Estimating Rainfall in Regions of Active Convection

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(Manuscript received 26 June 1972, in revised form 11 September 1972)

ABSTRACT

An attempt to estimate rainfall in convectively active regions using Kuo's parameterization scheme has been made. The precipitation in this model is given by $P=lQ_1/\Delta t$, where P is the precipitation per unit time, l the fraction of a synoptic area covered by deep active convection, Q_1 the amount of condensation heating according to moist adiabatic ascent, and Δt a time parameter related to the precipitating lifetime of the convective elements.

Investigation of the above equation when P, l and Q_1 were available indicated that an appropriate time parameter was 30 min and that the main contribution to the precipitation comes from the l parameter.

A case is presented where l is obtained from satellite observations. The resulting precipitation estimate appears quite reasonable. The potential for estimating precipitation over the tropical oceans is pointed out.

1. Introduction

Recently, Kuo's (1965) parameterization model of latent heat release in tropical cyclones has been used diagnostically in other types of synoptic situations (e.g., Krishnamurti, 1968; Krishnamurti and Moxim, 1971; Hudson, 1971). According to Kuo, the fraction of a grid area covered by active cumulus cloud is given by

$$l=\frac{I}{Q}\Delta t, \tag{1}$$

where I represents the net moisture input to the column extending from the surface to the top of the atmosphere, Δt is a convective-scale time parameter, Q is the net amount of moisture required to produce a convective cloud over the entire unit grid, and I includes evaporation and horizontal moisture convergence. For diagnostic application of the model Δt has been interpreted as the average precipitating lifetime of the convective elements in the grid area. The moisture term Q is defined by two vertical integrals: Eq. (2), the amount of moisture necessary to saturate the column and Eq. (3), the amount of moisture necessary to warm the column to the moisture and temperature values of the moist adiabatic of the cloud. The integrals (in gm cm^{-2}) are given respectively by

$$Q_2=\frac{1}{g}\int_{P_B}^{P_T}(q_s-q)dp, \tag{2}$$

$$Q_1=\frac{C_p}{gL}\int_{P_B}^{P_T}(T_s-T)dp, \tag{3}$$

where P_B and P_T represent the pressure of the lifting condensation level and the level of zero buoyancy, respectively. The remaining symbols are defined as follows:

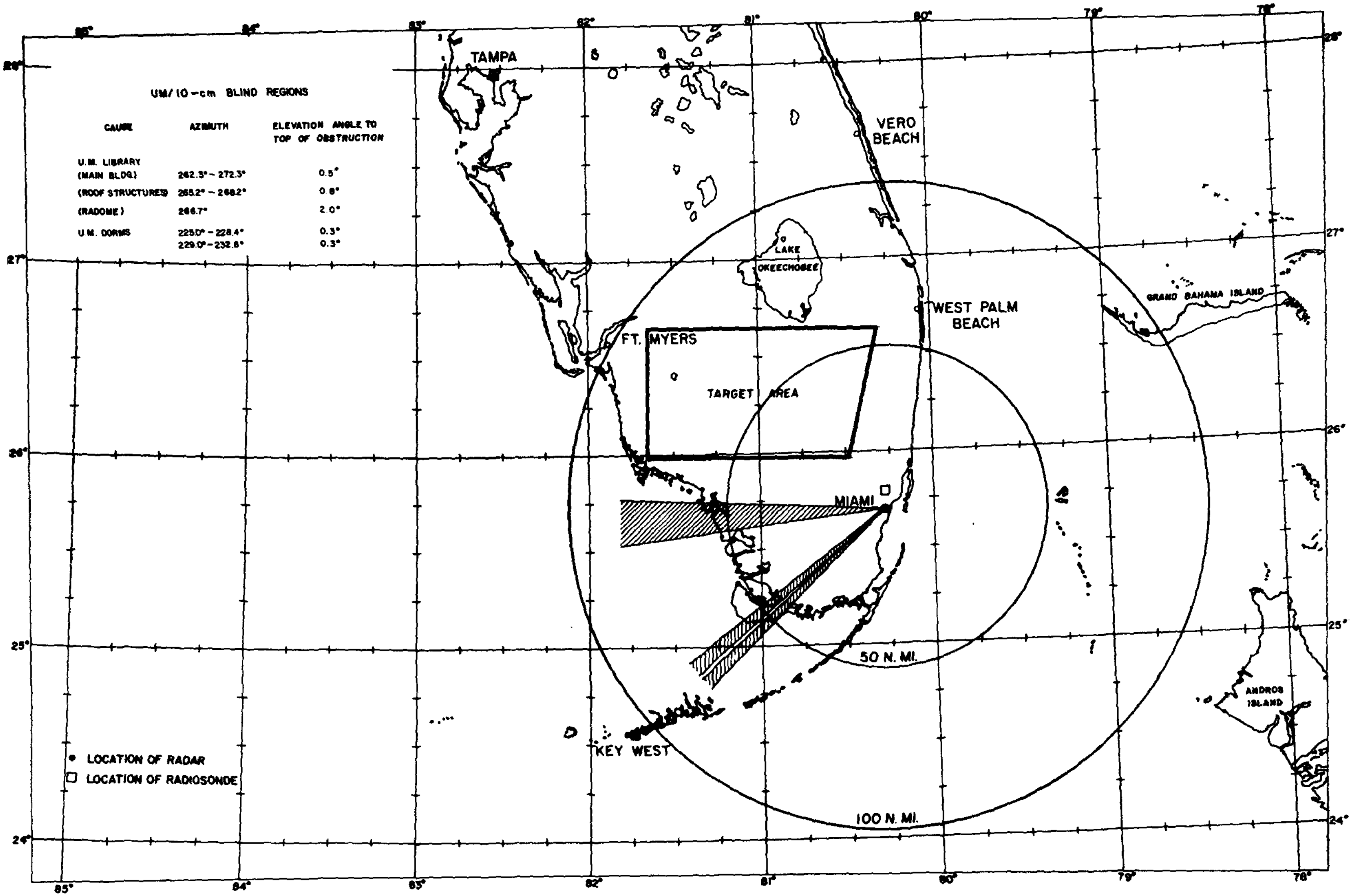
C_p	specific heat at constant pressure
g	acceleration of gravity
T	environmental temperature
T_s	temperature along the moist adiabat passing through lifting condensation level
q	environmental specific humidity
q_s	saturated specific humidity along the moist adiabat passing through the lifting condensation level
L	latent heat of condensation

