A Dual-Wavelength Radar Hail Detector[1]

P. J. ECCLES[2] AND D. ATLAS[2]

Dept. of the Geophysical Sciences, The University of Chicago

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ABSTRACT

It is proposed that the range derivative of the logarithm of the ratio of average echo powers from two (S- and X-band) synchronized and slaved radars would yield a highly reliable indication of the boundaries of hail shafts. In the presence of rain alone, and ignoring fluctuations, this derivative would always be positive and proportional to the incremental difference in attenuation at the smaller wavelength. In general, the derivative has the same sign as the hail concentration gradient and attains negative values on the far side of a hail shaft. Without hail, signal fluctuations are the only possible source of negative derivatives, and so of false alarms. Thus, a small negative threshold level would avoid the identification of the effect of signal fluctuations at the far side of a hail shaft; similarly a large positive threshold would avoid identifying regions of intense rain as the near side of a hail shaft. This approach is capable of detecting smaller concentrations of hail with greater confidence and in larger backgrounds of non-hail precipitation than the use of the dual-wavelength reflectivity ratio alone because 1) it requires a smaller hail reflectivity ratio, at the two wavelengths; 2) it is not affected significantly by attenuation, and 3) it is independent of absolute radar calibrations. The limitations of the technique are discussed.

1. Introduction

The unequivocal detection of hail and the determination of hail size by remote probes such as radar have been scientific and applied goals for over two decades. The initial impetus toward such an objective was the avoidance of hazardous hailstorms by aircraft. More recently, research on the mechanisms of hail growth and on artificial hail inhibition have required operational, real-time information on the presence and location of hail for effective seeding and for evaluation of such seeding. These remain the objectives of hail detection today.

The use of the difference in the wavelength dependence of the reflectivity of hail from that of rain as a technique to measure hail, by taking the simple ratio of the echo powers at two wavelengths [first proposed by Atlas and Ludlam (1961)], had been abandoned in the United States because the effects of hailstone surface wetness, sponginess, roughness, shape and orientation (Atlas, 1964) so complicate the theoretical reflectivity-wavelength dependence as to render virtually all generalizations, and measurements on hail size, meaningless. More recently, however, it has been exploited in the Soviet Union (Sulakvelidze *et al.*, 1965; Sulakvelidze, 1968). They use dual-wavelength observations at 3.2 and 11 cm and report remarkable success in identifying hailstorms. However, in view of the aforementioned difficulties, one must at least raise the question as to the sensitivity and false alarm rate of any hail detection approach based solely on the dual-wavelength signal ratio, if not to doubt the entire validity of such a method.

The present paper describes one possible configuration of an improved dual-wavelength technique for hail detection which should overcome the above limitations. It is shown, theoretically, that the method should provide a reliable indication of the presence of hail of any size greater than about 1 cm, but no other measure of the actual sizes.

2. General concept

We summarize the basic concept of the hail detector as follows. The average echo power P_2 and P_1 returned at 10 cm and 3 cm wavelengths, respectively, from matched beams illuminating a volume at range r[km] are given by

$$P_2=[C_2(Z_2+Z)/r^2]10^{-0.2\int_{r_0}^{r_0+s} q_2' H dr}, \tag{1}$$

$$P_1=[C_1(Z_1+Z)/r^2]10^{-0.2\int_{r_0}^{r_0+s} (A q_1' H) dr}, \tag{2}$$

where C_1 and C_2 are the radar constants and include the attenuation factors up to the range r_0 at the front of the storm and any calibration errors, s is the distance from r_0 to the observed volume, q' the attenuation coefficient (dB km^{-1} per gm m^{-3}) due to hail, H the hail ice concentration (gm m^{-3}), $Z_2+Z=Z_{10}$ is the sum

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[2] Present affiliation: National Center for Atmospheric Research, which is supported by the National Science Foundation.

of the hail (Z_2) equivalent reflectivity factor plus that of rain (Z) at λ_2, and $Z_1+Z=Z_3$ is the corresponding sum at λ_1. We choose λ_2 so that the attenuation contribution to A (db km^{-1}) from the longer wavelength will be negligible, in general, in rain. Note that attenuation may be neglected at 3 cm for ice crystals and snow. Taking the common logarithm of the ratio, we get

$$y=10\log[(P_2/C_2)/(P_1/C_1)],$$

$$=\underbrace{10\log\left(\frac{Z_2+Z}{Z_1+Z}\right)}_{F}+2\int_{r_0}^{r_0+s}\underbrace{[(q_1'-q_2')H+A]}_{\alpha}dr, \quad (3)$$

and

$$\frac{dy}{dr}=\frac{dF}{dr}+2\alpha. \quad (4)$$

Note that y is simply the decibel ratio of the 10- to 3-cm echo powers (after normalization for the all-embracing radar constants described above), F is the corresponding reflectivity ratio, and the last term in (3) is the 3-cm decibel attenuation to the observation point modified by the 10-cm attenuation by hail (if any). Indeed (3) represents the overly simple technique originally proposed by Atlas and Ludlam (1960) if one notes that Z_{10}/Z_3 takes on values greater than unity and thus produces $y>0$ whenever hail of diameter >1 cm is present, and is identically unity (or $y=0$) when only rain is present and *attenuation is neglected.*

