Stratospheric Mixing Estimated from High-Altitude Turbulence Measurements[1]

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ABSTRACT

Vertical diffusion coefficients in the stratosphere are estimated from data obtained in the High Altitude Clear Air Turbulence (HICAT) investigation. The HICAT data sample was obtained from 285 flights of over 800,000 km distance, containing 24,000 flight kilometers of turbulence between 14 and 21 km MSL, and is the only such collection of fine-scale, true gust velocities in the stratosphere.

One of the HICAT program objectives was to compute power spectral densities from each of the gust velocity component measurements over the wavelength range 30–15,000 m. The square roots of the integrals of these spectra for wavelengths <610 m, designated v_i (for the ith component of velocity), were computed and related to the dissipation of kinetic energy, ϵ, by use of the inertial range assumption in the form

$$\epsilon=[2v_i^2/(3a_i)]^{3/2}k.$$

Here k is the lower wavenumber limit of the spectral integration (2π/610 m), and a_i is equal to 0.5 for the longitudinal component and 0.65 for the lateral and vertical components, or 1.8 for the total velocity vector. With the aid of an assumption on the flux Richardson number in turbulence, the eddy heat diffusivity K_H is related to energy dissipation rate by $K_H=\epsilon/(3N^2)$, where N is the Brunt-Väisälä frequency.

Our calculations indicate that small-scale diffusion coefficients vary from order 4×10^3 cm^2 sec^{-1} over ocean regions to order 10^5 cm^2 sec^{-1} over high mountains when averaged over periods of perceptible turbulence, which include from 2.0 to 5.2%, respectively, of the total flight distance. The values decrease rapidly above 17 km except over mountainous terrain, where mixing appears to be pronounced up to and above 19 km in winter. The overall mean value over North America and Greenland is 190 cm^2 sec^{-1}, and the global mean is 120 cm^2 sec^{-1}, with an uncertainty of about half an order of magnitude.

1. Introduction

In this paper an estimate is made of the vertical diffusion coefficient for heat due to eddy mixing associated with small-scale turbulence. The estimate is made as a function of underlying terrain, with means deduced for the earth as a whole and for the geographical region of North America and Greenland. It is based on turbulence data obtained from aircraft of the Air Force HICAT program. The values obtained, if correct, are believed to be about equally valid for ozone or other more-or-less passive scalar quantities, and may be of a suitable magnitude for incorporation into three-dimensional numerical simulation models of the stratosphere, bearing in mind the very great intermittency of observed turbulence. They are not suitable for use in a zonally symmetric model because they do not include the effects of quasi-horizontal motion or mean circulations.

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2. Stratospheric turbulence data

By far the major source of definitive data on turbulent flow between 14 and 21 km MSL arises from the series of flights of an Air Force WU-2 sponsored by Project HICAT. Between 1964 and 1968, a total of 285 flight missions were conducted, covering more than 800,000 km over the continental United States, Alaska, Hawaii, Australia, New Zealand, the Carribean, Panama, eastern Canada, England and France. A comprehensive description of the program, including instrumentation, may be found in Crooks *et al.* (1967, 1968) and Ashburn *et al.* (1968, 1969, 1970), with important statistical summaries also to be found in Waco (1970, 1972). Some specific case histories have been studied in greater detail by Dutton (1969) and Lilly and Toutenhoofd (1969). The quality of the turbulence instrumentation on board the HICAT aircraft has yet to be equalled in any stratospheric aircraft installation.

A smaller and less complete set of statistical data on stratospheric turbulence was obtained from instrumentation built and installed by the Canadian National Research Council in an Air Force WB-57F during 1969 and 1970. Some aspects of these data were summarized in MacPherson and Morrissey (1969, 1970). Additional data from the WB-57F type aircraft with incomplete, but useful instrumentation were obtained during the Colorado Lee Wave Experiment of 1970 and were described in catalog form by Lilly *et al.* (1971), with reports on two case studies also appearing in Lilly (1971) and Lilly and Kennedy (1973), and Lilly and Lester (1974).

