Rise and Condensation of Large Cooling Tower Plumes[1]

Steven R. Hanna

Atmospheric Turbulence and Diffusion Laboratory, NOAA, Oak Ridge, Tenn. 37830

(Manuscript received 24 January 1972, in revised form 24 April 1972)

ABSTRACT

Formulas for the variation with height of the buoyancy, volume, and water vapor fluxes from large wet cooling towers are derived. The simple formulas developed by Briggs are suggested for estimating plume rise, if the possibility of the release of latent heat is accounted for in the definition of the initial buoyancy flux. The probability of whether condensation will occur is sensitive to small variations in moisture content and temperature of the environment. Verification of the theory is hampered by the scarcity of adequate measurements of cooling tower plumes.

1. Introduction

Large wet cooling towers are being constructed at an increasing rate for use at fossil or nuclear power plants. Hot water is dripped through these towers and evaporated in the upward moving air, which is driven either naturally by density differences or mechanically by fans [see the review article in *Industrial Water Engineering* (1970)]. Some of the natural draft towers are 100 m high with top openings 30 m or larger in radius and discharge from 100 to 1000 MW to the atmosphere. Water is evaporated at the rate of about four tons per hour per megawatt. Over 70% of the energy flux from these towers is in the form of latent heat, which is released to the atmosphere only when condensation occurs. Some of the general effects of cooling tower plumes on the atmosphere were discussed previously (Hanna, 1971a, b; Hanna and Swisher, 1971). In this paper, analytical methods of determining the final plume rise and the levels of initial condensation of plumes from cooling towers are outlined.

2. Background

According to Briggs (1969) the final rise H of dry plumes dominated by buoyancy in a stable atmosphere can be estimated by the formulas:

$$\text{Calm:} \quad H=5.0F_o^{1/4}s^{-3/8} \tag{1}$$

$$\text{Windy:} \quad H=2.9(F_o/Us)^{1/3} \tag{2}$$

where U is the wind speed, s the stability parameter $(g/T_p)(\partial\theta_e/\partial z)$, and F_o the initial flux of buoyancy $(g/T_p)w_oR_o^2(T_{po}-T_{eo})$. The variables g, T, θ, w_o and R_o are the acceleration of gravity, temperature, potential temperature, and initial vertical speed and radius of the plume. In all that follows, the subscripts p, e, o, s represent plume, environment, initial, and saturation variables, respectively. For simplicity, the factor π is neglected in flux calculations. These equations can be used as a starting point for the derivation of moist plume rise equations.

Analyses of moist plume rise were made by Morton (1957) for calm conditions and Csanady (1971) and Wigley and Slawson (1971) for windy conditions. Morton extended the model for dry rise proposed by Morton *et al.* (1956) in order to account for the continuity of water and the addition of latent heat during water phase changes. He found that in a uniform atmosphere (vertical gradients of environment specific humidity q_e and potential temperature θ_e equal to zero) all plumes will reach levels where condensation is possible. Because of the adiabatic decrease of temperature, the saturation specific humidity is eventually reached. During stable conditions the development of a convective cloud is found to be highly sensitive to environment specific humidity. His theory is adequate for estimating the level of condensation in a calm environment, although it is necessary to go through many detailed calculations for each particular case. Because point sources are assumed, Morton's theory does not necessarily apply at small heights above large source openings. The initial cross-sectional area of some cooling tower plumes is over 2000 m².

The work of Csanady and of Wigley and Slawson applies only to depths of the atmosphere less than about 100 m because, in order to simplify their equations, they neglected the variation of environment temperature T_e and saturation specific humidity q_{ps} with height. Consequently, they concluded that condensation occurs either near the stack or not at all, in conflict with Morton's (1957) finding that condensation

[1] Parts of this paper were summarized at the Air Pollution, Turbulence, and Diffusion Symposium, Las Cruces, N. M., 7–10 December 1971.

will occur in a plume rising in a uniform environment. In this paper the theory of Csanady and of Wigley and Slawson is extended to explain the behavior of bent-over vapor plumes at heights $\gtrsim 100$ m above the stack.