However, the problem arises that the presence of an unknown attenuation [the last term in Eq. (3)] due to liquid water content (LWC) can produce $y>0$ when $Z_{10}/Z_3=1$ and only rain is present, thereby producing a false indication of hail if only the echo ratio is used. We shall show later that the condition $y>0$ may also occur in the absence of rain when the two beams are unequal in size, or when system calibration errors are present. These problems are almost entirely overcome by using dy/dr [Eq. (4)] as the hail indicator rather than y alone. For example, when only rain or cloud liquid water is present, $Z_{10}/Z_3=1$, $F=0$, $dF/dr=0$, and dy/dr (the slope of the signal ratio) is directly proportional to the attenuation due to the LWC at the measurement range, *and is always positive.* Accordingly, dy/dr is at least a rough measure of the LWC,[3] an important by-product of the method (Eccles and Mueller, 1971). On the other hand, when hail is present, Z_{10}/Z_3 and F will increase with range ($dF/dr\geqslant 0$) at the leading edge of the hail shaft and $dF/dr\leqslant 0$ at its trailing edge; of course, the magnitude of dF/dr will depend upon the reflectivity gradients at the boundaries of the hail shaft. In any case, we see that dy/dr will increase in the positive direction at the hail leading edge and decrease at its trailing edge.

Indeed, unless M (the LWC) is excessive at the hail trailing edge, dy/dr will go negative; this is an *unequivocal* indicator of the hail trailing edge since dy/dr can only be zero or positive under all other circumstances. Even when M is large, the method will work wherever the gradients of M are not excessive. Under these circumstances a sharp *change* in dy/dr, or a large second derivative (d^2y/dr^2), is likely to occur at the hail boundaries. Thus, the trailing edge of the hail shaft will be readily detected. Indeed, Carbone *et al.* (1973) report that such trailing edge detection of hail has already been achieved in real time.

For detection of the leading edge of a hail shaft one first determines the probability distribution of M from rainfall statistics for the climatological region of the observations and selects that value which is exceeded say only 5% of the time; call this $M_{5\%}$. Further, let q_1 be the drop-size-distribution-dependent attenuation coefficient [dB km^{-1} per gm m^{-3}] for liquid water. Then if $dy/dr>2A=2qM_{5\%}$ (and is, of course, positive), we can have 95% confidence that this is due to entry into a hail shaft.

In summary, there are five major points to make about the behavior of dy/dr:

(i) At the trailing edge of a hail shaft, $dy/dr<0$ is a hail indicator which is subject only to statistical signal fluctuations.

(ii) At the leading edge of a hail shaft a value of $dy/dr>2q_1M_{5\%}$ indicates a high probability of hail.

(iii) In ordinary rain, and absence of hail, $dy/dr=2A$ yields a direct physical measurement of attenuation due to liquid water content.

(iv) Accurate absolute calibration of the radars is not needed since the radar constants, including the calibration errors, vanish when dy/dr is calculated in (v) below.

(v) A qualitative indication of the amount of hail may be obtained if the total profile of y is examined in the light of (i), (ii) and (iii), but this depends upon assumptions of the hail structure and radar cross sections.

While the use of dy/dr instead of y as a hail indicator may appear at first sight to be a trivial step, we emphasize that the latter may give rise to a high false alarm rate while the former will provide a high degree of confidence of hail detection.

3. Model calculation of the detectability of hail shafts in rain

Let us assume a parabolic distribution of the rain intensity with range through a convective storm. As we have pointed out above, y deduced from any profile is monotonically increasing and, in this case, is given by the smooth curve of Fig. 1. The term y is simply a

[3] Attenuation is not a unique function of LWC unless all of the attenuating particles are very much smaller than the 3-cm wavelength.

