Moisture and Temperature Limits of the Equilibrium Evapotranspiration Model

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ABSTRACT

Energy balance measurements of evapotranspiration from a developing corn crop are compared with daily equilibrium evapotranspiration estimates to examine the accuracy of the model and the environmental conditions under which it can be applied. Equilibrium estimates compared closely (a standard error of 6%) with the measured values when the surface was moderately dry, a condition which applied to 14 of the 24 days of the experiment. The ratio of actual evapotranspiration to available energy and the Bowen ratio are used to establish moisture and temperature limits for the model. The success of the model was related to a typical diurnal pattern of the difference between actual and equilibrium evapotranspiration which reflects expected variations of moisture stress during daytime hours. The performance of the model was nearly independent of the physical condition of the surface and the height of the required air temperature measurement. An equation is presented which permits easy calculation of equilibrium evapotranspiration from air temperature, net radiation, and soil heat flux data.

1. Introduction

In recent years there has been a sustained effort to develop reliable models of evapotranspiration. Physical models of this process which incorporate both energy balance and aerodynamic principles are known generally as the "combination" method. Penman (1948) developed the first combination equation to calculate potential evapotranspiration. With suitable refinements his model accurately estimates hourly and daily water loss from vegetation with an unrestricted water supply but fails when the surface becomes dry (Davies and McCaughey, 1968). The Penman model has been used extensively because it requires few and simple measurements. Unfortunately, the models which have developed for dry surfaces do not possess these desirable features.

Monteith (1965) and Tanner and Fuchs (1968) have extended the combination method to estimate actual evapotranspiration. The Monteith model utilizes a "surface resistance" parameter which can be determined by five methods (Szeicz and Long, 1969). However, four of these require a prior determination of the evapotranspiration rate and the fifth, which requires measurements of individual leaf resistance, is clearly unsuitable for use on a broad scale. The Tanner and Fuchs model employs a measure of surface temperature. This can be adequately measured for a well-defined surface such as bare soil (Fuchs *et al.*, 1969) but is nearly impossible to define for a three-dimensional surface such as a vegetation canopy.

Both of these models precisely define the surface response during the evapotranspiration process, but they are not yet suitable for general application. An alternative is to find a model which approximates the surface response while remaining operational. The equilibrium evapotranspiration model offers such a possibility. The model originates from a simplified combination approach presented by Slatyer and McIlory (1961) and is appealing because it is simple in both theory and practice. However, with the exception of a preliminary study by Denmead and McIlroy (1970), it has never been rigorously tested. This paper describes an examination of the accuracy of the equilibrium model and attempts to define the environmental conditions under which it can be applied.

2. The equilibrium evapotranspiration model

The energy balance of a land surface can be written as

$$R_n = LE + H + G, \tag{1}$$

where R_n is net radiation, LE the latent heat flux, L the latent heat of vaporization of water, E the evapotranspiration rate, H the sensible heat flux, and G the soil heat flux. Slatyer and McIlroy (1961) and Monteith (1965) have shown that LE can be defined by the combination equation

$$LE = \frac{S(R_n - G)}{(S+\gamma)} + \frac{\rho C_p}{r_a}(D_s - D_0), \tag{2}$$

where S is the slope of the saturation vapor pressure versus temperature curve at the mean of the wet bulb temperatures at the surface and in the overlying air, γ the psychometric constant [0.66 mb(°C)$^{-1}$], ρ air density, C_p the specific heat of air at constant pressure, D_z and D_0 the wet bulb depressions in the overlying air and at the surface, respectively, and r_a the aerodynamic resistance to the diffusion of water vapor between the surface and height z.