The reports by Ashburn *et al.* (1970) and Waco (1970) contain the most complete statistical summarizations of the HICAT data for estimations of probabilities, means and extremes of stratospheric turbulence and temperature fluctuations, although the analyses were undertaken more for aeronautical than climatological purposes. In this paper the various aforementioned published data were utilized. In addition, steps were taken to supplement and improve the data from a climatological point of view by increasing the sample size and by removing the bias contributed by pattern flying through a known turbulent region. The calculations given in Table 1 for the mean root mean square (rms) velocity components for wavelengths <610 m for various terrain categories show the effect of increasing sample size and removal of bias. The data in the upper part of Table 1 labeled "unweighted" were computed from all available true gust velocity measurements without removal of bias attributable to pattern flying; the data in the lower part of Table 1 labeled "weighted" were computed after a significant portion of the bias attributed to pattern flying was removed by considering a turbulence region as a single event. Thus, flights which were judged to be part of a pattern of flights through the same turbulent region were individually weighted according to their contribution (percent of total turbulent flight distance in the turbulent region) to the total event. In addition, the lower part of Table 1 is based on computations from a much larger data sample (866 runs) compared to the upper part (197 runs). This was accomplished by including in the sample rms values of the derived equivalent gust velocity (rms U_{de}). A time history of U_{de} was computed for each run from the time history of normal accelerations measured at the aircraft's center of gravity (Crooks *et al.*, 1967). A complete description of the derivation of U_{de} is found in Pratt and Walker (1954). The rms values of U_{de} were found to be highly correlated with and generally running about 10% lower than the rms vertical true gust

Table 1. Mean rms of velocity components (cm sec^{-1}) at wavelengths less than 610 m (2000 ft) for categories of terrain.

Underlying terrain	Velocity component: Vertical	Lateral	Longitudinal
		Unweighted	
Water	26.5	39.0	36.9
Flatland	26.8	40.2	34.8
Low mountains	37.8	46.9	46.0
High mountains	48.8	60.7	56.7
		Weighted	
Water	27.4	38.1	35.4
Flatland	26.8	37.5	34.8
Low mountains	31.7	42.4	39.6
High mountains	36.3	46.9	44.2

velocities. The values for rms lateral and longitudinal components in the unweighted sample (Table 1) were averaged directly from the rms true gust velocity data. In the weighted sample they were estimated by taking their average deviation from the vertical component in the unweighted sample.

From these and other supporting data in Section 3 of this paper it is clear that, even after pattern flying bias is removed, the intensity and frequency of occurrence of stratospheric turbulence is strongly controlled by underlying terrain.[5] Such a preponderance of mountain-associated turbulence does not apparently exist in the troposphere, at least over the United States (Collis *et al.*, 1969), although certain combinations of strong terrain features and mean flow conditions are known to produce high frequencies of turbulence (Harrison and Sowa, 1966). During the course of the HICAT project, this bias toward mountain areas became apparent, causing additional numbers of flights to be scheduled over mountain areas. In the end, 90% of the "greater-than-moderate" turbulence encounters occurred over high mountain areas.

3. Calculations of dissipation rates and eddy diffusivities

With the aid of certain approximations the HICAT data can be used to make estimates of the vertical mixing which results from stratospheric turbulence. Ashburn *et al.* (1970) show that the composite variance spectrum of velocity components with wavelengths $\lesssim 1000$ m follows the "$-5/3$ power law" predicted theoretically for the inertial range of turbulence. Thus, we assume variance spectra of the form

$$S_i(k)=a_i\epsilon^{2/3}k^{-5/3}, \tag{1}$$

[5] Ashburn *et al.* (1970) classify underlying terrain according to local terrain relief which is greater than 2100 m for high mountains, between 900 and 2100 m for low mountains, and less than 900 m for flatland. For the earth as a whole the proportion of water, flatland, low and high mountains is respectively 71%, 23%, 3% and 3%. For North America and Greenland the approximate corresponding proportions are 10%, 65%, 12% and 13%.

TABLE 2. Mean dissipation rates ($cm^2 sec^{-3}$) for turbulence intensity above HICAT instrumentation threshold for categories of terrain.