3. Basic equations governing moist plume rise

The theory of moist plume rise given below is based on work by Morton (1957) and Briggs (1969), who derive equations for the variation with height of the fluxes of volume (V), momentum (wV), buoyancy (bV), water vapor specific humidity (qV), and liquid water mixing ratio (σV). The volume flux V is assumed to equal wR^2 during calm conditions and UR^2 during windy conditions. The initial flux V_o equals $w_oR_o^2$ in both cases. The buoyancy parameter b is defined as $(g/T_p)(T_p-T_e)$. Thus, bV equals F. The liquid water mixing ratio σ is defined as mass of liquid water per unit mass of air. For example, if the liquid flux σV is multiplied by π times the plume air density ρ_p, then the mass of liquid water passing through a given plume cross section per unit time is obtained.

The set of governing equations for moist plume rise is written first for an unsaturated plume, and then for a saturated plume. Steady-state conditions are assumed.

Unsaturated plume ($q_p<q_{ps}$, $\sigma=0$)

$$\frac{\partial}{\partial z}(wV)=\frac{V}{w}[b+0.61g(q_p-q_e)], \tag{3a}$$

$$\frac{\partial}{\partial z}bV=-sV, \tag{4a}$$

$$\frac{\partial}{\partial z}(q_p-q_e)V=-V\frac{\partial q_e}{\partial z}. \tag{5a}$$

Saturated plume ($q_p=q_{ps}$)

$$\frac{\partial}{\partial z}(wV)=\frac{V}{w}[b+0.61g(q_{ps}-q_e)-g\sigma], \tag{3b}$$

$$\frac{\partial}{\partial z}(bV)=-sV-\frac{gLV}{c_pT_p}\frac{\partial q_{ps}}{\partial z}-\frac{gL}{c_pT_p}(q_{ps}-q_e)\frac{\partial V}{\partial z}, \tag{4b}$$

$$\frac{\partial}{\partial z}(\sigma V)=-V\frac{\partial}{\partial z}q_{ps}-\text{rainout}-(q_{ps}-q_e)\frac{\partial V}{\partial z}. \tag{5b}$$

The constants L and c_p are the latent heat required to evaporate one gram of water and the specific heat of air at constant pressure, respectively.

Equations for the variation of the radius with height follow from the entrainment assumption by Briggs (1969):

$$\left.\begin{aligned}\text{Calm:}\quad &\frac{\partial R}{\partial z}=0.2-\frac{Rb}{2w^2}\\ \text{Windy:}\quad &\frac{\partial R}{\partial z}=0.5\end{aligned}\right\}. \tag{6}$$

In addition, expressions for rainout and the variation of saturation specific humidity q_{ps} with height and temperature are required. In the numerical model for moist plume rise described by Hanna (1971a), the empirical formula for $q_{ps}(T,z)$ suggested by the World Meteorological Organization (1966) is used. In the analytical model mentioned in Section 5, the Clausius-Clapeyron and hydrostatic equations are used to derive an analytical expression for $q_{ps}(T,z)$. The empirical formula agrees with observations better and is more easily used in a computer program, but is too long and unwieldy for analytical use. Rainout is approximated in the numerical model by an empirical expression suggested by Simpson and Wiggert (1970). A distinction must be made between cloud water and rainwater in order to evaluate this expression, thus adding another equation to the set. For brevity, these highly empirical expressions will not be given here. Hanna (1971a) discusses them in detail. The accuracy of the results obtained using any numerical cloud model is uncertain, for it is not known whether the parameterizations based on observations of cumulus clouds apply to plume clouds.

Because of the nonlinearity of the equations and the difficulties introduced by the water phase change terms, analytical solutions to the general equations cannot be obtained. In the following sections, some special problems associated with moist plume rise will be treated. Enhancement of plume rise due to the release of latent heat is discussed. Criteria for determining whether condensation will occur are derived and three special cases are analyzed in detail.

4. Methods of estimating plume rise from cooling towers

If there is excess water vapor in the plume, or if some of the excess vapor in the plume condenses, the buoyancy flux in the plume can be greater than the flux due only to the temperature excess at the stack opening. The total possible initial buoyancy flux to be used in Eqs. (1) and (2) is then given by

$$F_o=gw_oR_o^2\left[\frac{T_{vpo}-T_{veo}}{T_{vpo}}+(q_{po}-q_{eo})\frac{L}{c_pT}\right], \tag{7}$$

where T_v is the virtual temperature, defined as $(T+0.61q)$, and it is assumed that all condensed water falls out of the plume. If the condensed water stayed in the plume, it would be necessary to account for the decrease in buoyancy due to the weight of the water drops. Table 1 lists the additional relative buoyancy resulting from the latent heat term in (7). Typical temperatures and humidities occurring during cooling tower operation are assumed. The plume is assumed to be initially saturated in all of the calculations in this paper.