Eq. (2) is impractical for general use because of the difficulty of measuring D_0, but it is instructive because it separates the basic energy sources. The first term on the right represents the net amount of radiant energy expended on evapotranspiration and the second the convective energy used from the atmopshere for that purpose. It is the convective term which is principally responsible for evapotranspiration differences between surfaces of different wetness. When a surface is wet or moist, $D_0=0$, and evapotranspiration proceeds at the potential rate. When the water supply to the surface is restricted, D_0 acquires a finite value and the actual evapotranspiration rate is less than the potential. Slatyer and McIlroy considered the special and apparently limited case when $D_z=D_0$, thereby eliminating the convective term. This reduces (2) to

$$LE=LE_{EQ}=S(R_n-G)/(S+\gamma), \tag{3}$$

in which E_{EQ}, the equilibrium evapotranspiration rate, can be calculated as a function of temperature and available radiant energy.

Interpretations of the physical meaning of (3) have varied. Monteith (1965) and Tanner and Fuchs (1968) have noted that it describes the evapotranspiration which would occur in a saturated atmosphere. This is the simplest case in which $D_z=D_0$ because both are equal to zero. Slatyer and McIlroy (1961) and Denmead and McIlroy (1970) suggested that (3) represents a lower limit to potential evapotranspiration. The former authors considered that the wet bulb depressions would be equal when the surface and the overlying air had reached a state of mutual adjustment with regard to moisture, and they suggested that the condition described by (3) should be referred to as "equilibrium" evapotranspiration. The position taken here is simply that the depressions may have finite values and may still be equal or nearly equal, in which case (3) will remain valid or stand as a good approximation of the actual evapotranspiration rate.

3. Experimental procedure

Control data for the experiment were provided by energy balance calculations of LE. This technique is based on (1) and consists of proportioning the available energy (R_n-G) into the components LE and H by measuring vertical temperature and humidity differences between two or more levels above the surface. Rearranging (1) and dividing by LE gives

$$LE=(R_n-G)/(1+H/LE)=(R_n-G)/(1+\beta), \tag{4}$$

where $\beta=H/LE$ is known as the Bowen ratio (Bowen, 1926). Both R_n and G are readily measured and it is possible to solve (4) by determining β as

$$\beta=\gamma(\Delta T/\Delta e), \tag{5}$$

where the difference in air temperature, ΔT, and the difference in vapor pressure, Δe, are measured between the same two heights. Substituting (5) into (4) gives

$$LE=(R_n-G)/(1+\gamma\Delta T/\Delta e), \tag{6}$$

in which ΔT can be measured directly and Δe can be calculated as

$$\Delta e=(S'+\gamma)\Delta T_W-\gamma\Delta T, \tag{7}$$

where S' is the slope of the saturation vapor pressure vs temperature curves at the mean of the wet bulb temperatures at the two levels, and ΔT_W is the difference in wet bulb temperatures at the two levels.

The required measurements to solve (3), (6) and (7) were obtained on 24 days during July 1969 at the Simcoe Horticultural Experiment Station in Southern Ontario. Observations were taken on a flat rectangular (210 m $\times$120 m) plot of sweet corn growing in sandy loam soil. Rows were 1 m apart and individual plants were separated by an average 15 cm in each row. The plants were 20 cm high when measurements were begun on 1 July and had reached a height of 105 cm on 25 July when measurements were terminated. Net radiation was measured with a shielded net radiometer (Swissteco, Type S–1) mounted 1 m above the crop surface. Soil heat flux was calculated as the sum of the measured flux at a depth of 5 cm and the calculated flux divergence between 5 cm and the surface using a procedure similar to that described by Fuchs and Tanner (1968). Five-junction thermopile units were used to measure ΔT and D at 10, 35 and 60 cm above the surface. The components of the measurement system have been described by Rouse and Wilson (1972). Bowen ratios were calculated hourly for the three air layers: 10–35, 35–60 and 10–60 cm. This procedure ensured a continuous record in the event of a technical failure at one level. Since the three calculated values of the latent heat flux usually differed by less than 0.03 cal cm^{-2} min^{-1}, daily totals of E were calculated as the sum of the median hourly values between 0500 and 2000 local time.