	$\bar{\epsilon}$			$\bar{\epsilon}_T$
Terrain	Vertical	Lateral	Longitudinal	Total
Water	3.64	9.08	11.8	6.57
Flatland	2.82	7.51	9.87	6.27
Low mountains	4.68	11.2	14.2	8.23
High mountains	10.9	23.3	29.0	16.8

where $S_i(k)$ is the one-dimensional variance spectrum of the ith component of velocity, and ϵ is the rate of kinetic energy dissipation by viscous processes; a_i is a dimensionless constant equal to about 0.5 for the longitudinal velocity component and 0.65 for the lateral and vertical components, or 1.8 for the total velocity vector. If we integrate Eq. (1) from k to infinity, we obtain an expression for the contribution to the variance associated with wavenumbers greater than k, say, $[v_i(k)]^2$. Thus, we have

$$v_i^2(k)=(\tfrac{3}{2})a_i\epsilon^{\frac{2}{3}}k^{-\frac{2}{3}}. \tag{2}$$

The mean dissipation rates were estimated by solving Eq. (2) for $k=(2\pi/610)\ m^{-1}$ to obtain

$$\bar{\epsilon}=\left(\frac{2}{3a_i}\right)^{\frac{3}{2}}\overline{[v_i^2(k)]^{\frac{3}{2}}}k, \tag{3}$$

and evaluating $\overline{[v_i^2(k)]^{\frac{3}{2}}}$ from summations of the HICAT data. The overbar indicates averaging over a large number of separate turbulence runs.

Table 2 contains estimates of $\bar{\epsilon}$ for various categories of terrain for turbulence above the HICAT instrumentation threshold (rms vertical component $=0.15$ m sec^{-1}). The column labeled $\bar{\epsilon}_T$ was computed for the vector velocity variance according to

$$\bar{\epsilon}_T=\left[\frac{2}{3a}\right]^{\frac{3}{2}}\overline{[\Sigma v_i^2(k)]^{\frac{3}{2}}}k, \tag{4}$$

where $\Sigma v_i^2(k)$ is the variance of the vector velocity, and $a=1.8$; $\bar{\epsilon}_T$ was selected for use in succeeding calculations for the sake of brevity. It is not known which estimate of energy dissipation is a more reliable result.

To proceed further, we assume that the majority of turbulence in the stratosphere is produced directly by shearing instability, even though the local shears may be developed by larger scale wave disturbances. The experiments of Thorpe (1972) suggest that not only the onset, but also the end result of such instabilities are characterized by gradient and flux Richardson numbers near $\frac{1}{4}$, so that the production of energy by Reynolds stresses operating on mean shears is just four times the loss due to up-gradient buoyant flux. We therefore assume that

$$\overline{\mathbf{V}'w'}\cdot\frac{\partial\bar{\mathbf{V}}}{\partial z}=-4g\overline{w'\theta'}/\bar{\theta}, \tag{5}$$

where the primed variables represent turbulent deviations from the average. The viscous dissipation of kinetic energy is the difference between the production and buoyant removal of energy, so that we estimate the buoyancy flux to be $\frac{1}{3}$ of the dissipation, i.e.,

$$\frac{g\overline{w'\theta'}}{\theta}=-\frac{\epsilon}{3}. \tag{6}$$

If we now define a diffusion coefficient for heat (K_H) by the usual gradient diffusion assumption

$$\overline{w'\theta'}=-K_H\partial\bar{\theta}/\partial z, \tag{7}$$

we see that K_H may be evaluated as

$$K_H=\frac{\epsilon}{3N^2}, \tag{8}$$

where N is the Brunt-Väisälä frequency

$$N^2=\frac{g}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z}. \tag{9}$$

We have evaluated $\bar{K}_H$ according to Eq. (8) assuming an isothermal stratosphere at $T=210$K ($N^2=4.6\times10^{-4}$ sec^{-2}). This assumption is justified on the grounds that the length and time scales corresponding to the overbar are much longer than those appropriate for turbulence patches. The variability of $\bar{K}_H$ is described as a function of underlying terrain (Table 3), terrain and altitude (Table 5) and terrain and season (Table 6).

Two values for $\bar{K}_H$ are given in Table 3 for four terrain types. The smaller value (second column) was computed by assuming that there is negligible contribution to dissipation and mixing from turbulence below the threshold of the HICAT measurement system (rms vertical velocity $=0.15$ m sec^{-1}). Specifically, $\bar{K}_H$ in the second column of Table 3 was com-

TABLE 3. Mean heat diffusivity coefficients ($\bar{K}_H$) computed from rms total velocity for categories of terrain (see text for clarification).

Terrain	$\bar{K}_H$ ($cm^2 sec^{-1}$) [turbulence intensity above instrumentation threshold]	$\bar{K}_H$ ($cm^2 sec^{-1}$)
Water	4800	100
Flatland	4600	110
Low mountain	6000	190
High mountain	12200	640

TABLE 4. Ratio (percent) of turbulent (vertical rms ≥0.15 m sec⁻¹) to total flight distance for categories of terrain, altitude and season.

