NOTES AND CORRESPONDENCE

Comments "On the Importance of Precision for Short-Range Forecasting and Climate Simulation"

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In a recent paper by Williamson and Washington (1973), the results of integration of the NCAR global circulation model with 48-, 24- and 21-bit mantissa arithmetic are compared with each other. Their conclusion is that the lower precision arithmetic does not have any detrimental effect on the accuracy of short-range forecasting with the current NCAR model. It is also suggested that first-order aspects of the climate seem to be quite insensitive to the precision difference, at least in the integration up to 80 days.

We would like to report that there have been cases where lower precision arithmetic required us to use special caution in order to avoid the deterioration of the numerical results. In hurricane simulation experiments as well as other projects at the Geophysical Fluid Dynamics Laboratory, the conservation of mass in a closed domain was perfect when the integration was made with a UNIVAC 1108 which has the floating point number accuracy of eight equivalent decimal digits or 27-bit mantissa. When the same model was integrated with an IBM 360 system, the accuracy of which is six to seven decimal digits or 24-bit mantissa, a very small but systematic decrease of mass was noticed. For example, the mass deficit estimated from an integration of a hurricane model was equivalent to an overall 1 mb decrease of sea level pressure after 2×10^4 iterations. Also the heat budget of the total system became inconsistent; namely, the total heat added to the system and that corresponding to the real temperature change of the model were slightly different from each other. In an extreme case, a small tendency can be even masked by a round-off error. For example, the change in the area mean total potential energy to be expected from heat budget components for a certain time step in a hurricane model was only 7.6×10^4 ergs cm^{-2}. While this amount of energy was added to the model, the model lost fictitiously more energy through round-off operations involved in the marching process for temperature. As a result, the variation of mean total potential energy of the model during that one time step turned out to be a decrease of 6×10^6 ergs cm^{-2}. Using the techniques to be described later, such difficulties could be practically eliminated. In relating to the precision problem, Harrison (1973) discussed thermal balance of a model, too.

Fig. 6 in Williamson and Washington's paper shows the zonal average pressure at the 6-km level at Day 80 in the control and 24-bit experiments. The pressure for the 24-bit case is higher than that for the control experiment at all latitudes by about 5 mb. Let us suppose that this difference is caused by precision difference. If sea level pressure distributions are assumed to be the same for the two experiments, the mean temperature below 6 km in the 24-bit case at Day 80 should be higher than that for the control experiment by more than 3K in order to yield the above-mentioned pressure difference. If temperature distributions are assumed to be the same for the two experiments, the 24-bit experiment should have the greater air mass at Day 80 by the amount equivalent to the 5-mb sea level pressure. It may be a subjective matter whether the above difference in the two experiments is regarded as climatologically significant or not. In some studies on climate change (e.g., Starr and Oort, 1973), variation in the mean temperature of the atmosphere in the amount even less than one degree in five years is discussed. At any rate, a systematic difference in their experiments with different precision is seen in the pressure field at the 6-km level, although it does not manifest itself in the zonal mean flow field as shown in their paper. It would be informative if the process, physical or computational, which caused the above difference could be clarified.

A few techniques which were tested and are implemented in some numerical models at GFDL to correct the problems associated with low precision arithmetic are listed below.

1) "Round up and down" a number before storing it in single precision location. The following is a for-

mula for this purpose, consisting of one subtraction, one multiplication and one addition:

$$R(D)=S(D)+2.0S[D-S(D)],$$

where D is a double precision number, $R(D)$ a rounded value (in single precision) of D, and $S(\ )$ a single precision value obtained by rounding-off or truncating the lower order mantissa of a double precision number.

This is one of the most effective ways to avoid the accumulation of round-off errors. It can be utilized when a small term comparable to or slightly greater than the round-off error is added to a large term. If an additive term is much smaller, additional problems will arise since it will be always rounded down.

2) Use double precision variables when it is particularly desirable to reduce the amount of round-off error or to increase accuracy of calculations. A proper scaling of a quantity may also be useful.

3) Consideration of computer arithmetic may be useful to improve the accuracy. The computer arithmetic is not always exactly commutative nor associative nor distributive. For example, suppose a variable y is computed for a given x by $(x-a)b$, where a and b are constants. If y is recomputed later by $xb-ab$, the value may be different from the previous value by an order of the round-off error.

The extra effort of programming for making arithmetic more precise as mentioned above, if it is minimized, would be justified in view of the enormous economy in the use of a lower precision computer.

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