ation is surprisingly large for the longitudinal wind component. Temporarily setting $p=0$ again and allowing for $h/l=0.45$ (curves c) produces an effect equivalent to increasing the horizontal separation of the axes: T_1 is raised and T_2 is lowered. Again the difference between curves a and c is quite large for T_1. Finally, the curves labeled d show the combined effect of the two additional factors considered here. The difference between curves a and d could be significant for correcting spectra in the inertial subrange. These differences would be apparent, for example, when checking for the isotropic $\frac{4}{3}$ ratio between crosswind and alongwind spectra or when using the inertial subrange form of the spectrum [Eq. (4)] to determine either the rate of dissipation or Kolmogorov's constant.

TABLE 1. Computed transfer functions for $d/l=0.6$, $p/l=0.2$, $h/l=0.45$.

	$T_1(k_1l)$ α (deg)				$T_2(k_1l)$ α (deg)				
k_1l	0	20	20	30	0	10	20	30	$T_3(k_1l)$
0.16	0.996	0.997	1.000	1.002	0.989	0.989	0.990	0.992	0.993
0.32	0.997	0.997	1.001	1.003	0.971	0.972	0.974	0.980	0.981
0.64	1.010	1.009	1.004	0.997	0.920	0.923	0.931	0.946	0.946
1.28	1.066	1.051	1.010	0.956	0.794	0.803	0.828	0.863	0.863
2.56	1.120	1.083	0.979	0.831	0.558	0.576	0.625	0.692	0.692
5.12	0.902	0.855	0.725	0.535	0.284	0.306	0.364	0.442	0.442
10.24	0.451	0.420	0.333	0.218	0.115	0.131	0.168	0.214	0.214
20.48	0.194	0.181	0.146	0.097	0.050	0.055	0.073	0.093	0.093

The computed transfer functions are tabulated in Table 1. Although listed to the third decimal place, the results are not quite that accurate due to the need to keep computer costs to a minimum. They are, however, accurate to $\pm 1\%$ and in most cases to ± 0.002 or better.

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Variability of Wind Over a Distance of 16.25 km

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ABSTRACT

The variability of wind with distance over New England during the period January–March, 1969, is compared to some general models and found to be larger than the models would predict. The difference is not unreasonable considering the location and the amount of cyclonic activity.

1. Introduction

A series of 41 paired, simultaneous AN/GMD-1 radiosonde flights, especially monitored and edited for accuracy, were made from Maynard and Bedford, Massachusetts, between 6 January and 3 April, 1969, a very active cyclonic period. The launch sites are 16.25 km apart. This note presents the observed variability of wind over the distance and relates the findings to models of atmospheric variability available in the literature. These models are generalizations from data taken under widely varying conditions of cyclonic activity.

2. Method

Variability is commonly expressed as the rms difference between a series of paired observations. Significance is attributed to this measure by assuming a normal probability distribution. This implies a mean difference of zero for the series.

When the mean difference of a set is not zero, as in this case, the rms statistic is an exaggeration of the appropriate probability parameter. A more suitable estimate is the standard deviation of the difference.

For wind in terms of orthogonal components, U and V, the standard deviation of the difference in winds with distance, S_d, is obtained from

$$(nS_d)^2 = n\sum (U_i - U_{i+d})^2 - [\sum (U_i - U_{i+d})]^2 + n\sum (V_i - V_{i+d})^2 - [\sum (V_i - V_{i+d})]^2. \quad (1)$$

3. Discussion

The total observed variability S_d, calculated from the 41 paired observations, is presented in Fig. 1. It is nearly constant up to 6 km, increasing at higher levels.

An estimate of the typical instrumental error which

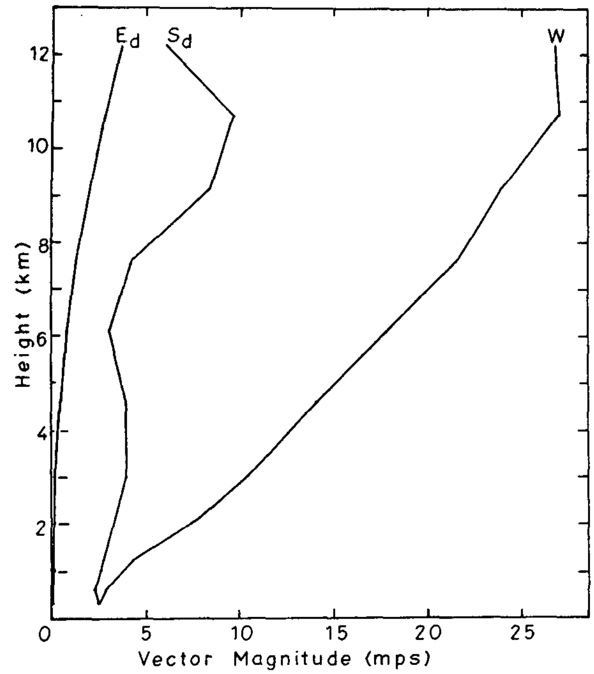

FIG. 1. Differences in wind between Bedford and Maynard: w, vector mean wind speed; S_d, observed variability; E_d, vector error.