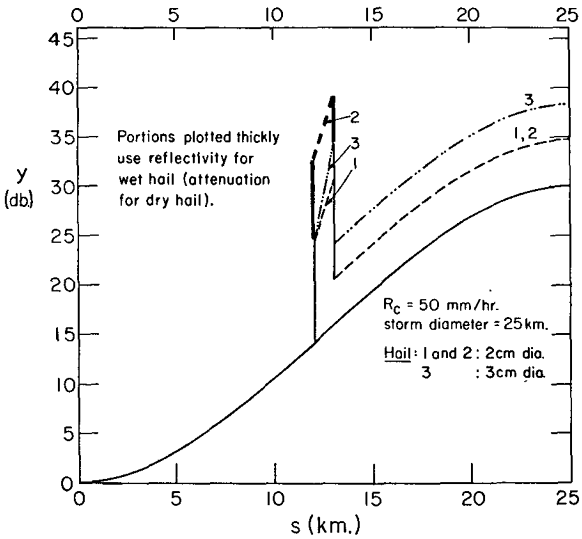

FIG. 1. Logarithm of the ratio of echo power y from 10- and 3-cm radars as a function of range s through model storms. The smooth curve represents y for a storm of 25 km diameter with a parabolic distribution of rainfall rate with range, a peak rainfall rate of 50 mm hr^{-1}, and no embedded hail. One km diameter hail-shafts of 2-cm dry hail and wet hail at a concentration of 2 gm m^{-3}, and 3-cm dry hail at a concentration of 4 gm m^{-3} are considered superposed on this storm at the position of peak rainfall intensity, yield the curves 1, 2 and 3, respectively, which all contain sharp discontinuities. Each numbered curve shows the specific radar signature of hail; that is, a large discontinuity and thus a large negative value of dy/dr on the far side of the hail shaft.

plot of the decibel attenuation, $\int A\,dr$, through the storm at the shorter wavelength (3.2 cm). Attenuation at the longer wavelength (10 cm) is neglected. The attenuation $A=q_R R$, where R is the local rainfall rate (mm hr^{-1}) assuming zero updraft, and $q_R=0.013R^{0.15}$ [dB km^{-1} per mm hr^{-1}] following Wexler and Atlas (1963). Because the attenuation term is almost directly proportional to R, dy/dr is virtually an image of the assumed parabolic rainfall distribution. The model also assumes that a shaft of hail is superimposed on the rainfall profile where it will be most difficult to detect, i.e., in the peak of the storm.

The assumed hail distribution has uniform size and concentration about the center of the rain distribution. The curves of y from detailed calculations on this model are displayed in Fig. 1 for a storm diameter of 25 km and for hail concentrations given in the legend (approximately 1 stone per 2 m^3). The reflectivity terms for hail, Z_1 and Z_2, are taken from Figs. 25 and 26 of Atlas (1964).

The superposition of a sharp-edged hail shaft on the rain causes an abrupt change in y at the leading and trailing edges of the shaft. Within the hail shaft dy/dr is larger than outside the shaft because of the extra term due to the hail attenuation; thus on the far side of the hail shaft y does not return to the value it would have in the absence of hail. Beyond the hail shaft the hail attenuation term acts like a constant for the remainder of the storm in which there is no hail. The attenuation term is $(q_1'-q_2')H\Delta r$, where H is the hail concentration (gm m^{-3}). The attenuation coefficients q_1' and q_2' are taken from Atlas and Ludlam (1960) assuming dry hail.

The presence of hail is clearly indicated by the negative value of dy/dr at the trailing edge of the hail shaft. The sharp rise on the leading edge of the shaft could be confused, if only rarely, with a sharp-fronted region of very intense rain. However, a value of the slope greater than that likely (say at the 5% point) for rain will indicate the high probability of the leading edge of a hail shaft. Note also that the change in y is very much larger for wet 2-cm hail than for dry 2-cm hail.

4. Minimum detectable hail concentration gradients in various rain backgrounds

a. Simplification and approximations

We have shown that large hail concentrations and concentration gradients, with reflectivity greater than, or of the order of, that from the local rain background ($Z_2>Z_1\gg Z$), are easily detected. However, hail detection must become increasingly difficult as the hail concentration is reduced (such that $Z\gg Z_2>Z_1$) since (3) yields $y+\Delta y\approx 10\log 1+$(attenuation terms), and $\Delta y/\Delta r$ is thus small. The properties of the radar signal for these barely detectable hail concentration gradients in a background of heavy precipitation can be derived if we realize that the term involving H (due to attenuation by small amounts of hail) can be neglected and that the larger of Z_1 and Z_2 will be very much less than Z for the rain. A simplified version of (4) is thus

$$\frac{dy}{dr}=4.3\frac{d}{dr}\left[\ln\left(1+\frac{Z_2-Z_1}{Z}\right)\right]+2A,$$

$$\approx 4.3\frac{d}{dr}\left(\frac{(Z_2-Z_1)}{Z}\right)+2A,$$

$$=\underbrace{\frac{4.3}{Z}\frac{d}{dr}(Z_2-Z_1)}_{\text{(I)}}+\underbrace{\frac{4.3(Z_1-Z_2)}{Z^2}\frac{dZ}{dr}}_{\text{(II)}}+\underbrace{2A}_{\text{(III)}}\ . \quad (5)$$

We interpret dy/dr as $\Delta y/h$ (the change in y in the distance $h=c\tau/2$ separating two contiguous pulse volume centers) through much of this paper; τ is the pulse duration and c the velocity of light.