It is clear from Table 1 that if all the initial excess vapor in the plume were to condense, the buoyant flux would increase by several hundred percent. But because the flux F_o is raised to the $\frac{1}{4}$ or $\frac{1}{3}$ power in Eqs. (1) or (2), a sixfold increase in F_0 increases the plume rise in calm conditions by only 57%, or in windy conditions by only 82%.

The estimates of plume rise made using (1) or (2) were compared with the plume rise obtained using a numerical model of cloud growth (see Hanna, 1971a), based on the work by Weinstein (1970) and Simpson and Wiggert (1970). The model includes equations for the vertical change of vertical speed, plume radius, temperature, water vapor, cloud water and rainwater. These are integrated numerically, assuming steady-state conditions and that each variable is constant over a cross section of the plume at any height. An initial radius R_o of 30 m, vertical speed w_o of 5 m sec^{-1}, the temperature and humidities in Table 1, and environment temperature gradients of 0 and 1K(100 m)$^{-1}$ are used.

For environment relative humidities of 60%, no cloud develops in the numerical model in the cases considered. Thus, the term in (7) involving latent heat is not important. The numerical estimates of plume rise are within 20% of the estimates using Eqs. (1) or (2). For environment relative humidities of 100% a cloud always forms in the upper layers of the numerical model. In this case, the numerical results are within 40% of the simple predictions of (1) or (2), even though the latent heat term is included in (1) in the highly simplified manner suggested in Eq. (7). However, there are many reasons for being careful in the application of numerical cloud growth models to cooling tower plumes. A major problem is that we possess very little information with respect to cloud microphysics in the mouth of the cooling tower or in the visible plume. Furthermore, to the author's knowledge, there are currently no observations of cooling tower plume rise available to validate these formulas. Data that are needed include initial buoyancy flux, initial cloud microphysics, vertical profiles of environment temperature and humidity, and observations of final plume rise.

5. Methods of estimating levels of condensation in plumes from cooling towers

Two general problems will be considered. The first involves whether saturation will occur at heights where the differences between the saturation specific humidities q_{ps} and q_{es} can be neglected. The second involves the formation of a cloud or the persistence of initial cloud water at small heights above the tower, where the difference between plume and environment saturation specific humidities is significant. The general equations for treating these problems will be discussed and some particular examples worked out.

TABLE I. Buoyancy due to latent heat release, divided by the buoyancy due to initial temperature differences. Typical cooling tower plumes and environmental initial temperature and environment relative humidities are considered. The plume is initially saturated.

T_{po} (°K)	T_{eo} (°K)	$\frac{L(q_{po}-q_{eo})}{c_p(T_{po}-T_{eo})}$ 100% relative humidity	60% relative humidity
305	275	2.3	2.5
315	285	3.9	4.2
305	285	2.9	3.4
315	295	4.6	5.7
305	295	3.6	5.4

Qualitatively, condensation will occur if there is sufficient water emitted from the cooling tower to saturate the initial volume flux plus the flux of air entrained into the plume. The initial flux of water Q_o (in mass per unit time) in the plume is given by

$$Q_o=\rho_o(q_{po}+\sigma_{po})V_o, \tag{8}$$

where ρ_o, q_{po}, σ_{po} and V_o are the initial air density, specific humidity (assumed saturated), cloud water mixing ratio, and volume flux ($w_oR_o^2$), respectively. Note again that π is factored out of all flux calculations. For simplicity, it is assumed that supersaturation never occurs. The flux of water Q_s at level z that is necessary to saturate the plume, which now includes both initial and entrained air, is given by

$$Q_s(z)=\rho(z)\{V_oq_{ps}(z)+[V(z)-V_o][q_{ps}(z)-\bar{q}_e(z)]\}. \tag{9}$$

When the fluxes Q_o and $Q_s(z)$ are equal, saturation is achieved. The average specific humidity $\bar{q}_e(z)$ of the air at level z which has been entrained into the plume during its rise to this level is defined by

$$\bar{q}_e(z)=\left\{\int_0^z q_e(\eta)[\partial V/\partial\eta]d\eta\right\}\Big/[V(z)-V_o]. \tag{10}$$

Because the specific humidity q_e generally decreases with height, the average value $\bar{q}_e$ is usually greater than q_e at any height. Consequently, it is possible for Q_s to drop below zero as the plume temperature approaches the environment temperature. Plumes that are dry initially will thus condense near the level of natural cloud formation in the atmosphere, if they possess sufficient buoyancy to rise to this level.