Physically, Q_2 represents the storage of moisture in the volume and Q_1 the latent heating due to the condensate precipitating out. In fact, the precipitation rate is given by

$$P=\frac{C_p l}{\Delta t L}\int_{P_B}^{P_T}(T_s-T)\frac{dp}{g}=\frac{Q_1 l}{\Delta t}. \tag{4}$$

When the model is used diagnostically one must specify the value of Δt. This has been done by the investigators cited above in an arbitrary fashion; resulting estimates ranged from 15 min to 2 hr. Since it is seen from Eq. (1) that l is directly proportional to Δt, the choice of an appropriate Δt is important.

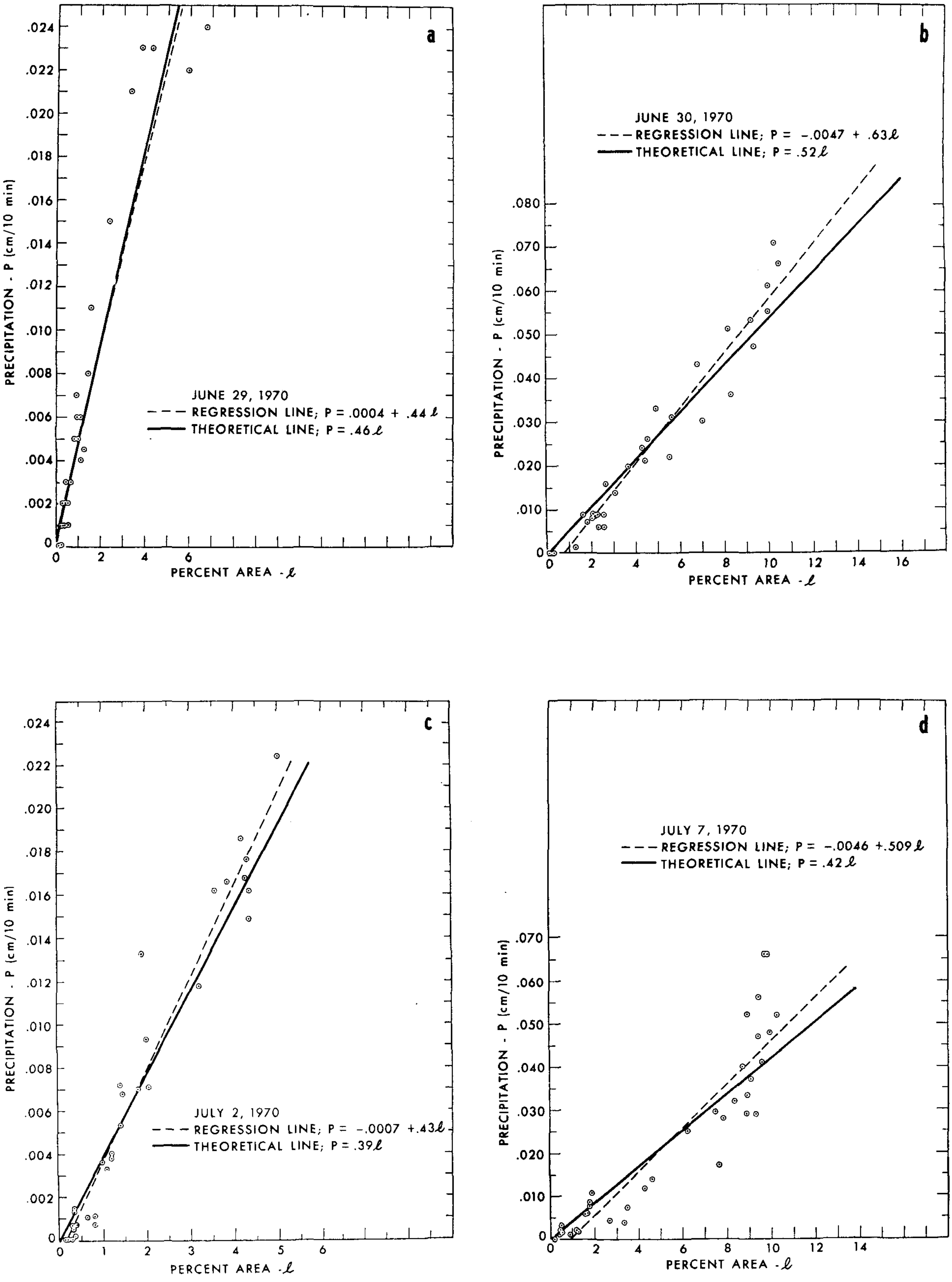
The parameterization model is restricted to conditionally unstable areas and regions of low-level convergence, making it particularly applicable to tropical disturbances.