makes up a part of the observed difference between two winds was obtained from the expression

$$E_w = 0.4633119 \times 10^{-5} h / \sin^2\alpha, \qquad (2)$$

for an error in wind as determined by AN/GMD-1 equipment; the error in the difference between two winds is given by $E_d^2 = E_1^2 + E_2^2$. The errors are in m sec^{-1}, h is the height in feet, and α the elevation angle of the radar antenna. This estimate of wind error is based on the specified tracking accuracy of the radar.[1] Values of E_d are shown in Fig. 1 and demonstrate that the observed variability is unlikely to be entirely due to wind error. True wind variability $\hat{S}_d$ can be expressed as $\hat{S}_d^2 = S_d^2 - E_d^2$. Thus, even at 12.2 km, where E_d attains its greatest value of 3.57 m sec, $\hat{S}_d$ is 4.80 m sec^{-1} as compared to the observed difference of 5.98 m sec^{-1}.

Wind statistics for the period and area of the soundings are presented in Figs. 1 and 4. The vector mean wind speed **W** was obtained by averaging the components for all flights and both launch sites and then combining the components to obtain the vector modulus. The standard vector deviation σ was obtained from the component standard deviations: the variances σ^2 were averaged for the two stations and the component averages added to obtain the average vector variance. It is apparent from Figs. 1 and 4 that some correlation exists between variability and the mean wind vector and standard vector deviation.

One of the most widely used models for variability S based on G. I. Taylor's statistical theory of turbulence is

$$S = kq^p, \qquad (3)$$

where q may be either time or distance and p is usually assigned a value of 0.5. Arnold and Bellucci (1957) provide a good discussion of this model. They examined data from numerous investigations to obtain a general expression for the variation of wind with distance, i.e., $S_d = 0.53 d^{\frac{1}{2}}$ for S_d in m sec^{-1} and d in km. Since d is known (16.25 km) and S_d has been obtained from the data, values of k can be calculated for each level for this Bedford-Maynard experiment. These are shown in Fig. 2a. They increase from about 0.6 near the surface to over 2.0 at 9 and 10 km. The variability in this sample is consistently underestimated by the values provided by Arnold and Bellucci for the general model. Also, it is not constant at all altitudes, as is implied by the model.

A similar condition was noted with the time variability model on a different set of soundings at Bedford (Lenhard *et al.*, 1963). In that case, as well as this, the values of S and of k increased with increasing wind speed. The same sort of increase of k with wind speed was noted by Eriksson (1961) with time varying data. The linear regression of k on W was determined and is shown in Fig. 2b. A correlation of 0.81 was obtained. The equation for space variability then becomes

$$S_d = (0.3192 + 0.0533W) d^{\frac{1}{2}}. \qquad (4)$$

This is equivalent to the Arnold and Bellucci estimate only when the mean vector wind is 3.95 m sec^{-1}.

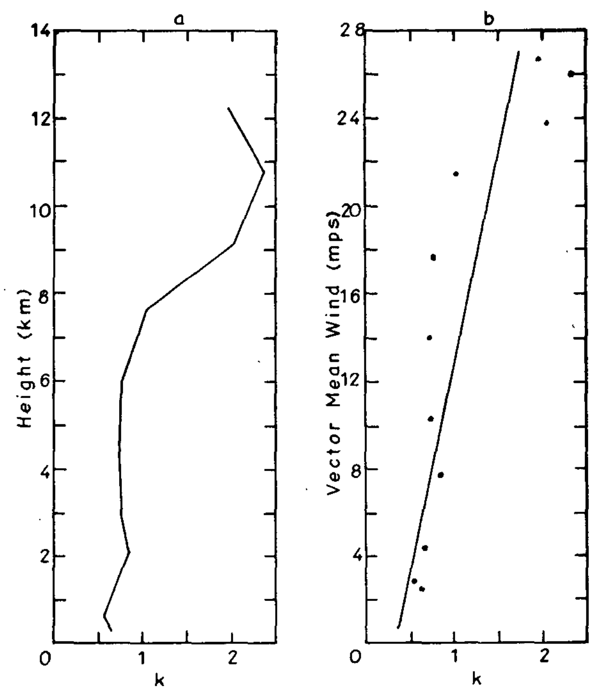

FIG. 2. Behavior of k in $s = kd^{0.5}$ as functions of altitude, a., and vector mean wind speed w, b.