Since it is nearly impossible to ascribe a wet bulb temperature to a vegetation canopy, it is necessary to approximate the true value of S in (3). A reasonable and convenient approximation is provided by using the dry bulb temperature of the overlying air. Error introduced by this approximation will normally be small since the value of S is used to calculate $S/(S+\gamma)$ in (3), and the latter factor varies relatively slowly with

TABLE 1. Summary of daily evapotranspiration data for July 1969.

Date	R_n (mm)	G (mm)	E (mm)	E_{EQ} (mm)	$E/(R_n-G)$	$E_{EQ}/(R_n-G)$	T* (°C)	D* (°C)
1	5.42	0.64	3.41	3.26	0.71	0.68	18	6.0
2	3.91	0.42	2.57	2.35	0.73	0.67	18	4.5
3	5.43	0.68	3.13	3.23	0.66	0.68	19	3.2
5	5.52	0.66	4.49	3.54	0.92	0.73	22	3.1
6	6.61	0.65	3.38	3.87	0.57	0.65	16	4.3
7	6.00	0.63	2.78	3.65	0.52	0.68	18	6.1
8	5.53	0.66	2'44	3.35	0.50	0.69	18	5.8
9	5.51	0.85	3.16	3.34	0.68	0.72	22	6.5
10	4.05	0.41	3.75	0.267	1.03	0.73	23	3.8
11	4.88	0.73	3.40	3.13	0.82	0.75	24	5.3
12	5.31	0'55	3.79	3.52	0.80	0.74	25	6.5
13	5,52	0.56	3.59	3.64	0.72	0.73	25	6.5
14	5.89	0.61	3.77	3.90	0.71	0.74	25	6.3
15	6.04	0.62	3.83	4.07	0.71	0.75	26	6.7
16	5.41	0.48	3.62	3.81	0.74	0.77	27	5.2
17	3.32	0.19	2.64	2.35	0.84	0.75	26	3.0
18	2.29	−0.13	2.72	1.70	0.89	0.70	21	2.5
19	3.35	0.01	2.66	2.31	0.80	0.69	19	1.9
20	5.32	0.38	3.60	3.55	0.73	0.72	22	3.3
21	5.14	0.19	3.73	3.61	0.75	0.73	22	3.4
22	5,24	0.21	4.34	3.65	0.86	0.73	22	4.7
23	6.08	0.35	4.82	4.25	0.84	0.74	23	6.0
24	4.14	0.20	3.02	2.85	0.77	0.72	22	2.5
25	3.99	0.09	3.66	2.88	0.95	0.74	23	3.9

* Arithmetic mean of all hourly values.

temperature. Denmead and McIlroy (1970) used the dry bulb temperature of the air at 85 cm to calculate S and considered that this produced a small underestimate of the true value of $S/(S+\gamma)$ during unstable atmospheric conditions. In the present experiment the mean hourly dry bulb temperature at 60 cm has been used to calculate S using a solution suggested by Dilley (1968) which gives

$$S=[25{,}029/(T+237.30)^2][\exp(17.269T/237.30)], \quad (8)$$

where T is the dry-bulb temperature (°C) at 60 cm above the crop. Hourly values of E_{EQ} were then calculated from (3) and were summed to obtain daily totals.

4. Results

a. Performance in relation to surface moisture conditions

A preliminary analysis of all daily values of E and E_{EQ} (Table 1) showed a considerable scatter in the data, clearly indicating that the equilibrium model cannot be used indiscriminately. However, the equilibrium estimates were remarkably accurate on 14 of the 24 days, and further examination of the data indicated that the performance of the model was dependent on the prevailing surface moisture condition. It was possible to divide the data into three groups as shown in Fig. 1.

In the first group the equilibrium estimates were much smaller than the measured values. This applied to days when rain occurred, days following a night rain, and days with short periods of reversed sensible heat flux. In all of these situations the surface was effectively wet.