Terrain			
Water	Flatland	Low mountains	High mountains
2.0	2.4	3.2	5.2
Altitude (km)			
13.7–15.2	15.2–16.8	16.8–18.3	>18.3
2.9	4.0	2.6	1.6
Season			
Winter*		Summer**	
Water and flatland	All mountains	Water and flatland	All mountains
2.0	4.4	2.3	4.9

* November–April.
** May–October.

puted by multiplying $\bar{K}_H$ for turbulence above the threshold by the percent of turbulent to total flight distance given in the upper portion of Table 4. The assumption of negligible contribution to $\bar{K}_H$ by turbulence below the threshold is supported by a calculation of the contribution to $\bar{K}_H$ by low-intensity turbulence below threshold for water and high mountain terrain; the amount and distribution of low-intensity turbulence below threshold was estimated for those terrain categories by fitting a least-squares exponential function to the rms observations (Fig. 1). In both cases the average K_H for the hypothetical below threshold values was 3 cm² sec⁻¹, which is sufficiently close to zero when compared to $\bar{K}_H$ for turbulence above threshold (second column of Table 3) to support the original assumption that it was zero. Although it is understood that other hypothetical distributions could yield larger values for the contribution to $\bar{K}_H$ by below-threshold turbulence, there is no support for their existence.

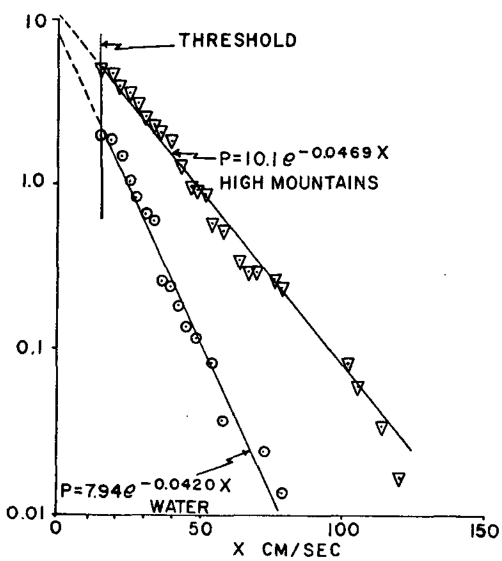

FIG. 1. Percent of flight miles with rms vertical gust velocity (for wavelengths <610 m) equalling or exceeding X for turbulence over water and high mountains.

TABLE 5. Mean heat diffusivity coefficients ($\bar{K}_H$) computed from rms total velocity for categories of terrain and altitude (see text).

Altitude (km)	Terrain	$\bar{K}_H$ (cm² sec⁻¹) [turbulence intensity above instrumentation threshold]	$\bar{K}_H$(cm² sec⁻¹)
13.7–15.2	Water and flatland	7500	200
	All mountains	4000	150
15.2–16.8	Water and flatland	4600	190
	All mountains	9400	450
16.8–18.3	Water and flatland	4100	90
	All mountains	8300	350
>18.3	Water and flatland	2600	14
	All mountains	8100	390

Table 5 shows the large difference in $\bar{K}_H$ above 16.8 km (55,000 ft) for mountains compared to flat terrain. Table 6 shows significantly larger values of $\bar{K}_H$ over mountains for winter compared to summer.

By weighting the values of $\bar{K}_H$ given for various terrain categories according to the distribution of terrain over North America and Greenland (Ashburn *et al.*, 1970), we have estimated $\bar{K}_H$ to be 190 cm² sec⁻¹ in the annual average. The corresponding figure for the entire earth is 120 cm² sec⁻¹. Similarly, $\bar{\epsilon}$ is estimated to be 0.25 cm² sec⁻³ for the stratosphere over North America and Greenland and 0.16 cm² sec⁻³ over the entire earth.

These values of dissipation rates and diffusion coefficients show the weaker and more sporadic mixing processes of the stratosphere as compared to the troposphere. Kung (1966) evaluated the dissipative losses of kinetic energy in the vicinity of the tropopause over North America and Greenland to be several cm² sec⁻³, a full order of magnitude larger than the above

TABLE 6. Mean heat diffusivity coefficients ($\bar{K}_H$) computed from rms total velocity for categories of terrain and season (see text).