UM/10-cm BLIND REGIONS

CAUSE	AZIMUTH	ELEVATION ANGLE TO TOP OF OBSTRUCTION
U.M. LIBRARY (MAIN BLDG.)	262.3° – 272.3°	0.5°
(ROOF STRUCTURES)	265.2° – 268.2°	0.8°
(RADOME)	266.7°	2.0°
U.M. DORMS	225.0° – 228.4° 229.0° – 232.6°	0.3° 0.3°

FIG. 1. Target area over which the radar and rainfall estimates were made by the Experimental Meteorology Laboratory during their 1970 cloud seeding experiment (after Herndon *et al.*, 1971).


a
JUNE 29, 1970
– – – REGRESSION LINE; P = .0004 + .44 ℓ
—— THEORETICAL LINE; P = .46ℓ
PRECIPITATION - P (cm/10 min)
0
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.024
PERCENT AREA - ℓ
0
2
4
6
b
JUNE 30, 1970
– – – REGRESSION LINE; P = –.0047 + .63ℓ
—— THEORETICAL LINE; P = .52ℓ
PRECIPITATION - P (cm/10 min)
0
.010
.020
.030
.040
.050
.060
.070
.080
PERCENT AREA - ℓ
0
2
4
6
8
10
12
14
16
c
JULY 2, 1970
– – – REGRESSION LINE; P = –.0007 +.43ℓ
—— THEORETICAL LINE; P = .39ℓ
PRECIPITATION - P (cm/10 min)
0
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.024
PERCENT AREA - ℓ
0
1
2
3
4
5
6
d
JULY 7, 1970
– – – REGRESSION LINE; P = –.0046 +.509ℓ
—— THEORETICAL LINE; P = .42ℓ
PRECIPITATION - P (cm/10 min)
0
.010
.020
.030
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.060
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PERCENT AREA - ℓ
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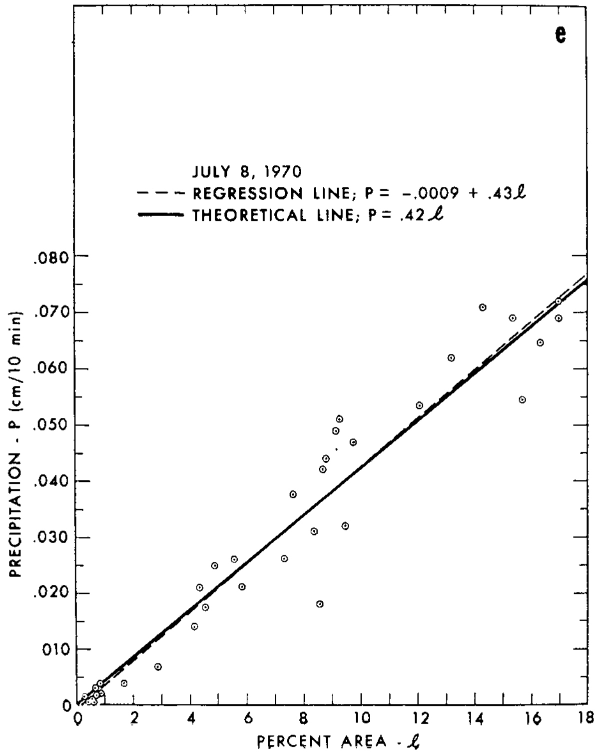