A second group of data was composed of three days (6–8 July) when the equilibrium estimates were much too large. Several factors indicate that the rate of water supply to the surface was severely restricted during this period. All of the days were clear and sunny and the crop canopy was not yet closed. The daily Bowen ratios all exceeded a value of 0.7 (Fig. 2) and gravimetric soil moisture measurements taken in an auxiliary program (Rouse and Wilson, 1972) showed that these were the driest soil conditions in the first nine days of July (Fig. 3). Moreover the proportional difference between LE and LE_{EQ} steadily increased from 6 to 8 July, corresponding to an expected decrease in the water supply

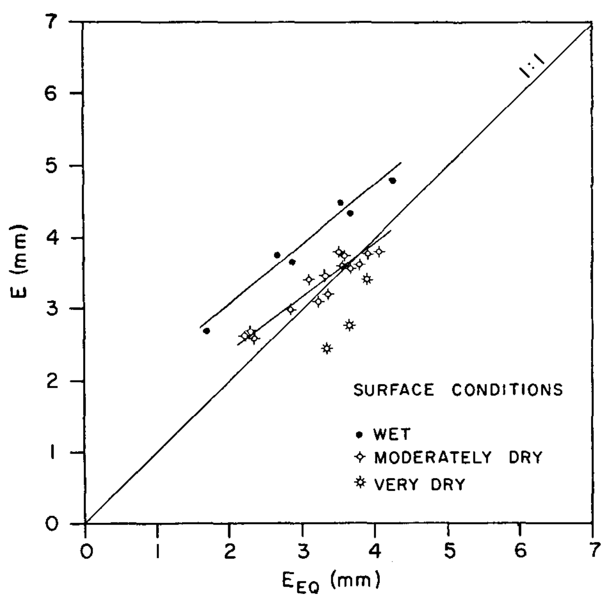

FIG. 1. Relationship between daily values of E and E_{EQ}.

rate to the evaporating surfaces. This type of deviation from equilibrium evapotranspiration conditions did not occur again after 8 July, despite similar weather conditions and even drier soil. However, the growth rate of the crop increased shortly after this very dry period and the plants began to shade most of the bare soil. Consequently, it appears that the inadequate performance of the model was caused by a severe moisture stress which resulted from the presence of unshaded bare soil.

The remaining data applied to moderately dry conditions and in this case the points are clustered around the $LE=LE_{EQ}$ line in Fig. 1. The standard error of the $LE=LE_{EQ}$ relationship was only 0.21 mm day^{-1} which represents 6% of the average evapotranspiration for these days.

The results of regression and correlation analyses for various groups of the data are shown in Table 2. The equilibrium estimates were highly correlated with the measured values for both moderately dry and wet days, indicating the general importance of the radiation term in (2). It is also noteworthy that there were positive intercept values for both of these moisture classes. A positive value is to be expected for wet surfaces since the equilibrium model neglects the atmospheric term in (2). In the case of moderately dry surfaces it indicates that the model tends to produce underestimates at low evapotranspiration rates and overestimates at high rates.

The excellent performance of the equilibrium model for predicting daily totals for moderately dry days is a reflection of good agreement between hourly values, and it can be related to a typical diurnal pattern of the differences between LE and LE_{EQ}. The diurnal trends of these parameters on 12 and 15 July are shown in Fig. 4. These two days present contrasting situations since the measured total slightly exceeded the equilibrium total on 12 July, whereas on 15 July the situation was reversed. The daily patterns of the differences are, however, typical of this group. On sunny days LE was usually greater than LE_{EQ} until mid-morning when there was a reversal that lasted for a varying length of time. By mid- to late-afternoon LE would again be greater than LE_{EQ} or else the two values would be nearly equal. On cloudy days the two curves were always close throughout the day.

