\overline{u'w'}\partial\bar{u}/\partial z$) could not be computed from the data that were available. However, the probable relative magnitudes of these terms can be estimated in this case with the knowledge that the turbulence was embedded in a large-amplitude standing wave (Fig. 4). Since the mean streamlines exhibited a large vertical oscillation, the direction of maximum shear probably varied radically in the vicinity of the maximum turbulence. The shear of the mean vertical velocity along the flight path should approach the same magnitude as the vertical shear of the longitudinal wind component. Thus, upon intersecting the rotor, the shear feeding of energy should be accomplished alternately by the terms

$$-\overline{u'w'}\frac{\partial\bar{w}}{\partial x} \text{ and } -\overline{u'w'}\frac{\partial\bar{u}}{\partial z}.$$

Assuming that u' and w' are of the same order of magnitude and that (\bar{w}/\bar{u}) is a constant which is of the order unity on the flanks of the rotor, then

$$\left| \overline{u'w'}\frac{\partial\bar{w}}{\partial z} \right| \approx \left| \overline{u'w'}\frac{\partial\bar{w}}{\partial x} \right| \approx \left| \overline{u'^2}\frac{\partial\bar{u}}{\partial x} \right|. \quad (9)$$

Note that although the terms may attain similar magnitudes within the rotor environment, they may differ at the same location.

The above argument implies that the convergence term (Fig. 6) may at times be as large as the vertical

shear term. Since the latter is normally considered one of the more important terms of the energy budget (Lumley and Panofsky, 1964; Dutton, 1969), the convergence term may be equally important in the present case, at least locally. From Fig. 6 it can be seen that the advection term may also be locally important, but that the flux divergence term is of secondary importance.

The major cause of the intermittency of the record in Fig. 5 is the intersection of the flight path with the upper part of a rotor (the major burst). The previous discussion of the magnitude of the convergence term ($-\overline{u'^2}\partial\bar{u}/\partial x$) implies that a major part of the shear production of energy in the rotor is accomplished alternately by the horizontal and vertical shears of the \bar{w} and \bar{u} components, respectively. This emphasizes the fact that the mesoscale wave structure is included in the mean flow. The turbulence is being carried away from the rotor along mean streamlines which pass through the flight level, thus contributing to the intermittency of the record collected in horizontal flight near the base of a stable layer. The minor turbulence burst located upstream of the major occurrence appears to be associated with the top of another rotor, according to pilot's and observer's logs, while downstream, the turbulence appears to be due to a series of short, wave-like perturbations.

An estimate of the importance of interactions between the mean and turbulent (filtered) components is found in the comparison of the records of $\overline{u'u'}$ and $\overline{u'^2}/2$ in Fig. 7. Only those portions of the records near the major turbulent burst are shown. The large relative values of $\overline{u'u'}$ indicate that the interactions cannot be

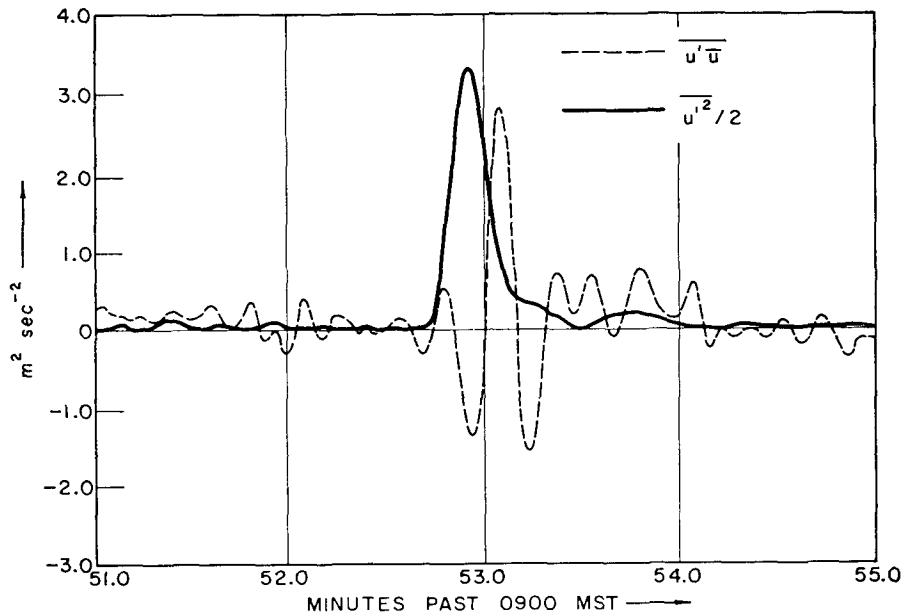


FIG. 7. Distribution of the interaction term ($\overline{u'u'}$) and the turbulent kinetic energy ($\overline{u'^2}/2$) in the vicinity of the rotor.

neglected in the computation of the instantaneous energy budget. It is of interest to note that $\overline{u'u}$ attains large values despite the spectral gap induced by the filtering scheme (Fig. 2c).

4. Summary and conclusions

It has been proposed that the physical causes of the intermittency of turbulence in the free atmosphere be investigated by separating the flow field into a mean and a deviation, not by the usual Reynolds averaging scheme, but by applying a set of numerical filters to the data. In this way one may take advantage of the ability of well-instrumented aircraft to sense significant changes over small distances. Some of the physical aspects of individual turbulent bursts can be deduced through the use of the concept of a filtered mean in the computation of the turbulent kinetic energy budget. For example, the location, intensity and dimensions of individual turbulent bursts can be specified in terms of the spatial distribution of turbulent kinetic energy. Also, the importance of various physical processes can be examined by considering the relative magnitudes of terms of the energy budget.

The greatest difficulty associated with using the proposed analytic technique is that the finite bandwidth over which the cutoff occurs in practical numerical filter applications does not allow the exact satisfaction of the Reynolds averaging rules. However, from the results of this study, it appears that most of the rules can be approximately satisfied by requiring that the low-pass (mean) and high-pass (deviation) filters do not overlap and that they have sharp cutoffs. Furthermore, unless a gap of specific width exists in the spectrum of the filtered variables at the scale of the filter cutoff, important interactions will arise between the mean and turbulent flows.

In the absence of some sort of a spectral gap, the selection of the scale at which separation should occur is arbitrary. Considering the speeds of modern aircraft and the scales of the phenomenon responsible for CAT, a separation scale of 1 km seems appropriate. However, it should be kept in mind that the proposed technique is perfectly general and can be applied to the study of other time- or space-dependent phenomena where a different separation scale may be desirable. For example, if an investigator wishes to separate undulance (wave motion) from classical turbulence, it would be advisable to select the cutoff frequency on the basis of the cross spectrum analysis of w and T (Busch, 1969).

Although several difficulties have been encountered in the present study, it has been demonstrated that the proposed technique is capable of illuminating some of the small-scale physical characteristics of intermittent turbulence in the free atmosphere. Computation of a space-dependent (filtered) mean appears to be a feasible approach to a phenomenon which varies greatly in space.

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