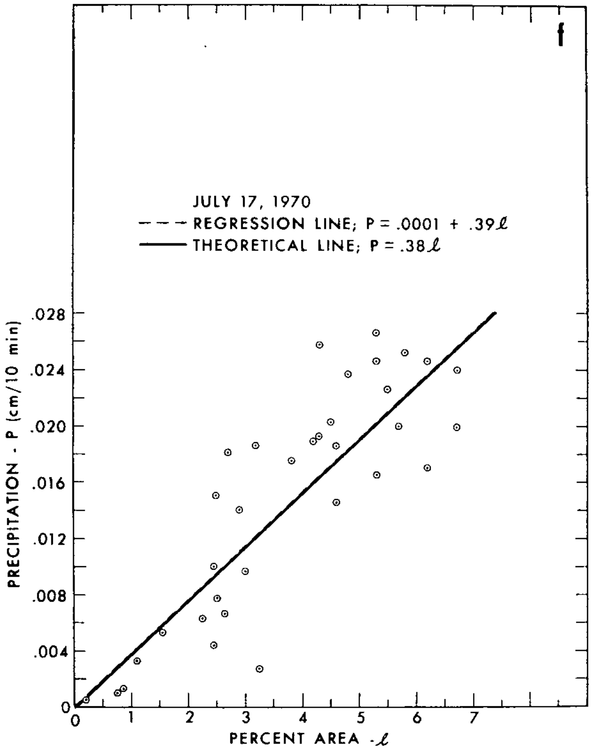

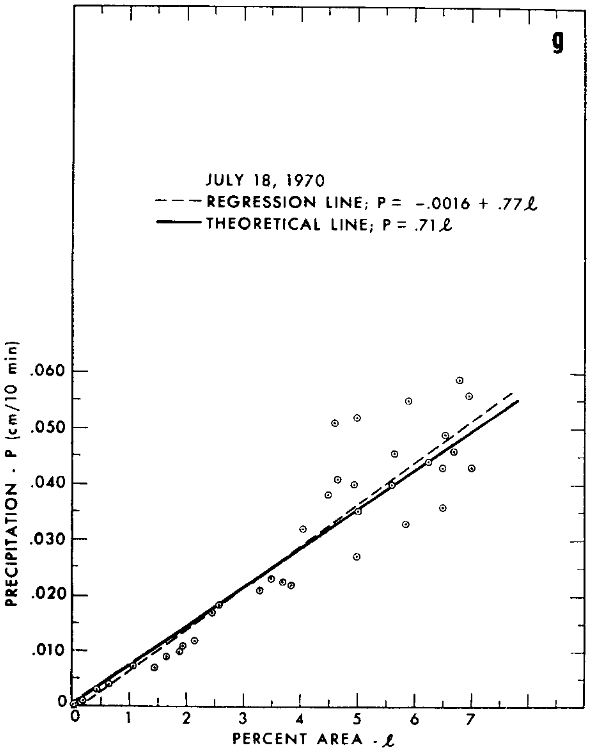

Figs. 2a–g. A plot of both the theoretical and least-square lines between percent area covered by convection and precipitation rate per 10-min period for the seven cases of Table 1.

TABLE 1. Parameters necessary for estimating Δt.

Case no.	Date	Average P parameter [cm (10 min)$^{-1}$]	Average l parameter	Q_1 parameter (gm cm^{-2})	P/l	Calculated Δt (min)
1	6/29/70	0.006	0.013	1.26	0.46	27
2	6/30/70	0.024	0.046	2.00	0.52	38
3	7/2 /70	0.007	0.018	1.50	0.39	38
4	7/ 7/70	0.023	0.055	1.26	0.42	30
5	7/ 8/70	0.030	0.071	1.50	0.42	36
6	7/17/70	0.014	0.037	1.20	0.38	32
7	7/18/70	0.030	0.042	1.26	0.71	18

The purposes of this paper are to present estimates of Δt obtained by solving Eq. (4) when directly observed values of P, Q_1 and l were available, and to present evidence that satellite observations can provide one of the essential parameters necessary for the computation of precipitation using Eq. (4). It should be emphasized that the approach used here differs significantly from other diagnostic applications in that an independent measure of l is used rather than calculating it through the moisture balance equation, as is usually done. This is an important point, since it eliminates the need for detailed knowledge of the wind and humidity field and allows for the application of remotely sensed data to Eq. (4).

2. Estimating the convective scale time parameter

The data used in establishing the magnitude of the convective scale time parameter came from the NOAA Experimental Meteorology Laboratory cloud seeding effort over the Florida Peninsula during the summer of 1970. The data consisted of 10-min intervals of radar echo coverage and radar estimates of rainfall for a 2700 (n mi)^2 area south of Lake Okeechobee (Fig. 1). These estimates were made as much as 1 hr prior to seeding and continued for about 5 hr after initial seeding. The calibration of the radar and a test of radar estimates of precipitation and raingage measurements have been fully described by Woodley and Herndon (1970) and Herndon *et al.* (1971). On the basis of their reports it appears that the areal rainfall rates as determined by the radar analysis are the most reliable source of rainfall data available.