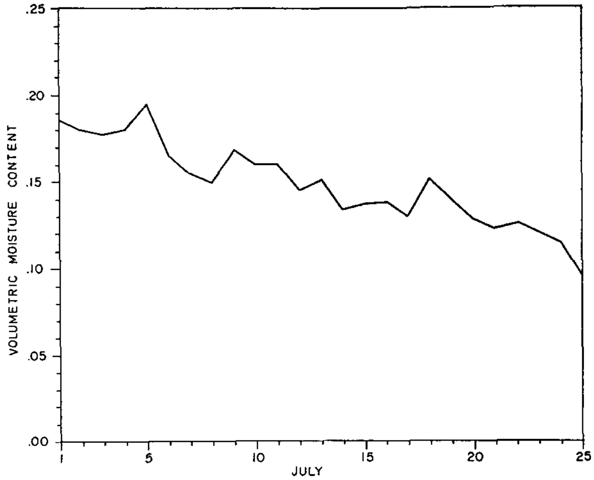

Fig. 3. Variation of soil moisture content in the 0–25 cm surface layer.

These patterns are apparently related to the rate of water supply to the evaporating surfaces. Under low radiation conditions the rate of water supply, augmented by dew in the mornings, was adequate to maintain an evaporative flux equal to or greater than the equilibrium rate. When there was a demand for a large flow rate during peak radiation conditions, the supply became limiting and caused the reversal at mid-day. The peak rate of water supply apparently decreased from 12 to 15 July because the reversal persisted longer at the later date. This is to be expected since the last previous rainfall occurred on 10 July.

The key to the success of the equilibrium model was the mid-day reversal in which LE_{EQ} was greater than LE. The overestimates produced during the reversal compensated for the underestimates produced in the early morning and late afternoon. This resulted in better prediction for daily rather than hourly periods on moderately dry days. When the surface was wet after a rain, the reversal either did not occur or did not last long enough to produce satisfactory daily estimates. With a very dry surface the reversal persisted throughout most of the afternoon so that the daily equilibrium estimate was much too large.

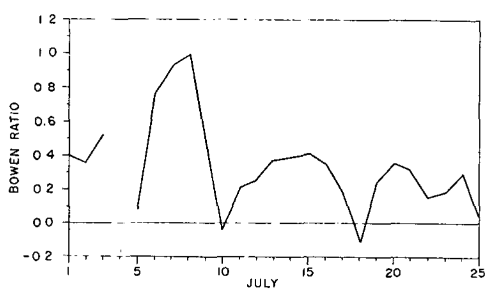

Fig. 2. Daily Bowen ratio during July 1969.

Table 2. Regression and correlation constants for relationships between E and E_{EQ} of the form $E=a+bE_{EQ}$ (mm day^{-1}).

Description	a	b	Correlation coefficient	Standard error
All days	1.356	0.634	0.65	0.46
Wet surface	1.382	0.829	0.99	0.12
All except wet	1.143	0.635	0.74	0.31
Moderately dry	0.920	0.738	0.95	0.13