Season	Terrain	$\bar{K}_H$ (cm² sec⁻¹) [turbulence intensity above instrumentation threshold]	$\bar{K}_H$ (cm² sec⁻¹)
Winter	Water and flatland	4600	90
	All mountains	10700	470
Summer	Water and flatland	4900	110
	All mountains	3200	160

estimates. Only over high mountain regions does the dissipation in perceptible regions of turbulence (from Table 2) exceed, on the average, the *mean* dissipation at jet-stream levels. Since the static stability in the stratosphere is 4–5 times higher than that in the troposphere, Eq. (8) indicates that turbulent mixing coefficients differ by two orders of magnitude between the upper troposphere and stratosphere.

4. Error analysis

There are several possible sources of error in the above computations. The HICAT data may be somewhat biased in the direction of increased turbulent frequency and intensity due to the flight motivation which was, in fact, to seek out turbulence. In view of the removal of bias associated with pattern flying, as discussed in the previous section, the only remaining bias of this sort should be that due to forecasting skill. In the HICAT program it was found, however, that the most effective forecast technique utilized the empirical relationship between terrain height and turbulence frequency, and the stratification into terrain classes should largely remove that effect.

Some error may arise from assuming an inertial range spectrum to compute dissipation rate. The error would probably be in the direction of underestimating dissipation, since the measured spectral slopes were usually a little flatter than $-5/3$, especially for the vertical component. Also, as shown by Table 1, the ratios of vertical, lateral and longitudinal turbulence amplitudes differed substantially from the 1, 1, $\frac{3}{4}$ ratios predicted for isotropic turbulence. If the longitudinal component alone were used, as is sometimes done in similar studies, Table 2 shows that the dissipation estimate would be larger by about a factor of 2.

In estimating the heat flux and diffusion coefficient, the arguments leading to Eqs. (5) and (6) may be inadequate. There are no direct atmospheric data against which to test these predictions. The nearest approach is the evaluation of buoyant losses of energy in penetrative convection (Lenschow, 1973), in which the ratio corresponding to $\frac{1}{3}$ in Eqs. (6) and (8) is closer to $\frac{1}{10}$ or less. There are good reasons to expect the ratio to be larger in the present case, however. The buoyant energy losses occur through the process of entrainment of stably stratified air into a turbulent layer. In the case of penetrative convection this occurs only at the top of the convective layer, while an elevated turbulent layer has two sides, thus doubling the entrainment region. In addition, the energy generation region in shear-induced turbulence tends to be near the edge of the shear zone, i.e., near the entrainment region, so that the turbulence is probably more effective in producing entrainment than is thermal convection, where the principal energy generation is in the middle of the layer. Nevertheless, Eq. (8) may well be in error on this account by up to a factor of 2.

5. Conclusions

The most striking result of the above evaluations is the generally small amplitude of our calculated diffusion coefficients. Reed and German (1965), Gudiksen *et al.* (1968) and Luther (1973) evaluated vertical eddy diffusion coefficients from calculations of heat and tracer transports and generally found values an order of magnitude higher than ours. We do not believe that this discrepancy necessarily implies any major error in either of these types of calculations. All of the above studies were made from zonally- and time-averaged data and therefore include the effects of transports from all scales of motion. The quasi-horizontal motions on scales of thousands of kilometers completely dominate the lateral transports and may produce an apparent vertical diffusion by some kind of higher order interaction. It seems, therefore, that small-scale turbulence is not the major cause of vertical transport of heat or trace materials for the global atmosphere, although it is the only process which can finally produce fine-scale molecular diffusion. On the other hand, Mahlman (1973)[6] has stated that for his three-dimensional simulations of the stratosphere, in which the quasi-horizontal processes are explicitly represented, vertical diffusion coefficients of similar or smaller magnitude to ours are found appropriate.

The other significant result of our study is the strong preference of stratospheric turbulent mixing events to occur over mountainous terrain. From evidence of case study analysis (Lilly and Toutenhoofd, 1969; Lilly, 1971; Lilly and Kennedy, 1973) it is clear that these events are induced by high-amplitude "breaking" gravity waves. The generation mechanism of non-mountain turbulence is less clear. Levels below 15 km, which have the highest frequency of non-mountain turbulence, are sometimes in or near the tropopause where large synoptic-scale vertical shears exist. At higher levels, turbulence is probably induced by large-amplitude gravity waves, which can be radiated upward from strong tropospheric convection, shear-induced turbulence in the troposphere, or possibly ageostrophic motion fields associated with cyclogenesis or inertial instability.

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