It is helpful to normalize the water fluxes by the initial water flux. Thus, we define the dimensionless flux A as the fraction of the initial water flux Q_o that is condensed at any level, i.e.,

$$A=\frac{Q_o-Q_s(z)}{Q_o}=1-\frac{\rho(z)}{\rho_o}\frac{V(z)}{V_o}\frac{[q_{ps}(z)-(1-V_o/V)\bar{q}_e(z)]}{(q_{po}+\sigma_{po})}. \quad (11)$$

If A is calculated to be less than zero, no condensation has occurred, while if A is greater than one, the flux of cloud water in the plume is greater than the flux of initial water. The flux of cloud water, $\rho\sigma V$, in the plume at any height z is thus equal to

$$\rho(z)V(z)\sigma(z)=AQ_o, \quad (12)$$

subject to the condition $Q_s/Q_o<1$.

The saturation specific humidity q_{ps} of the plume is a function of the plume temperature T_p and the height. Integration of the Clausius-Clapeyron equation and use of the hydrostatic and ideal gas laws yields the following approximate equation for the saturation specific humidity:

$$q_{ps}=q_{eso}\exp(+gz/R_d\bar{T})\exp\left[\frac{L}{R_v\bar{T}^2}(T_e-T_{eo})\right]\times\exp\left[\frac{L}{R_v\bar{T}^2}(T_p-T_e)\right], \quad (13)$$

where R_d and R_v are the gas constants for dry air and water vapor, respectively, and $\bar{T}$ is the average temperature through the depth of the plume. The exponent is broken in this way so that the second exponential term is a function only of the vertical variation of environment temperature and the third term is a function of the plume parameters. In addition we have used the approximation

$$\frac{\rho(z)}{\rho_o}=\exp(-gz/R_d\bar{T}). \quad (14)$$

The variation with height of the dimensionless temperature difference can be obtained through the definition

$$\frac{T_p-T_e}{T_{po}-T_{eo}}=\frac{FV_o}{F_oV}. \quad (15)$$

The vertical variation of the relative volume flux V/V_o and relative buoyancy flux F/F_o can be calculated from (3a), (4a) and the following approximations to Eq. (6):

$$\text{Calm:}\quad R=R_o+0.2z \quad (16)$$

$$\text{Windy:}\quad R=R_o\left(\frac{w_o}{U}\right)^{\frac{1}{2}}+0.5z \quad (17)$$

The factor (w_o/U) is necessary so that the volume flux UR^2 at the initial height equals the initial volume flux $w_oR_o^2$. The relative volume flux for the windy case is then

$$\text{Windy:}\quad \frac{V}{V_o}=\left[1+0.5\frac{z}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^2. \quad (18)$$

The ratio V/V_o is plotted as a function of z/R_o for several values of U/w_o in Fig. 1. Integration of (4a) yields the following approximate formula for the variation of the buoyancy flux with height:

$$\text{Windy:}\quad \frac{F(z)}{F_o}=1-\frac{s}{b_o}\left(\frac{w_o}{U}\right)^{\frac{1}{2}}\frac{R_o}{1.5}\times\left\{\left[1+0.5\frac{z}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^3-1\right\}. \quad (19)$$

From (18) and (19) it can be seen that for constant environment conditions (U and s constant) and constant initial volume flux $w_oR_o^2$ and buoyancy flux F_o, the ratios V/V_o and F/F_o are independent of the initial radius R_o of the source.