In addition to the radar coverage a special 1800 GMT sounding was available for each of the seven cases studied. These cases were 29 and 30 June 1970 and 2, 7, 8, 17 and 18 July 1970.

The fraction of the target area covered by convective elements was obtained directly from the radar echo coverage. The values so obtained were then scaled downward by 50%. This was done to reflect the fact that much of the echo area produces little rain. The 50% scaling factor was based on the finding that on the average 90% of all the precipitation comes from 50% of the echo area (Woodley *et al.*, 1971).

To maintain physical consistency with the model, it was assumed that when one observes precipitation there is net moisture convergence, i.e., $I>0$. Since the area is quite small that is probably a valid assumption.

The procedure used for determining Δt was to average each 10-min observation of echo area coverage and rainfall for the day in question, calculate Q_1 from the available rawinsonde data, and then compute Δt from Eq. (4). The results of the computation of Δt for seven cases are presented in Table 1.

For the cases studied, Δt ranges from 18 to about 38 min, with an average value of $\sim$31 min. Included in Table 1 is the ratio $P/l \equiv Q_1/\Delta t$. This ratio represents the slope of the linear relationship between P and l given by Eq. (4). This relationship was tested by fitting a least-squares line to the 10-min observations for each case. The deviation of the least-squares line from the line predicted by Eq. (4), here called the theoretical line, provides a measure of the validity of the parameterization model. The results are presented in Figs. 2a–g where both the regression and theoretical lines are plotted. The pertinent statistics are summarized in Table 2. There is a good linear relationship between P and l, with correlation coefficients significant at the 0.1% level. Also included in the table are the 99% confidence limits on the slope (b) and the P intercept (p_0) of the least-squares line. Except for case 2, the results indicate that the P intercept is not significantly different from the theoretical slope. There is no readily available explanation as to why case 2 is significantly different from the theoretical results. However, if we exclude that case the average Δt is still $\sim$30 min.

If, based upon the above analysis of the individual cases, we accept the validity of the model, the next logical question to be answered is will the average value of 30 min for Δt provide a realistic measure of the convective scale time parameter to be used in future studies? In order to gain some insight into this question the precipitation for each case was computed using 30 min for Δt. The results summarized in Table 3 show that the magnitude of the percentage change varies from zero to about 40%. Because of the great difficulty

TABLE 2. Summary of the least-squares statistics.

Case no.	Date	Regression line	Correlation coefficient	99% confidence limits on slope (b)	99% confidence limits on P intercept (P_0)	Theoretical line
1	6/29/70	$P= 0.0004+0.44l$	0.95	$0.37<b<0.51$	$-0.001 <P_0< 0.002$	$P=0.46$
2	6/30/70	$P=-0.0047+0.63l$	0.97	$0.56<b<0.70$	$-0.009 <P_0<-0.001$	$P=0.52$
3	7/ 2/70	$P=-0.0007+0.43l$	0.98	$0.38<b<0.48$	$-0.002 <P_0< 0.001$	$P=0.39$
4	7/ 7/70	$P=-0.0046+0.51l$	0.92	$0.40<b<0.61$	$-0.01 <P_0< 0.003$	$P=0.42$
5	7/ 8/70	$P=-0.0009+0.43l$	0.97	$0.38<b<0.48$	$-0.006 <P_0< 0.004$	$P=0.42$
6	7/17/70	$P= 0.0001+0.39l$	0.85	$0.28<b<0.50$	$-0.004 <P_0< 0.005$	$P=0.38$
7	7/18/70	$P=-0.0016+0.77l$	0.92	$0.61<b<0.92$	$-0.0088<P_0< 0.0005$	$P=0.71$

in obtaining an accurate measure of precipitation, particularly of the convective type, it seems fair to conclude that a Δt of 30 min will yield satisfactory estimates of precipitation and thus provides a realistic measure of the convective-scale time parameter.

In view of its physical interpretation 30 min does not appear to be unrealistic. Indeed, observations made during the Thunderstorm Project (Byers and Braham, 1949) indicate that the mature stage of a thunderstorm lasts about 15–30 min. This stage produces the bulk of the precipitation. And more recently Shmeter (1969) pointed out that the mature stage lasts about 20–50 min. Thus, the value arrived at using the parameterization model, while based on a small number of cases, has some observational support. A study based on a larger number of cases obviously would be desirable.

Examination of the Q_1 parameter and the l parameter for the seven cases as presented in Table 1 indicates relatively little variation in Q_1 while the l parameter varies by about a factor of 5.5. This suggests that the main control over the precipitation rate as calculated by Eq. (4) will be the l parameter and that satisfactory estimates of precipitation could be arrived at by using an average value of Q_1. This is important because it implies that if l can be estimated from satellite data then the model can be extended to tropical oceanic regions where the only estimates of rainfall are from highly unrepresentative island observations. The appropriate Q_1 parameter would be provided by climatological data.