Eq. (5) consists of three terms: (I) depends on the hail reflectivity gradient; (II) depends on both the total hail reflectivity and the background reflectivity gradient; and (III) is due to attenuation.

First of all we propose to ignore (II) in (5) with respect to the first term. The proof that (II) cannot alter the results significantly is given in Eccles and

TABLE 1. x as a function of sweep false alarm probability

Sweep false alarm probability, $P(s)$	0.01	0.025	0.05	0.1
x	2.33	1.96	1.64	1.28

Atlas (1969). Thus, we formally calculate the hail that is of minimum detectability in uniform rain, i.e., $dZ/dr=0$, if we use

$$\frac{dF}{dr}=\frac{4.3}{Z}\frac{\Delta(Z_2-Z_1)}{\Delta r}. \qquad (6)$$

In examining (6) we note that $Z_2>Z_1$ for spherical hail of virtually any diameter greater than 1 cm, and for any condition of wetness (with the exception of dry stones with $D\approx 4.7$ to 5.1 cm) and for any size distribution (Atlas and Ludlam, 1960). With few exceptions, the same is true for any shape and orientation (Atlas and Wexler, 1963) and any degree of sponginess (Battan and Herman, 1962). The detection criteria in those exceptional cases in which $Z_2<Z_1$ will be noted later and it will be shown that the method will indicate hail even under these conditions. Obviously, however, detection fails if $Z_1=Z_2$ at both r and $r+\Delta r$, i.e., when $(d/dr)(Z_2-Z_1)=0$.

b. Calculations of the effect of weather "noise"

While we have shown that a negative value of dy/dr is a strong indicator of the trailing edge of a hail shaft, we have thus far ignored fluctuations in average echo power. These must be considered because even in the case of *uniform* precipitation the average echo power fluctuates with range. Accordingly, the value of $y=10\log(P_2C_1/P_1C_2)$ may decrease ($\Delta y/\Delta r<0$) with range simply as a result of signal fluctuations and thereby produce a false indication at this location on just one sweep. This should be distinguished from the overall observer-system false alarm since the observer will ignore a random array of spots corresponding to these false indications associated with noise, and identify a patterned array of such indications as associated with the real far-side of a hail shaft.

In light rain, when $\Delta y/\Delta r$ has an expected value near zero, the signal fluctuations still occur and about 50% of the calculated values of $\Delta y/\Delta r$ will be below zero regardless of how many independent signals are averaged, because zero is its average value. Clearly, a rigid demand that "$\Delta y/\Delta r<0$ indicates hail" guarantees a most undesirable sweep false alarm probability of about 50%. Therefore, it is much more realistic to relax the decision level to some negative value, $-L$, requiring "$\Delta y/\Delta r<-L/\Delta r$ indicates hail" so that even with small numbers k of independent samples in the average power from "rain-only" pulse volumes, the cumulative probability $P(s)$ of $\Delta y/\Delta r$ being less than $-L/\Delta r$ will be small [$P(s)$ is called the sweep false alarm probability]. The number of independent samples k and the value of the level $-L$ to achieve any arbitrarily small sweep false alarm probability is considered in Appendix I of Eccles and Atlas (1969). Srivastava and Carbone (1971) have also considered the statistics of false alarm detection and obtain results that essentially agree with those quoted here.

Using (6), Eq. (4) may be rewritten as

$$\Delta y=\frac{4.343}{Z}\Delta(Z_2-Z_1)+2Ah, \qquad (7)$$

where Δy represents the change in y when the signals from two contiguous pulse volumes, spaced $\Delta r=h$ between centers, are compared. The term $\Delta(Z_2-Z_1)$ is the change in (Z_2-Z_1) in this distance.