Integration of Eq. (3a), neglecting water vapor and using assumption (17), yields the relative volume flux for the calm case:

$$\text{Calm:}\quad \frac{V}{V_o}=\left(1+0.2\frac{z}{R_o}\right)\times\left\{1+\frac{15}{8\mathrm{Fr}_o}\left[\left(1+0.2\frac{z}{R_o}\right)^2-1\right]\right\}^{\frac{1}{2}}, \quad (20)$$

where Fr_o is the initial Froude number $w_o^2/2b_oR_o$. Solutions to Eq. (20) for various values of Fr_o are also plotted on Fig. 1. For cooling tower plumes with the characteristics described earlier, the ratio V/V_o increases with height at a greater rate for windy conditions than for calm conditions if the air speed is greater than about 1 m sec^{-1}. For simplicity, (20) can be closely approximately by

$$\text{Calm:}\quad \frac{V}{V_o}=\left[1+0.2\left(\frac{15}{8\mathrm{Fr}_o}\right)^{1/5}\frac{z}{R_o}\right]^{5/3}. \quad (21)$$

Using (21), Eq. (4a) can now be integrated, yielding

$$\text{Calm:}\quad \frac{F}{F_o}=1-\frac{15}{8}\frac{sR_o}{b_o}\left(\frac{15}{8\mathrm{Fr}_o}\right)^{-1/5}\times\left\{\left[1+0.2\left(\frac{15}{8\mathrm{Fr}_o}\right)^{1/5}\frac{z}{R_o}\right]^{8/3}-1\right\}. \quad (22)$$

It is interesting to note that the length $\mathrm{Fr}_o^{1/5}R_o$, or equivalently $(V_o^3/\mathrm{F}_o)^{1/5}$, scales the lengths z and b_o/s in Eqs. (21) and (22). This is an important scaling length for the analysis of calm plume rise.

The preceding equations can be substituted into (11) in order to estimate the fraction A of the initial total water flux Q_o that has condensed at any level. For any given set of initial and environment conditions, the fraction A is a function only of height. As mentioned in Section 2, simplified solutions for calm conditions at great heights were given by Morton (1957), and for windy conditions close to the tower by Csanady (1971), and Wigley and Slawson (1971). Solutions for windy conditions at great heights and calm conditions close to the tower are discussed next.

a. Specific case of a well-mixed environment at great heights

The simplest case for analysis occurs when the environment is well-mixed, i.e., for windy conditions when the potential temperature θ_e and mixing ratio q_e do not vary with height. In this case the average mixing ratio $\bar{q}_e(z)$ equals q_{eo}, the surface value. Furthermore, it develops that at heights $\gtrsim 200$ m above the tower the temperature difference (T_p-T_e) can be neglected. The following approximation for the saturation mixing ratio then applies:

$$q_{ps}(z)=q_{eos}\exp(-4.5gz/R_d\bar{T}),\ z>200\text{ m}. \tag{23}$$

For simplicity we let the scaling height of the atmosphere, $R_d\bar{T}/g$, equal 8000 m and the initial liquid water content, σ_o, equal zero. From (11), we obtain the following analytical expression for the fraction A of the initial water vapor flux that is condensed at level z:

$$A(z)=1-\exp(-z/8000\text{m})\left[1+0.5\frac{z}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^2 \times\left\{\frac{\exp(-z/1800\text{m})-\dfrac{q_{eo}}{q_{eos}}\left\{1-\left[1+0.5\dfrac{z}{R_o}\left(\dfrac{U}{w_o}\right)^{\frac{1}{2}}\right]^{-2}\right\}}{\dfrac{q_{po}}{q_{eps}}}\right\}. \tag{24}$$

Consider the typical case when radius $R_o=30$ m, the ratio $U/w_o=4.0$, the ratio $q_{po}/q_{eos}=3.2$, and the initial environment relative humidity $q_{eo}/q_{eos}=0.8$. The fraction A is calculated to increase from zero to unity between heights of about 370 and 410 m. In other words, condensation begins at a height of 370 m. At the height of 410 m the environment air would have begun to condense irrespective of the initial water flux of the plume. This is the level of natural cloud formation of the environment air and provides a simple upper limit to the level of first condensation in this problem. This upper limit is defined by the height z_L at which the quantity in braces in Eq. (24) becomes zero, i.e., for which

$$\exp(-z_L/1200)\text{m}=\frac{q_{eo}}{q_{eos}}\left\{1-\left[1+0.5\frac{z_L}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^{-2}\right\}. \tag{25}$$

The second fraction on the right-hand side of (25), which is much less than unity for large z_L/R_o, can usually be neglected. For a well-mixed environment the height z_L can then be approximated by

$$z_L\approx(1800\text{m})\ln\left(\frac{q_{eos}}{q_{eo}}\right). \tag{26}$$

The actual level of first condensation approaches this limit as $U/w_oR_o^2$ increases and q_{po}/q_{eos} decreases.