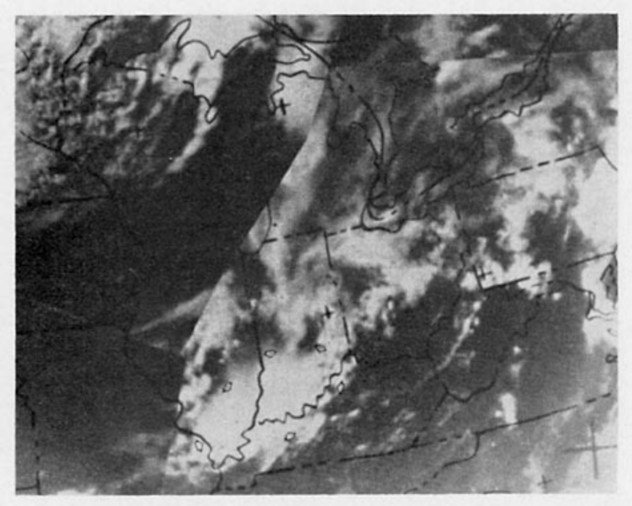

FIG. 3. Satellite view of the disturbance over Illinois and Indiana on 3 July 1970. Arrows delineate main cloudiness associated with disturbance.

3. Analysis of satellite data

In this section it is demonstrated that an estimate of l can be arrived at using satellite data and that a realistic estimation of precipitation can thus be obtained using Eq. (4).

The case selected for study was a disturbance which occurred in southern Illinois and Indiana on 3 July 1970 at about 0630 GMT. Fig. 3 shows a satellite view several hours after the actual time for which the study was performed. The picture is shown merely to give some idea of the size and extent of the clouds associated with the disturbance. This case was selected because of the conjunction of excellent satellite coverage in the infrared region (10.5–12.5 μm) with extensive radar coverage. There was also a nearby rawinsonde and the opportunity to obtain precipitation data from an hourly raingage network.

Surface and radar observations indicated considerable convective activity. There were reports of radar echoes greater than 50,000 ft. The 0000 GMT sounding at Peoria, Ill., was used to obtain a value of 1.42 gm cm^{-2} for Q_1. The lifting condensation level was at about 850 mb and the level of zero buoyancy occurred at about 150 mb with a temperature of 206K.

To deduce where the regions of deep active convection were located in the disturbance, high-resolution infrared (IR) window channel (10.5–12.5 μm) data available

TABLE 3. Comparison between the mean precipitation and precipitation calculated using $\Delta t=30$ min.

Case no.	Date	Mean P [cm(10 min)$^{-1}$]	Calculated P [cm(10 min)$^{-1}$]	Percentage difference
1	6/29/70	0.006	0.0055	−9
2	6/30/70	0.024	0.031	29
3	7/ 2/70	0.007	0.009	28
4	7/ 7/70	0.023	0.023	0
5	7/ 8/70	0.030	0.036	20
6	7/17/70	0.014	0.015	7
7	7/18/70	0.030	0.018	−40

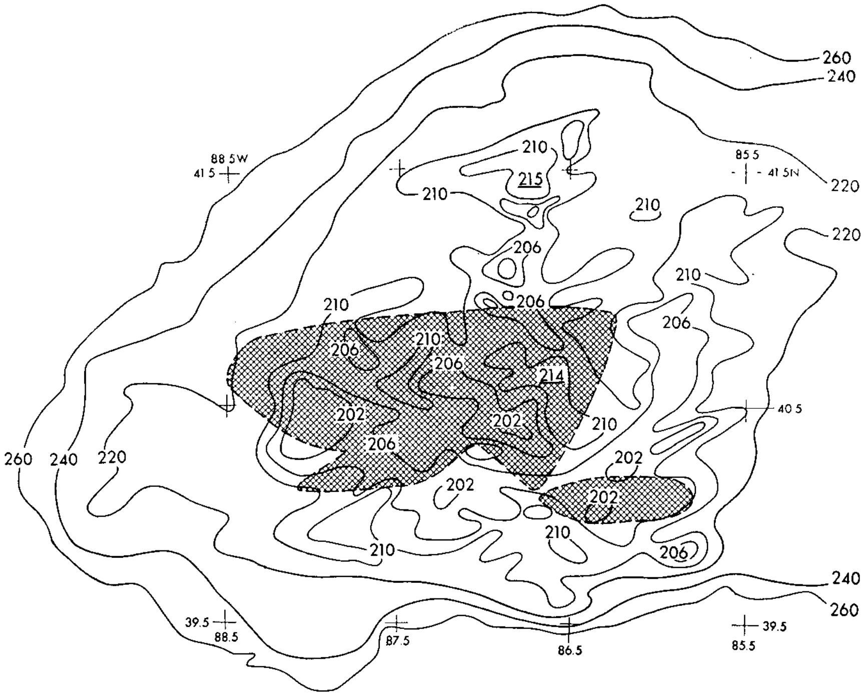

FIG. 4. Infrared temperature analysis of the disturbance with superimposed radar echos, at about 0645 GMT 3 July 1970.