Eccles and Atlas (1969) show that Δy is very nearly normally distributed with standard deviation, $8.7/(k-2)^{0.5}$. Accordingly, a decision level

$$-L=-8.7x/(k-2)^{0.5}\ [\text{dB}] \qquad (8)$$

must be set to detect values of Δy due to hail concentration gradients; that is, if $\Delta y<-L$, in the mathematical sense, it is very likely that hail exists in the pulse volumes being examined. Here x is a constant which depends on the desired sweep false alarm probability and is listed in Table 1. For example, if $x=1.96$ in (8), then Δy is less than $-L$ in 2.5% of the pulse volumes which have no hail simply as a result of signal fluctuations. An equivalent statement is that under the same conditions the probability that $\Delta y<-L$ is 2.5% in any one pulse volume. This decision level, $-L$, is accurate to 1% of the dB value for $k>10$ and $\Delta y<4$ dB. A simple form of (8) which is accurate to 5% of the dB value, with similar limits on k, is

$$-L=-\frac{8.7x}{(k)^{0.5}}\ [\text{dB}]. \qquad (9)$$

Knowing that $\Delta y<-L$ indicates the trailing edge of a hailshaft with a false alarm rate of only $100P(s)$ [percent], we find, by substituting (8) into (7), that detectable values of $\Delta(Z_2-Z_1)$ are given by

$$\Delta(Z_2-Z_1)<-\frac{Z}{4.343}\left[\frac{8.7x}{(k-2)^{0.5}}+0.026R_r^{1.15}h\right], \qquad (10)$$

where $q_R=0.013R^{0.15}$ [dB km^{-1} per mm hr^{-1}] is simply the precipitation attenuation coefficient at X band assuming zero updraft, and $A=q_RR$.

For the special case of the comparison of the averages of an infinite number of independent signals from two contiguous pulse volumes ($k=\infty$), which means that there are no statistical fluctuations in Δy, (10) applies in the form

$$\Delta(Z_2-Z_1)_\infty=\frac{300R_r^{1.5}}{4.3}(0.026R_r^{1.15}h), \qquad (11)$$

where we have used $Z=300R^{1.5}$ (Joss *et al.*, 1968). Thus,

$$\Delta(Z_2-Z_1)_\infty=CR_r^{2.65}. \qquad (12)$$

This applies for any value of Δr (for instance $\Delta r=h$) and is the minimum difference in hail reflectivity at the two wavelengths which can be detected regardless of the number of independent samples averaged. The constant C will increase as h, the pulse length, increases; thus, we conclude, if k is very large, that h should be as small as possible.

Given the quantities on the right-hand side of (10) we can calculate $\Delta(Z_2-Z_1)$ and, using a transformation, we can convert this to a minimum detectable hail concentration gradient. To generate this transformation we follow Atlas and Ludlam (1960) and select a hail size range of 1 cm and assume that all sizes of stones within such a range occur in equal numbers. Atlas and Ludlam calculate the reflectivity of 1 gm m^{-3} of hair of size D which has a scattering cross section equal to that of the mean for a 1-cm interval centered at D. This realistic approach smooths out the severe oscillations which appear in the corresponding curves for monodisperse hail, as shown by Atlas and Ludlam (1961, Figs. 4 and 5). This appears to represent the sort of averaging which probably occurs in nature. The results of these calculations for wavelengths of 3.3, 4.67 and 10 cm are displayed in Fig. 2.

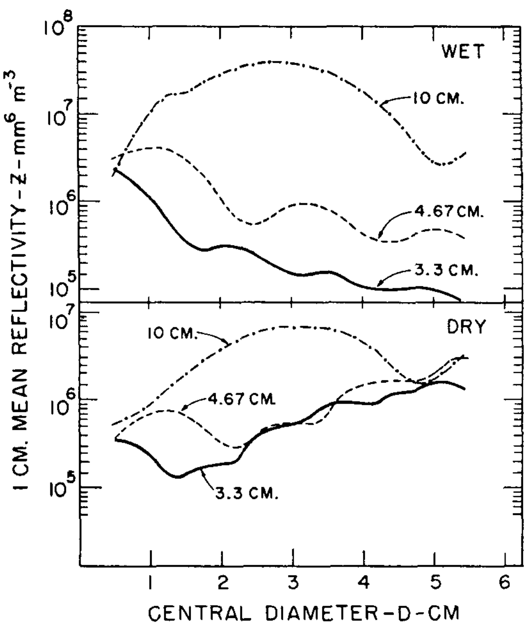

FIG. 2. Reflectivity Z at three wavelengths of 5 gm m^{-3} of wet or dry hailstones whose diameter lies within ± 0.5 cm of the diameter D, when the numbers within any small size interval are equal throughout the 1-cm size range (after Atlas and Ludlam, 1960).