For certain conditions condensation will occur through the entire depth of the plume. For example, with the same special conditions given above, with the exception that $U/w_o=1$, the fraction A in the plume will always be greater than zero. Then the level at which all the initial water has been condensed is still the natural condensation level z_L.

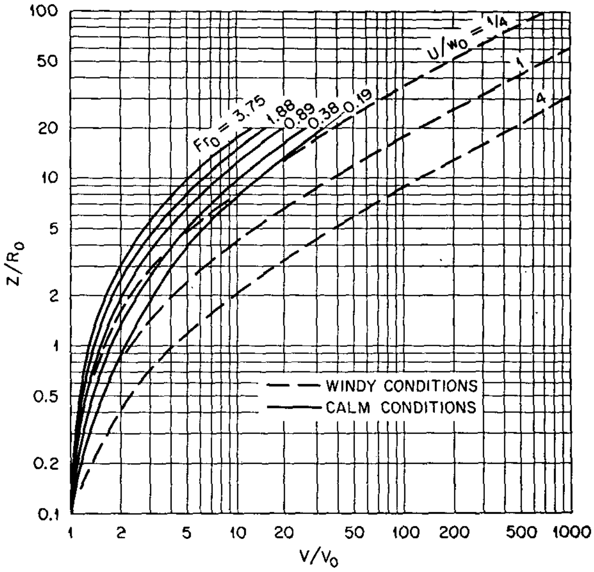

FIG. 1. Variation with dimensionless height, z/R_o, of the ratio of volume flux, $V(z/R_o)$, to initial volume flux, V_o, for windy and calm conditions, and typical values of u/w_o and Froude number $\text{Fr}_o=w_0R_0/2F_o$.

b. Specific case of windy, stable environment at plume equilibrium height

In this case, we assume that the environment mixing ratio q_e and potential temperature θ_e vary linearly with height, i.e.,

$$q_e=q_{eo}-\alpha z, \tag{27}$$

$$\theta_e=\theta_{eo}+Bz, \tag{28}$$

or in terms of actual temperatures,

$$T_e=T_{eo}+[B-0.01(^\circ\text{C m}^{-1})z].$$

The integration of Eq. (10) for a windy environment shows that the average specific humidity $\bar{q}_e(z)$ at great heights can be approximated by

$$\bar{q}_e(z)\approx q_e(\tfrac{2}{3}z)=q_{eo}-\tfrac{2}{3}\alpha z. \tag{29}$$

Let us first be concerned with whether condensation will occur at the equilibrium height z_E of the plume, i.e., the height at which the plume temperature T_p equals the environment temperature T_e. Briggs (1969) suggests that z_E is about 2.4 $(F_o/US)^{1/3}$, or about $\frac{5}{6}$ of the total plume rise, which is defined as the height at which the vertical speed w becomes zero. Substituting into (2) from (28) we obtain

$$z_E = 2.4\left[\frac{(T_{po}-T_{eo})w_oR_o^2}{UB}\right]^{1/3}.$$

The fraction A of the initial water flux Q_o that condenses at the equilibrium level is thus

$$A(z_E) = 1-\exp(-z_E/8000\text{m})\left[1+0.5\frac{z_E}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^2 \times\left\{\frac{\exp\{(L/R_v\bar{T}^2)[B-0.01(^\circ\text{Cm}^{-1})z_E]\}-\frac{q_{eo}}{q_{eos}}\left(1-\frac{2}{3}\frac{\alpha z_E}{q_{eo}}\right)\left\{1-\left[1+0.5\frac{z}{R_o}\left(\frac{U}{w_o}\right)^{\frac{1}{2}}\right]^{-2}\right\}}{\frac{q_{po}}{q_{eos}}}\right\}. \quad (30)$$

As an example, consider an isothermal atmosphere [$B=0.01(^\circ\text{C m}^{-1})$] with $\alpha/q_{eo}=1/2000$ m. Let

$$(T_{po}-T_{eo})=20\text{C} \qquad U=20 \text{ m sec}^{-1}$$
$$w_o=5 \text{ m sec}^{-1} \qquad q_{po}/q_{eos}=3.2$$
$$R_o=30 \text{ m} \qquad q_{eo}/q_{eos}=0.8$$