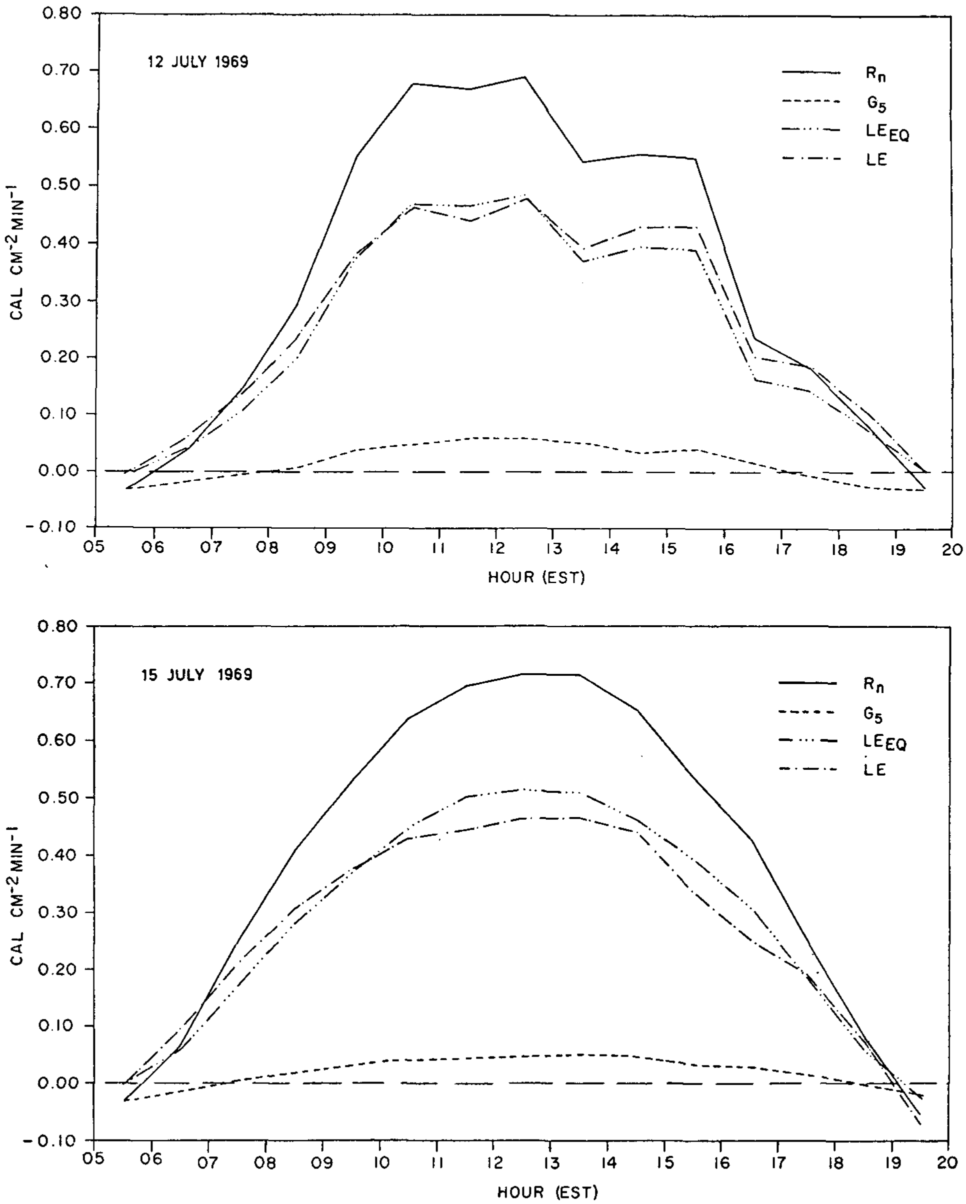

FIG. 4. Energy balance components and equilibrium evapotranspiration on 12 and 15 July 1969.

b. Moisture and temperature limits

From the previous discussion it is clear that the equilibrium model represents a response to certain particular environmental conditions. The two factors to consider in this regard are the surface moisture condition and the air temperature.

An indirect measure of the surface moisture condition is the ratio $LE/(R_n-G)$. When moisture stress is low the proportion of available energy used for evapotranspiration will be high. For example, Davies and McCaughey (1968) found that a daily average of 86% of the available energy was used to promote potential evapotranspiration from perennial rye grass at Simcoe. This value can be used for comparison in the present study.

Fig. 5 shows a plot of the ratio E/E_{EQ} vs $LE/(R_n-G)$, using daily totals, with the data separated according to the moisture conditions specified earlier. The lowest value of the ratio $LE/(R_n-G)$ during wet conditions was 0.84 which corresponds closely with the value of Davies and McCaughey. On the three very dry days the the ratio $LE/(R_n-G)$ did not exceed 0.57. For both wet and very dry conditions E_{EQ} differed from E by at least 12%. On moderately dry days $LE/(R_n-G)$ ranged between 0.66 and 0.84, but values greater than 0.80 applied to days when (R_n-G) was relatively small.

Consequently, one might assume that the values of 0.65 and 0.80 define the limits within which the model was generally applicable. Within this range the daily equilibrium estimates had a maximum error of only 10%.