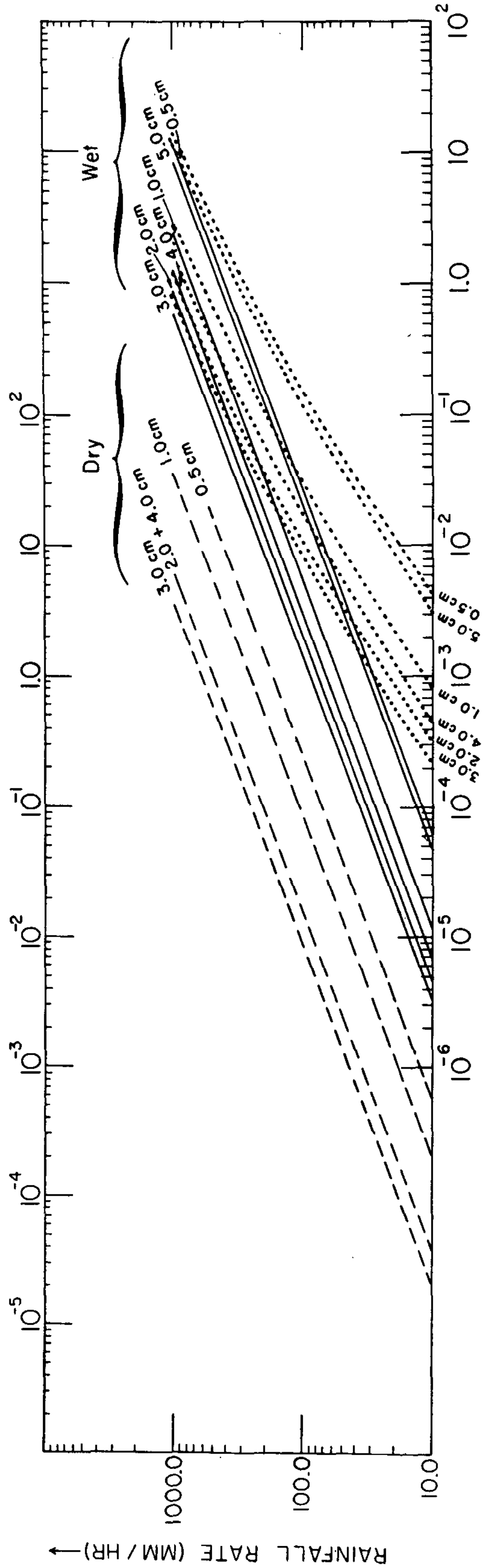

FIG. 3. Minimum detectable hail concentration gradients for spherical hail size distributions consisting of various 1-cm diameter wide distributions of hail, the central diameter being indicated as a parameter on the lines. For the straight lines, k, the number of independent estimates of the power returned, is virtually infinite. Zero updraft is assumed in the pulse volume. The curves assume that $k=24$ for volumes of wet hail. There is essentially no difference between the curves and the straight lines ($k=\infty$) for hail in heavy rain backgrounds. The abscissa for minimum dry hail concentration gradients (gm m^{-3} per pulse length) is along the upper boundary, and that for wet hail along the lower.

The straight lines of Fig. 3 are plots of minimum detectable hail concentration gradients of various 1-cm diameter wide distributions of hail whose central diameter is indicated as a parameter on the lines. The abscissa for minimum dry hail concentration gradients is along the upper boundary of the figure and that for wet hail along the lower. Note that as the hail increases in diameter it becomes more easily detected until it reaches a maximum at about 3 cm when the minimum detectable hail concentration starts to increase again. This sort of behavior is expected from Fig. 2 which shows a minimum difference Z_2-Z_1 for small and large hail, with zero values being found for wet hail $\sim$0.6 cm in diameter and dry hail $\sim$5 cm. For these latter cases we have already stated that the hail is not detectable.

When k decreases, the detection of hail must take place at a less sensitive level, i.e., $|L|$ must be larger [see Eq. (8) and Table 1]. The curves on Fig. 3 are an example of such a calculation for $k=24$ and show that a tolerable decrease in sensitivity of the hail detector occurs if quick scans are required. Once again the curves are plots of minimum detectable wet hail concentration gradients of various 1-cm wide distributions of hail whose central diameter is indicated as a parameter on the curves. There is essentially no change for detection sensitivities for hail in heavy rain backgrounds; and in light rain backgrounds the change is commonly only an order of magnitude.

In deriving Fig. 3 we are specifying a negative $\Delta y/\Delta r$. In all situations where $Z_2>Z_1$ in the hailshaft $\Delta y/\Delta r$ goes negative at the trailing edge of the hailshaft. However, in those rare situations where $Z_1>Z_2$ in the hailshaft $\Delta y/\Delta r$ goes negative at the leading edge. This applies to the 0.5-cm stones indicated in Fig. 3 for wet hail. It will also apply in rare cases when the shape, structure, and wetting conditions are such that $Z_1>Z_2$ in the hailshaft when there will be a step *decrease* in y on entering the shaft and a step *increase* on leaving it. However, the important signature of the hail (the negative value for dy/dr) would still have been observed, although at the opposite boundary of the shaft from which it would normally be expected.