In this case the equilibrium level z_E is 185 m and the fraction A of the initial water flux Q_o that is condensed at that level is calculated to be -1.6. A negative value of A implies that $Q_o<Q_s$, i.e., condensation is not occurring at the height. If the surface relative humidity were to increase to 0.91, A would equal zero and condensation would begin to occur at the equilibrium level.

c. Condensation near the cooling tower

At levels near the tower ($z\lesssim 100$ m above the tower) the environment temperature T_e and mixing ratio q_e can often be assumed constant. The saturation mixing ratio q_{ps} of the plume, given by (13), can then be approximated in this region by

$$q_{ps}=q_{eos}\exp[(L/R_v\bar{T}^2)(T_p-T_e)],\ z<100\text{ m}. \quad (31)$$

Furthermore, air density variations with height can be neglected in this shallow layer. The equation for the fraction A of the initial water flux Q_o that has condensed at height z is given by

$$A=1-\left(\frac{V}{V_o}\right) \times\left\{\frac{\exp[(L/R_v\bar{T}^2)(T_p-T_e)-\frac{q_{eo}}{q_{eos}}\left[1-\left(\frac{V_o}{V}\right)\right]}{\frac{q_{po}}{q_{eos}}}\right\}. \quad (32)$$

Eqs. (17) or (20) and (18) or (21) can be used to estimate V/V_o and (T_p-T_e) for substitution in Eq. (32) for either windy or calm conditions.

For comparison, the fraction $A(z)$ of initial water flux Q_o that is condensed at level z is calculated for both calm and windy conditions in Table 2.
Condensation occurs near the cooling tower in both of these particular examples. However, during calm conditions the cloud would persist above heights of 90 m, while the cloud during windy conditions would re-evaporate at this height corresponding to a horizontal distance of about 100 m downwind of the tower. It is important to note that when A remains constant with height, the concentration σ of water drops decreases in proportion to the ratio V_o/V.

If the initial relative humidity in these two cases is less than about 0.7, instead of 0.8, the fraction A is less than zero and no condensation occurs. Thus, this criterion is also sensitive to slight changes in the initial conditions and environment parameters.

These techniques can be used to determine whether an initial flux of cloud water will evaporate close to the tower. The successful application of these formulas depends on the accuracy with which the initial flux of cloud water is measured.

d. Corrections to plume rise in case of incomplete condensation

Using the formulas derived in this section it is possible to calculate the final rise of plumes whose initial

TABLE 2. Fraction A of initial water flux Q_o that is condensed at height z, for $T_{po}-T_{eo}=20$C, $w_o=5$ m sec^{-1}, $R_o=30$ m, $q_{po}/q_{eos}=3.2$, $q_{eo}/q_{eos}=0.8$.

	Z(m) 0	15	30	45	60	75	90
A ($U=5$ m sec^{-1})	0	0.09	0.13	0.12	0.10	0.05	0
A ($U=0$)	0	0.08	0.11	0.12	0.15	0.15	0.14

water vapor content is not completely condensed. One makes a first estimate of plume rise, H_1, based on Eq. (7) with the latent heat not included. The fraction A of the initial water that is condensed at H_1 is calculated from Eq. (11). The calculated value of A is then multiplied by the total possible latent heat flux and the result added to the buoyancy flux in (7). The new buoyancy flux is used to calculate the second estimate H_2 of plume rise. The iterative procedure is repeated until the plume rise H and the fraction A of initial water that is condensed, approach constants.

As an example, consider the case:

$$T_{po}-T_{eo}=20\text{C} \qquad q_{po}/q_{eos}=3.2$$
$$U=w_o=5 \text{ m sec}^{-1} \qquad s=\tfrac{1}{3}\times 10^{-3} \text{ sec}^{-2}$$
$$R_o=30 \text{ m} \qquad \alpha/q_{eo}=1/(2000 \text{ m})$$
$$q_{eo}/q_{eos}=0.90$$

In this case the first estimate of plume rise, H_1, without including latent heat effects, is 350 m. The value of the fraction A at this height is calculated to be 0.34, from which the second estimate, H_2, of plume rise is calculated to be 440 m. The value of A at this height is 0.45. Consequently, H_3 is 470 m. By continuing this iteration it is found that plume rise H is about 480 m and an amount of water equal to about one-half ($A=0.5$) of the initial water is condensed at that height. In order to experimentally verify this technique, it is clearly important to measure vertical profiles of water content in the plume and the environment, as well as vertical profiles of temperature.