from the Nimbus 4 satellite were analyzed. The data have a resolution of about 4 n mi at the sub-satellite point. However, in the form analyzed in this study, the data were averaged over about two high-resolution spots. Fig. 4 shows an IR temperature analysis of the disturbed area. The temperature analysis is restricted to temperatures ⩽ 260K, with the most detailed analysis made for temperatures <210K. Superimposed on the analyses is a schematic representation of the radar echoes available from the WSR-57 radar at Chicago. The radar data are within minutes of the satellite data. Attention is directed to the qualitatively good agreement between the radar echoes and the region of temperatures <210K. Other radar echoes from the Evansville, Ind., and the St. Louis, Mo., radars (not shown on the diagram) indicate similarly good agreement to the area south of the echo distribution presented. In view of a variety of errors in both the radar and satellite data as well as differing resolutions one should not expect a one-to-one agreement.

In order to obtain a quantitative measure of the area covered by deep convection it was assumed that all temperatures <206K, the temperature at the level of zero buoyancy, are the result of penetrative convection. Thus, the percentage frequency of those temperatures provides a direct measure of the l parameter.

The assumption suffers from a variety of deficiencies which are difficult to assess. First, because of the resolution of the instrument we are restricted to the largest convective elements or groups of convective elements. Second, we may be underestimating the amount of convection, since we cannot identify convective elements that do not penetrate the thick cirrus shield, regardless of resolution.

It is not known how serious these deficiencies are. However, they may be ameliorated somewhat since some of the temperatures <206K are probably the result of sampling variations and changes in thickness and heights of cirrus, and not related to penetrative convection. This would tend to overestimate the l parameter and thus compensate for the other deficiencies. More importantly, however, is the probability that it is the largest of convective elements that produce the most significant amounts of rainfall. Thus, any underestimation we make may not be too serious.

A frequency distribution of IR temperature for the region bounded by 85.5 to 88.5W and 39.5 to 41.5N is presented in Fig. 5. There were a total of 572 observations. The value 201K represents all those temperatures <202K. The remaining abscissa values are the midpoints of the temperature range for which frequencies were obtained. The mode centered at about 209–211K probably represents high thick cirrus clouds. Applying the assumption stated earlier the l parameter is deduced to be 13%. The area covered by all the radar echoes (including those which are not shown on Fig. 4) when treated as described for the Florida data indicate an area coverage of about 14%. At first glance the agreement appears remarkably good; however, it is obvious that there is not a one-to-one representation. Neverthe-

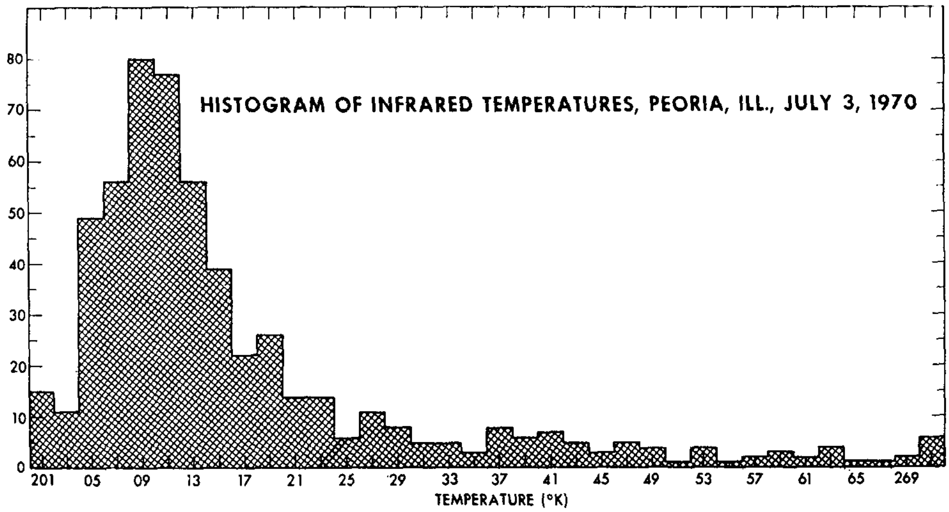

Fig. 5. Histogram of temperature over the area bounded by 39.5–41.5N and 85.5–88.5W.

less, the comparison indicates that for a first estimate, meaningful estimates of l are possible using satellite data.

The computation of the rainfall rate using this estimate of l, a Δt of one-half hour, and Q_1 arrived at from the Peoria, Ill., sounding yields a precipitation rate of 0.15 inch hr^{-1}.