In short, because the 10-cm reflectivities in the hail size range of interest are generally greater than at 3 cm, we expect a negative $\Delta y/\Delta r$ at the trailing edge and a positive $\Delta y/\Delta r$ at the leading edge. On very rare occasions the reverse may be true. Nevertheless, since $\Delta y/\Delta r$ cannot go negative except as a result of statistical fluctuations, a suitably large negative value of $\Delta y/\Delta r$ is an unequivocal indication of hail. Whether the negative $\Delta y/\Delta r$ corresponds to the leading edge or the trailing edge depends on the presence of an opposite sign $\Delta y/\Delta r$ nearby.

5. The essence of the hail detector

The desired quantity is dy/dr from (4), or, in terms of the steps available to us, $\Delta y/\Delta r$, where Δy is simply obtained by subtracting y at r from $y+\Delta y$ at $r+\Delta r$. Note that is is not necessary to calibrate the radars for accurate power measurement since, if there is an error of a factor γ in P_2 and β in P_1,

$$\begin{aligned} y &= 10(\log P_2-\log P_1)+10(\log C_1-\log C_2) \\ &\qquad +10(\log\gamma-\log\beta), \\ &= 10(\log P_2-\log P_1)+\text{constant}. \end{aligned} \tag{13}$$

This means that when the ratio of the powers is calculated any calibration error can be regarded as part of the radar constants. These constants are identical for all volumes separated by Δr along the beam, and vanish when Δy is formed. Thus,

$$\Delta y=10(\log P_2'-\log P_1')-10(\log P_2-\log P_1). \tag{14}$$

In essence, then, a hail detector must be able to calculate Δy and show on some form of display, the locations where Δy is negative, or large and positive, since Δr is always positive.

One form of the hail detector is comprised of two separate but synchronized radars of different wavelengths whose square-law-detected video signals in each range of interest are digitized and fed to integrators. Thus, the integrators each contain an accurate average of the echo powers as is required for (1) and (2). The quantity y, in (3) and (13), can be formed by a digital division of two such average powers and by taking the logarithm in a further digital operation. The quantity Δy is formed by subtracting two values of y from two pulse volumes along the beam separated by Δr as is required by (14).

A detailed description of a system design based on these concepts, of which two versions have been successfully constructed, is described elsewhere (Eccles and Atlas, 1969). This system includes a control to set the detection level, $-L$, for hail at the far side of the shaft. It also includes a large positive level for comparing $\Delta y/\Delta r$ with $2qM_{5\%}$ (see Section 2) so that the near sides of hail shafts are also indicated. Such hail indications appear in real time and can be inserted into the video input of standard PPI and RHI scopes.

6. Discussion

The full sensitivity of the proposed hail detector can be achieved only if the two radar beams are reasonably narrow, equal and collimated.

It is clear that as the hail volume becomes smaller than the pulse volume, the reflectivity Z of the background precipitation will make an increasingly large contribution to both Z_2+Z and Z_1+Z; the ratio $(Z_2+Z)/(Z_1+Z)$ accordingly decreases. Thus, the important quantity F in (3) and (4) decreases. It is therefore essential that the beam dimensions be as small as the smallest hail volumes which one desires to detect. Although many hail shafts may have a horizontal width less than 1 mi, corresponding to the width of a 1° beam at 57 mi, beams $\lesssim 1°$ require antennas $\gtrsim 25$ ft diameter at 10 cm wavelength. Thus, practical considerations limit us to beamwidths of $\sim 1°$.

The need for equal size beams is also implicit in (3). One of our basic assumptions is that $(Z_2+Z)/(Z_1+Z) = Z_{10}/Z_3 = 1$ in the absence of hail and that the difference in average echo powers, normalized for the radar constant, is due entirely to attenuation [Eq. (3)]. However, if the beams are unequal this is not generally true because the echo power is proportional to the distribution of reflectivity across the beam, weighted by the two-way radiation pattern. When the reflectivity distribution is highly peaked somewhere within the beam, the average echo power may be greatly reduced and so indicate an apparent reflectivity much less than the peak value (Donaldson and Tear, 1963). If the two beams differ in size, this "averaging down" effect will differ, as Donaldson and Tear show. Identical reflectivity profiles can only arise if the higher resolution radar is farther from the storm so that the radars illuminate identical pulse volumes. When the locations of the radars coincide, the broader beam will downgrade the peak reflectivities by the greater amount; and unfortunately, it is usually the 10-cm radar which has the wider beam. Thus, when hail is present, the Z_{10}/Z_3 ratio will generally be reduced simply by an increase in the 10-cm beamwidth. Clearly we cannot afford any such contaminating effects since the Z_{10}/Z_3 ratio may already be precariously close to unity in a background of intense non-hail precipitation.

