Comments on "A Comparison of Wet and Dry Bent-Over Plumes"

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Wigley and Slawson (1972) suggest that it is necessary to modify Csanady's (1971) theory of moist bent-over plume rise so that the atmosphere stability parameter N^2 involves the lapse rate relative to the wet adiabatic lapse rather than to the dry adiabatic lapse. We believe that their modification is redundant, since the difference between dry adiabatic and saturated adiabatic processes *within the plume* is already accounted for in the "energy" equation by means of a latent heat release term. Furthermore, this modification inflates the growth of the buoyancy flux without regard to how much latent heat is available. How can one account for plume processes by redefining an atmospheric parameter?

The "energy" equation, really a buoyancy flux equation, has been derived for an uncondensed plume in many different ways (Taylor, 1945; Priestley and Ball, 1955; Morton *et al.*, 1956; Slawson and Csanady, 1967; Hoult *et al.*, 1968; Briggs, 1969): it is Eq. (3) of Wigley and Slawson (1972) with the λS term removed. It can be derived by assuming the conservation of potential temperature of both plume gases and entrained air, and integrating the flux of buoyancy over a plane intersecting the plume. In the case of a condensed plume, the potential temperature can be increased locally by the release of latent heat associated with local condensation, i.e.,

$$d\theta = \theta_a \lambda d\sigma, \quad (1)$$

where $\lambda = L/(C_p T_a)$, σ is the mass fraction of liquid water in the plume, and the excess temperature of the plume is assumed small compared to the absolute ambient temperature. This adds the λS term, i.e.,

$$U \frac{d}{dx} F = -M N^2 + U \frac{d}{dx} (\lambda S). \quad (2)$$

The latent heat release, i.e., the saturated adiabatic process, is entirely accounted for the λS term, and the "dry plume" definition of N^2 [Eq. (5) of Wigley and Slawson] is not altered:

$$N^2 \approx \frac{g}{T_a} \frac{\partial \theta_a}{\partial z} = \frac{g}{T_a} (\Gamma_{ad} - \Gamma). \quad (3)$$

This result has also been derived by several different methods; see Morton (1957), Weinstein (1970), Simpson

and Wiggert (1970), Csanady (1971), and Hanna (1972). In all these derivations, the term corresponding to N^2 , the "stability parameter," is a measure of *ambient* atmospheric potential density stratification only, and has nothing to do with anything going on *inside* the plume.

Wigley and Slawson suggest (without proof) that, for the condensed portion of a moist plume, this atmospheric stability parameter "must be modified because of the differences between dry adiabatic and saturated adiabatic processes." But these processes are going on inside the plume and are already accounted for by the λS term. For this reason, we believe that their suggestion that Γ_{ad} , the dry adiabatic lapse rate, must be replaced by Γ_{as} , the wet adiabatic lapse rate, is incorrect.

As a result of Wigley and Slawson's incorrect modification of N^2 , their main conclusions have no sound physical basis. For example, the authors say that because of their modified form of N^2 , "one would expect that a wet plume in a neutral atmosphere would behave much like a dry plume in an unstable atmosphere in which the lapse rate was given by $\Gamma = \Gamma^* = \Gamma_{ad} + (\Gamma_{ad} - \Gamma_{as})$." But according to their recommended Eq. (5a) for a wet plume, the following comparison results:

$$U \frac{d}{dx} F = -M \frac{g}{T_a} (\Gamma_{ad} - \Gamma_{as}), \quad \text{dry plume, } \Gamma = \Gamma^*,$$

$$U \frac{d}{dx} F = U \frac{d}{dx} (\lambda S) - M \frac{g}{T_a} (\Gamma_{ad} - \Gamma_{as}), \quad \text{wet plume, } \Gamma = \Gamma_{ad}.$$

The two equations are obviously not identical. They would be approximately identical only if λS were small, i.e., the latent heat release were much less than the sensible heat flux. We would expect that such a plume, being merely "damp," would behave more like a dry plume in a neutral atmosphere, i.e., $N^2 \approx 0$ and $F \approx \text{constant}$.

We maintain that, whether the plume is wet or dry, the stability parameter is correctly defined by Eq. (3).

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