These limits can also be expressed in terms of the Bowen ratio. It is possible to write (3) as

$$LE_{EQ}=\frac{R_n-G}{1+\gamma/S}, \tag{9}$$

which is equivalent to (4) with $\beta=\gamma/S$. Thus, the equilibrium model is essentially an energy balance technique, with the Bowen ratio determined as a function of air temperature. Since the equilibrium model applied within definite $LE/(R_n-G)$ limits, it follows from (9) that the model must apply within a certain range of β values. Substituting $LE(R_n-G)$ values of 0.65 and 0.80 into (9) gives β values of 0.54 and 0.25, respectively.

Further re-arrangement of (3) gives

$$\frac{LE_{EQ}}{R_n-G}=\frac{S}{S+\gamma}, \tag{10}$$

which shows that the proportion of the available energy used for equilibrium evapotranspiration is determined by the values of $S/(S+\gamma)$ and, therefore, by air temperature. The variation of this factor with temperature is shown in Fig. 6 from which three facts emerge. First, temperature values of 17 and 32C correspond to $LE/(R_n-G)$ values of 0.65 and 0.80, respectively, and these form the thermal limits to the model's applicability within the specified moisture range. Second, changes in $S/(S+\gamma)$ are conservative in the 17–32C temperature range; a temperature change of 1C alters the ratio by

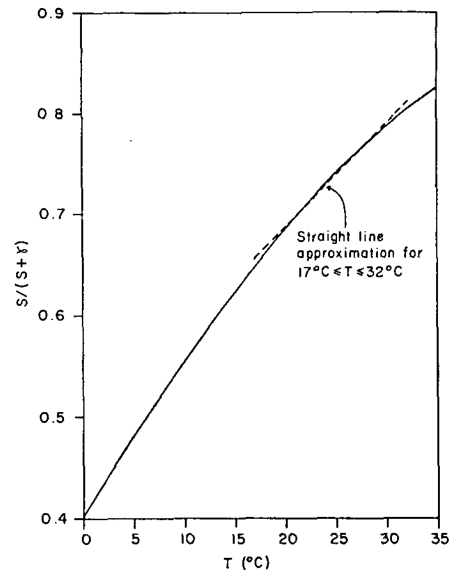

FIG. 6. Variation of $S/(S+\gamma)$ with temperature.

only 0.01. Therefore, although the proportionality factor is temperature-dependent and air temperature changes with height, the actual height of the temperature measurement is not critical. Finally, it is apparent from Fig. 6 that the value of $S/(S+\gamma)$ can be calculated with good accuracy from a straight line approximation to the curve. In the 17–32C range the equation of the straight line is

$$S/(S+\gamma)=0.48+0.010T, \tag{11}$$

which has a standard error of less than 0.01. Substituting (11) into (3) gives

$$LE_{EQ}=(0.48+0.010\,T)(R_n-G), \tag{12}$$

which permits quick and accurate estimation of the equilibrium evapotranspiration rate.

The effectiveness of the derived limits is apparent from the plot of hourly values of LE and LE_{EQ} shown in Fig. 7. Taken as a whole the data are quite widely scattered. However, most of the scatter disappears when only the data from the moderately dry days is selected. It is also apparent that the relationship between LE and LE_{EQ} is curvilinear. The two values generally agree when LE is between about 0.3 and 0.4 cal cm^{-2} min^{-1}, but the model's estimates are consistently too small at lower evapotranspiration rates and too large at higher rates. The same type of relationship was found by Denmead and McIlroy (1970) who reported general agreement up to about 0.37 cal cm^{-2} min^{-1}. This kind of relationship is to be expected for moderately dry surfaces since $LE/(R_n-G)$ typically reaches a daily minimum when LE and $S/(S+\gamma)$ achieve maximum values during mid-day. The fact that the data points cross the $LE=LE_{EQ}$ line means that $(D-D_0)$ was positive when LE was small in the