[1] Unpublished letter, Hq. 4th Weather Group, Air Weather Service, USAF, 20 November 1959. Subject: Accuracy of Meteorological Upper Air Data.

An alternative method of examining variability is by means of the correlation of two sets of winds. This is related to the variability by

$$S^2=\sigma_1^2+\sigma_2^2-2r_{12}\sigma_1\sigma_2, \quad (5)$$

where S is the variability, σ_1 and σ_2 are the standard vector deviations of two sets, and r_{12} is the correlation coefficient between the two sets. Where variability with time is concerned, $\sigma_1=\sigma_2$ and the relationship becomes

$$S_t^2=2\sigma^2(1-r_t). \quad (6)$$

Where variability with distance is concerned, $\sigma_1^2\neq\sigma_2^2$ in general so the relationship for Bedford (B) and Maynard (M) is

$$S_d^2=\sigma_B^2+\sigma_M^2-2r_d\sigma_B\sigma_M. \quad (7)$$

Durst (1954) studied the variability of winds, with special emphasis on the time variability. He provided a model of the decay of correlation with time:

$$r=e^{-at}, \quad (8)$$

where $a=6.9\times10^{-6}$ sec^{-1}. Durst also indicated a 1:1 correspondence between time and space variability with the variability at 3 hr equivalent to that at 50 n mi separation. The separation between Bedford and Maynard is 8.77 n mi, corresponding to 1894.5 sec, which yields a value of $r_t=0.987$. Eqs. (6) and (7) can be combined by equating S_t with S_d. If σ is equated to the larger of σ_B or σ_M, it can be shown that $r_t>r_d$, necessarily. It was also possible to calculate r_d from the data

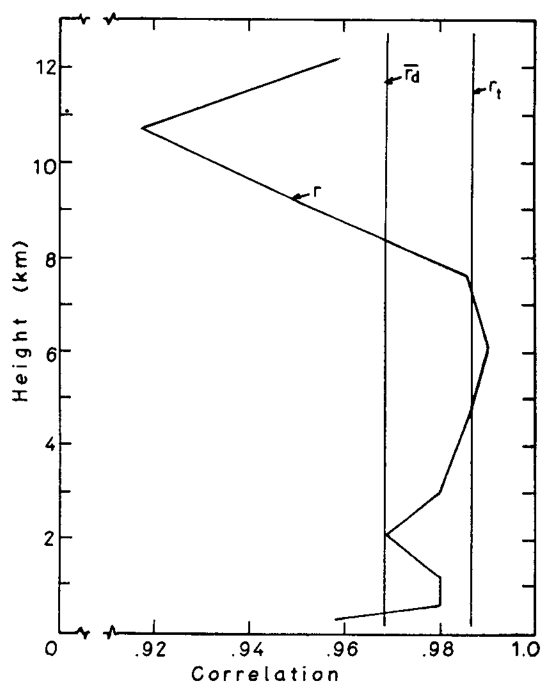

FIG. 3. Correlation of wind between Bedford and Maynard: r, correlation calculated from data; $\bar{r}$, average of calculated correlation; r_t, correlation from relation $r_t(3\text{ hr})=r_d(50\text{ n mi})$.

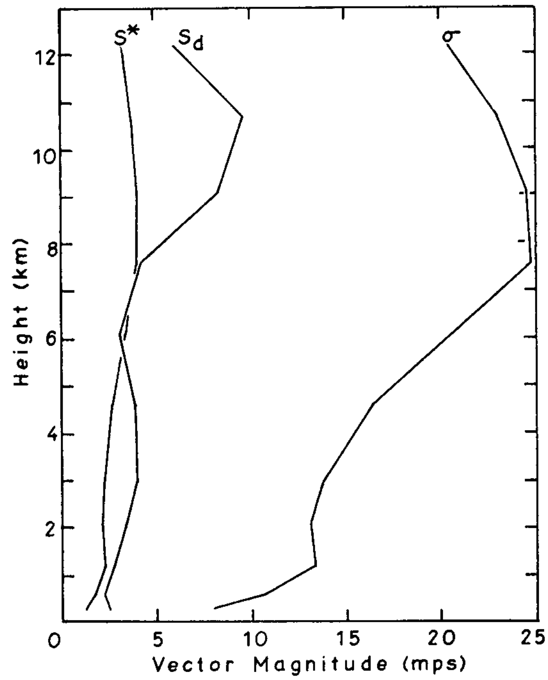

FIG. 4. Winds at Bedford and Maynard: σ, average standard deviation; S_d, observed variability between the two stations; S^*, variability calculated from $S^{*2}=0.026\sigma^2$.