Finally, for a real-time display, it is essential that the radars are observing the same volumes. Collimation errors of one-half a beamwidth may introduce averaging-down effects, though Carbone (1972) indicates the reverse of this for some combinations of hail and storm precipitation distributions. However, provided that the azimuth and elevation of each antenna are recorded together with the storm data, post-processing on a digital computer at identical azimuth and elevation positions automatically eliminates collimation errors.

It should also be noted that the dual-wavelength system has the important additional capability of discriminating a variety of targets according to their wavelength dependence. For example, it could distinguish between precipitation echoes, for which $\eta \propto Z\lambda^{-4}$, and refractivity fluctuations, either in clear air or on the boundaries of clouds and storms, for which $\eta \propto \lambda^{-\frac{1}{3}}$. Thus, the hail detector could have the added feature of being a clear air echo detector, a lightning (plasma) echo detector, and a detector of any other target whose echo has a wavelength dependence which differs from that of precipitation. These are exciting capabilities which go far beyond hail detection and are worthy of investigation in their own right.

Other experimental methods for estimating the high probability of hail occurrence, such as the high-effective-reflectivity method (Ward *et al.*, 1965; Geotis, 1963; Dennis *et al.*, 1971) or the detection of large values of depolarization of the microwave return (Barge, 1970) can only detect large hail or asymmetric hail. Furthermore, the theory of such detection has not been carried to the point of calculating the sweep false alarm probability (of "detecting" hail where there is none) for all rain backgrounds, and there are no estimates of the smallest hail concentration detectable by such methods.

7. Summary and conclusions

A specific computable radar signature of hail, which is a negative value of the range derivative of the logarithm of the ratio of the average echo powers measured by two synchronized and slaved radars of different wavelengths (e.g., 3 and 10 cm) forms the basis for a proposed new method of hail detection. When rain alone is present, and the averages of echo power are sufficiently accurate, this derivative is always positive and proportional to the incremental difference in attenuation at the smaller wavelength, and thus to the liquid water content M. On the other hand, hail of diameters $\gtrsim 1$ cm will be detected, whether wet or dry, spongy or solid, and spherical or not, except in the presence of very heavy rain, because the derivative goes negative, in general, if there is a negative hail concentration gradient; that is, on the far side of a hail shaft. The derivative has this property because the wavelength dependence of the equivalent reflectivity factor (Z) of hail is different from that of rain, and the hail Z is generally considerably larger at 10 cm than at 3 cm. A value of this negative range derivative less than $-L/h$, a level set with (8), should thus be a strong indicator of hail. The derivative is positive with positive hail concentration gradients and if it exceeds values expected from extreme concentrations of rain alone then this too is a hail indicator which marks the near side of the hail shaft.

The particular cases of wet and dry solid spherical hail are treated in detail and it is shown that the new system should be able to detect hail concentration gradients as small as 3×10^{-4} gm m^{-3} of 2 cm diameter wet hailstones per pulse length (150 m) in a background of 10 mm hr^{-1} rain with as few as 24 independent samples of the echo power. Where hail is entirely absent on the far side of the shaft, this is the actual concentration which is detectable within the adjacent pulse volume 150 m closer to the radar. The text provides the means for evaluating the detectable hail concentration gradients in other background rains.

The full sensitivity of the technique may be achieved only with small and equal beams at both wavelengths. Large beams would permit an excessive background of non-hail precipitation to obscure the wavelength differences expected for hail alone. A larger beam at the longer wavelength would also work to obscure the hail wavelength dependence. Thus, the success in hail detection reported by Sulakvelidze *et al.* (1965) with large, unequal size beams, and without the other advantages of the present method, provides support for the conclusion that the new approach should be considerably more sensitive and operate to substantially greater ranges.

We must, however, acknowledge the possibility of unanticipated problems, in detection of hail under all of the conditions treated in this paper, resulting from either unknown features of the nature of hailstorms and their boundaries or from still unrecognized features of the signals and their statistics. Nevertheless, the present theory provided a sufficiently solid foundation for confidence in the successful performance of an instrument based on these concepts, for two such dual-wavelength radars to be constructed. One of these has detected hail in real time showing that the dual-wavelength method of detecting hail is successful (Carbone *et al.*, 1973).

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