6. Conclusions

For moist plumes in calm or windy conditions, it is suggested that plume rise H can be estimated using Briggs' formulas, given in Eqs. (1) and (2), provided that the possibility of latent heat release is included in the definition of initial buoyancy flux. If condensation does not occur, then the initial buoyancy flux F_o used in Eqs. (1) and (2) equals the flux due to virtual temperature differences. If all the initial vapor is expected to condense then the additional latent heat flux should be added to the flux due to temperature differences. This heat may cause the plume to rise during stable conditions from 20 to 100% higher than it would have due to the initial sensible heat flux, for typical cooling tower plumes. If it is uncertain whether all the initial water vapor will condense, then plume rise can be estimated using an iterative procedure. In this procedure, dry plume rise H is first estimated. Then the fraction of the initial water vapor condensed at that height is calculated. This fraction is used to recalculate H, and so on. It is possible that under certain conditions the moist plume may become convectively unstable even in a stable environment. Once the plume reaches the level of natural cloud formation of the environment, the initial conditions have little influence on plume growth, and the problem is the same as one which has concerned cloud physicists for years.

The level of first condensation can be estimated for a wide variety of problems using the general criteria developed in Section 5. Condensation occurs when the initial flux of water drops and vapor is sufficient to saturate the initial volume flux plus the volume flux of entrained environment air. Slight variations in relative humidity or environment temperature can cause a difference between no condensation or a deep cloud. The criteria show, as expected, that condensation is most likely when the difference between the saturation and environment mixing ratios is a minimum. This condition is most likely to occur in a cold environment, but can possibly occur in a warm, humid environment.

Most of this discussion is not tied to any observations. The reason for this is that, to the author's knowledge, no set of measurements at cooling towers sufficient to test these theories has been made. It is important to measure initial fluxes of volume, buoyancy, water vapor and liquid water, as well as the variation of environment temperature and mixing ratio with height. Plume measurements should include temperature and vapor and liquid water content. From these variables plume rise and condensation levels could be estimated.

Acknowledgments. The author is indebted to Gary A. Briggs for many helpful discussions.

This research was performed under an agreement between the Atomic Energy Commission and the National Oceanic and Atmospheric Administration.

REFERENCES

Briggs, G. A., 1969: *Plume Rise.* AEC Critierical Review Series, TID-25075, Nat'l. Tech. Information Science, Springfield, Va., 82 pp.

Csanady, G. T., 1971: Bent-over vapor plumes. *J. Appl. Meteor.*, **10**, 36–42.

Hanna, S. R., 1971a: Meteorological effects of cooling tower plumes. Available from Cooling Tower Institute, 3003 Yale St., Houston, Texas, 17 pp.

——, 1971b: Cooling tower plume rise and condensation. *Proc. Air Pollution, Turbulence, and Diffusion Symp.*, Las Cruces, N. M., 7–10 December 1971.

——, and S. D. Swisher, 1971: Meteorological considerations of the heat and moisture produced by man. *Nuclear Safety*, **12**, 114–122.

Morton, B. R., 1957: Buoyant plumes in a moist atmosphere. *J. Fluid Mech.*, **2**, 127–144.

——, G. I. Taylor and J. S. Turner, 1956: Turbulent gravitational convection from maintained and instantaneous sources. *Proc. Roy. Soc. London*, **A234**, 1–23.

Simpson, J., and V. Wiggert, 1970: Models of precipitating cumulus towers. *Mon. Wea. Rev.*, **97**, 471–489.

Weinstein, A. I., 1970: A numerical model of cumulus dynamics and microphysics. *J. Atmos. Sci.*, **27**, 246–255.

Wigley, T. M. L., and P. R. Slawson, 1971: On the condensation of buoyant, moist, bent-over plumes. *J. Appl. Meteor.*, **10**, 263–259.

World Meteorological Organization, 1966: *International Meteorological Tables.* WMO-188-TP-94, Geneva, Switzerland (see introduction to Tables 4.6 and 4.7).