and the values are shown in Fig. 3. The average of r_d for all altitudes is 0.969 but variation from level to level is considerable. At 6.1 km r_d exceeds r_t as shown in Fig. 3. This could be accounted for by a minor deviation from the general relationship between space and time or by a variation in the relationship expressed in Eq. (8) such as a variation of a with height or season as indicated by Ellsaesser (1960).

This discrepancy could also be accounted for by the balloons having drawn closer together at 6.1 km altitude. To fit Durst's models this would have been a separation of 6.88 n mi. A check of balloon positions revealed that this was not the case.

As a final comparison, the preceding assumption concerning the standard vector deviation was used with the value of $r_d=0.987$ implied by Durst's study to compute an estimated variability S^* from $S^*=2\sigma^2(1-r_d)$. The results are presented in Fig. 4. This estimate is lower than the actual S_d at all altitudes except at 6.1 km.

4. Conclusion

In summary, the mesoscale (16.25 km) upper wind variability during the first three months of 1969 over Massachusetts is somewhat larger than would be inferred from models examined. The departure does not appear to be unreasonable for the time and location and amount of cyclonic activity. Also, the increase in variability with altitude to the altitude of maximum speed near the tropopause is more than the models predict. This characteristic has been noted in other data and failure to account for such a trend appears to be an inadequacy of the models.

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Autocorrelations of Detected Radar Signals

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The purpose of this note is to review some useful relations among signal autocorrelation functions that have largely been overlooked in recent years, at least in the meteorological literature. Originally given (although without proof or elaboration) by Kerr (1951) and Lawson and Uhlenbeck (1950), these relations provide a connection between the relative velocity distribution of the scatterers and the autocorrelation functions of the output signals from three idealized detectors: linear, quadratic and logarithmic. They are fundamental in establishing the decorrelation time of detected signals in weather radars, and hence the number of independent samples in a given measurement. They also show, in effect, how to derive a limited amount of Doppler information from incoherent radar measurements.

The relations are expressed by three equations:

$$g_I(\tau)=g^2, \tag{1}$$

$$g_A(\tau)=\frac{2E(g)-(1-g^2)K(g)-\pi/2}{2-(\pi/2)}, \tag{2}$$

$$g_L(\tau)=\frac{6}{\pi^2}\sum_{k=1}^{\infty}\frac{g^{2k}}{k^2}, \tag{3}$$

where g_I, g_A and g_L denote, respectively, the autocorrelation functions of fluctuations (about the mean) in signal intensity, amplitude and intensity level (i.e., intensity measured on a logarithmic scale). They are measurable, respectively, at the output of quadratic, linear and logarithmic detectors. In Eq. (2), $K(g)$ and $E(g)$ denote the complete elliptical integrals of the first and second kind.

In each equation the variable g is itself an autocorrelation function, defined most conveniently in terms of the Doppler spectrum associated with the target. Let $F(f)$ denote this spectrum. Then g is given by

$$g(\tau)=|G(\tau)|, \tag{4}$$

where

$$G(\tau)=\int_{-\infty}^{\infty}F(f)e^{2\pi if\tau}df. \tag{5}$$

Eq. (1), for the case of signal intensity, is well known. According to Kerr (1951, 556–557) it was first derived by A. J. F. Siegert in an unpublished memorandum at the MIT Radiation Laboratory. Derivations have since been given by Fleisher (1953) and others. Many years ago Hilst (1949) carried out experiments to measure the relative velocity distribution of meteorological scatterers, basing his observations on this equation. Since then several authors have used (1) to show the relation between velocity measurements by Doppler and by incoherent radar (e.g., Atlas, 1964).

The argument, as given for example in Rogers (1963), is as follows. Associated with $g(\tau)$ in (4) there is a power spectrum defined by

$$\Phi(f)=\int_{-\infty}^{\infty}g(\tau)e^{-2\pi if\tau}d\tau. \tag{6}$$

According to (1), this spectrum may be obtained from the output of a quadratic detector by a three-step process: compute the autocorrelation function, take the square root, and transform to the frequency domain. This procedure was used by Hilst (1949) and later by Stone and Fleisher (1956), who applied a small correction to the autocorrelation to account for the use of linear, rather than quadratic, detection. While $F(f)$, the Doppler spectrum, portrays the entire radial