Because of the inadequacy of hourly raingage networks to sample convective type precipitation accurately, it is not possible to verify the calculated precipitation rate. However, it was possible to obtain some measure of the reality of the computation by an examination of the hourly rainfall records for stations contained in the area for which computations were made. This was done for the 2-hr period 0500–0700 GMT, during which time 11 stations out of the 33 in the area reported precipitation. The data are listed in Table 4. The average rainfall rate for those stations was ~0.30 inch hr^{-1}. If we assume that the stations are uniformly distributed over the area, the area-normalized rainfall rate is 0.10 inch hr^{-1}. While it is clear that these two rates are not strictly comparable, the results do indicate that the model does yield a representative measure of the precipitation when satellite data are used to infer the l parameter.

Table 4. Observed rainfall rate for the period 0500–0700 GMT 3 July 1970.

Station name	Rainfall rate (inch hr^{-1})
Fowler, Ind.	0.30
West LaFayette 6NW, Ind.	0.30
Burlington, Ind.	0.20
Tipton Highway Garage, Ind.	0.21
Lebanon Water Works, Ind.	0.35
Crawfordville Power Plant, Ind.	0.10
Jamestown 1 SSW, Ind.	0.25
Indianapolis Riverside, Ind.	0.42
Indianapolis WB AP, Ind.	0.31
Rantoul, Ill.	0.55
Danville, Ill.	0.27

4. Concluding remarks

Using radar observations and conventional rawinsonde data, it was possible to calculate an average convective-scale time parameter from a relatively simple parameterization model. The fact that the line predicted by the parameterization model fit the observations quite well gives credence both to the physical reality of the model and to the computed time parameter.

The analysis also indicated that the main contribution to the rainfall using Eq. (4) was the l parameter. Although this was based on a small number of cases it does confirm our intuition which expects more precipitation the more active and widespread the convection.

It was also demonstrated that a realistic estimate of the l parameter could be obtained using satellite data. This was a first attempt and techniques for improving the estimate of l as derived from satellite data are being investigated. One approach under examination is the simultaneous use of brightness and infrared data to help more readily distinguish the convective areas. There is also the possibility of using the approach that Woodley *et al.* (1972) have recently demonstrated to arrive at the l parameter. Their approach essentially correlates enhanced brightness with radar echoes.

Nevertheless, the results from this first attempt have been quite encouraging and suggest that meaningful rainfall estimates can be made over the tropical oceans utilizing satellite observations.

It should be pointed out that this model may, under some circumstances, provide a quantitative estimate of precipitation that may be superior to radar estimates of precipitation. This can be the case particularly when one is attempting to obtain estimates of precipitation over oceanic areas from shipborne radars, because, in general, one will be dealing with uncertain rainfall rates versus radar reflectivity relationships and probably will not have any raingages suitable for calibrating the radars. In view of the importance of obtaining a quantitative measure of rainfall during the GARP Atlantic Tropical Experiment this method may prove to be of importance.

Acknowledgments. I would like to thank Dr. J. Winston and Mr. A. Krueger for their valuable discussions concerning this work as well as their review of the manuscript. I also wish to thank Mrs. Z. Licausi and Mrs. F. Dishman both of whom typed various portions of the manuscript and Mr. L. Hatton for drafting the figures. I would also like to thank Dr. W. L. Woodley of the Experimental Meteorology Laboratory for providing the radar and rainfall data.

REFERENCES

Byers, H. R., and R. R. Braham, Jr., 1949: *The Thunderstorm*. U. S. Department of Commerce, Washington, D. C., 287 pp.

Herndon, A., C. L. Courtright, W. L. Woodley and J. Fernandez-Partagas, 1971: A South Florida radar-raingage comparison for 1970. NOAA Tech. Memo. ERL OD-7, 22 pp.

Hudson, H. R., 1971: On the relationship between horizontal moisture convergence and convective cloud formation. *J. Appl. Meteor.*, **10**, 755–762.

Krishnamurti, T. N., 1968: A calculation of percentage area covered by convective clouds from moisture convergence. *J. Appl. Meteor.*, **7**, 184–195.

——, and J. R. Moxim, 1971: On parameterization of convective and non-convective latent heat release. *J. Appl. Meteor.*, **10**, 3–13.

Kuo, H. L., 1965: On formation and intensification of tropical cyclones through latent heat release by cumulus convection. *J. Atmos. Sci.*, **22**, 40–63.

Shmeter, S. M., 1969: Structure of fields of meteorological elements in a cumulonimbus zone. Hydrometeorological Council of Ministers USSR, *Trans. Central Aerological Obs.*, No. 88, IPST Cat. No. 5696.

Woodley, W., and A. Herndon, 1970: A raingage evaluation of the Miami reflectivity-rainfall rate relation. *J. Appl. Meteor.*, **9**, 258–264.

——, J. Norwood and B. Sancho, 1971: Some precipitation aspects of Florida showers and thunderstorms. *Weatherwise*, **24**, 106–119.

——, B. Sancho and A. H. Miller, 1972: Rainfall estimation from satellite cloud photographs. NOAA Tech. Memo. ERL OD-11, 43 pp.