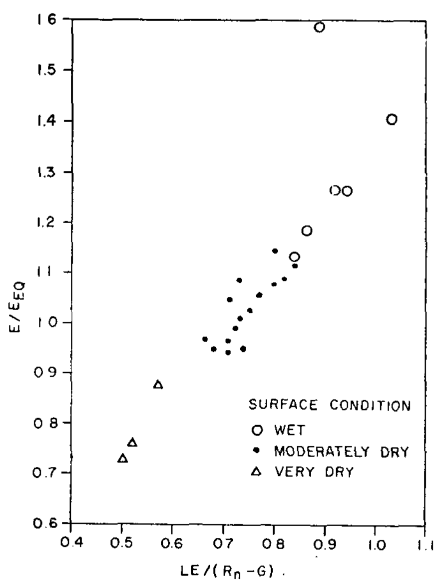

FIG. 5. Relationship between E/E_{EQ} and $LE/(R_n-G)$ for daily periods.

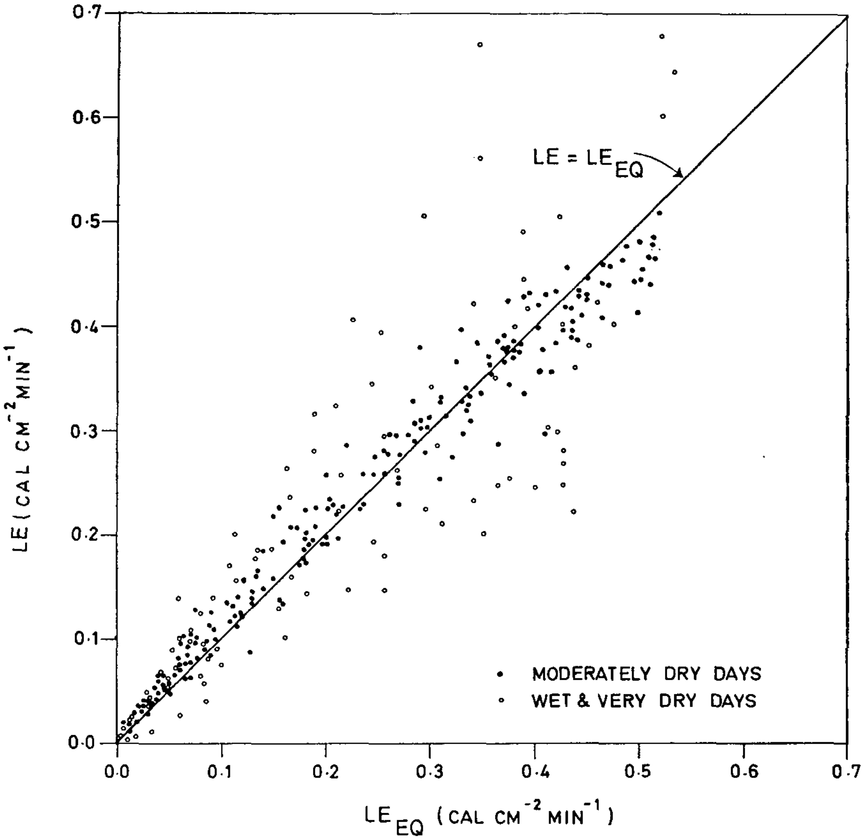

FIG. 7. Relationship between hourly values of LE and LE_{EQ}.

morning and evening hours, and it became negative during water stress conditions at mid-day.

5. Conclusions

This study shows that the equilibrium evapotranspiration model can be applied when evapotranspiration proceeds at less than the potential rate and when the water supply to the vegetation is not severely restricted. The exceptional performance of the model in this study certainly suggests that it should be tested further. In particular, the moisture and temperature limits should be examined more closely for different surface types and in different weather conditions. Since the model performed well throughout a major growth phase of a corn crop, its response seems to be nearly independent of the physical condition of the surface. However, the model appears to be more reliable for a vegetated surface than for bare soil because a severe water stress condition appears sooner when the soil is not shaded. The model is well suited for general application since it requires only a few simple measurements and calculations, is applicable over a fairly wide range of moisture and temperature conditions, and is insensitive to the height of the temperature measurement.

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