Intercomparison of Meteorological Measurement Systems by Data-Adaptive Complementary Filtering

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ABSTRACT

The paper presents a technique for designing a self-optimizing filter to combine noisy measurements of the same physical quantity as measured by different instruments into a single time-series representative of the measured quantity (signal). The procedure minimizes the output power (sum of the squared amplitudes) of a filter that is run over the data, subject to a set of constraints on filter coefficients to characterize signal information. Practical applications of such a data-adaptive weighting algorithm are also illustrated.

1. Introduction

A large number of meteorological research efforts are based on the post-processing of an extensive space-time sampling of atmospheric variables from a multiple set of measurement systems. For instance, field experiments like NHRE (National Hail Research Experiment) and GATE (GARP Atlantic Tropical Experiment) use several aircraft with varying performance capabilities, avionics, and meteorological instrumentation to gather data for understanding the thermodynamics of atmospheric circulation. Because of the great importance of basic aircraft measurements of wind, temperature, etc., in obtaining derived thermodynamic parameters like energy and momentum fluxes in the main field program, special intercomparison experiments are usually designed to provide the necessary information on inter-instrument bias and noise characteristics of the various measurements.

Duchon (1968) and Holland and Acheson (1973) have addressed the problem of intercomparison of two independent data systems using cross-spectral analysis. This paper concerns the use of multi-channel, coherent cross-correlation techniques to combine independent measurements of the same physical quantity as measured by various systems into a single “best-estimate” representative of the measured quantity during intercomparison experiments. A “quality-factor” for each measuring system is determined through a set of weighting coefficients obtained solely from the information content of the measured data.

2. The intercomparison problem

A fairly common method of evaluating output data from measurement systems involves a comparison of output differences to produce mean-difference estimates. The efforts of Duchon (1972) with respect to NHRE are along these lines. The mean-difference estimate includes the difference that could be expected due to natural variability. When we compare two systems making independent simultaneous measurements of the same quantity, we may speak of a “comparative variance”

$$\sigma_{a,b}^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{ai} - x_{bi})^2, \quad (1)$$

and a bias determined from the mean of the differences

$$b = \frac{1}{N} \sum_{i=1}^{N} (x_{ai} - x_{bi}), \quad (2)$$

where \(x_{ai}\), \(x_{bi}\) are independent measurements from two systems \(a\), \(b\). Using the standard definition of variance

$$\sigma_a^2 = \frac{1}{N} \sum_{i=1}^{N} [(x_{ai} - \bar{x}) - (x_{ai} - \bar{x})]^2, \quad (3)$$

and substituting from Eq. (2), we have

$$\sigma_{a,b}^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{ai} - x_{bi})^2 - b^2. \quad (4)$$

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Combining Eqs. (4) and (1) gives
\[ \sigma^2_{a,b}(c) = \sigma^2_{a,b} + b^2. \] (5)

Thus, we see that the comparative variance incorporates instrumental and random variability and the bias \( b \).

From these statistical results it is not known whether the difference between systems would be the same as, larger than, or smaller than that which would be observed had the systems been identical. Most importantly, to measure accuracy, a knowledge of the “true” value is required. Such information is usually not available for meteorological measurements. Therefore, we have to ask ourselves: Can we provide a cross-calibration by combining two or more data sources of comparable sampling density to obtain the “best estimate” representative of the true value? In order to do that, it is clear that the methodology should be based on the idea that observations contain information about some underlying pattern in the time-series. The purpose of the method then is to distinguish between the correlated and uncorrelated portions of the observed data traces.

3. Data-adaptive complementary filtering

Adaptive filtering is a procedure that can be used to determine a set of weights for the time-series from observed correlations in the data set. Much of the early work on adaptive filtering was performed in the field of electrical engineering as a means to isolate “white noise” from a signal pattern through a set of weights operating on observed data. Although the theoretical basis of adaptive filtering has been presented elsewhere (Wiener, 1949; Kalman and Bucy, 1961; Widrow, 1966), it is worthwhile to point out that the real power of adaptive filtering comes in having a rule that can be used to adjust the weights consistent with the observed variances in the measurements themselves.

The ensuing discussion will be based on the terminology and concepts reviewed in an earlier paper (Govind, 1972). A multi-channel system is characterized by \( N \) input channels of time-series data (Fig. 1). A finite record of simultaneously observed measurements from \( N \) data sources can be represented by a vector of inputs
\[ x_t = \text{col}(x_{1t}, x_{2t}, \ldots, x_{Nt}), \] (6)
for discrete samples at time \( t \). Without loss of generality, we shall use \( x_{it} \) as deviations at time \( t \) from the average after removing any linear trend.

The output \( y_t \), which is the “best estimate” of the true value is constructed as the moving summation of separate channels:
\[ y_t = \sum_{i=1}^{N} \sum_{s=0}^{m} f_i(s)x_i(t-s), \] (7)
where the weights \( f_i(0), f_i(1), \ldots, f_i(m) \) are the Robinson filter coefficients [for more details see Robinson (1967) and Govind (1972)] for \( m \) lags.

The filter coefficients are customarily designed to minimize the energy \( E \) in the filtered output:
\[ E = \sum_{t} y_t^2. \] (8)

If, in addition, we demand that the filter pass that portion of the input that is common to \( x_{1t}, x_{2t}, \ldots, x_{Nt} \), this is equivalent to a set of linear constraints on the output energy filter.

Consider a design specification featuring the fol-

![Fig. 1. Simple schematic of the problem of multi-channel filtering.](image-url)
lowing constraints on the \( m \) filter coefficients:
\[
\begin{align*}
    f_1(0)+f_2(0) + \cdots + f_N(0) &= 1, \\
    f_1(1)+f_2(1) + \cdots + f_N(1) &= 0, \\
    f_1(2)+f_2(2) + \cdots + f_N(2) &= 0, \\
    \vdots \\
    f_1(m)+f_2(m) + \cdots + f_N(m) &= 0.
\end{align*}
\] (9)

Such a scheme should yield (in a symbolic sense):
\[ y_i = s_i T \text{ (terms involving } n_1, n_2, \ldots, n_N, \text{ excluding } s_i). \]

Thus, there is no distortion on the common part \( s_i \) of \( x_{1i}, x_{2i}, \ldots, x_{Ni} \) by complementing several channels. The "constraint-set" of Eq. (9) can be expressed compactly in matrix notation as
\[
C = FH^T.
\] (10)

In Eq. (10), \( C \) is the \( 1 \times (m+1) \) row matrix
\[
C = \begin{bmatrix}
0 & 0 & \cdots & 0
\end{bmatrix},
\] (11)
and \( H \) is the \( (m+1) \times N \) \((m+1)\) matrix
\[
H =
\begin{bmatrix}
1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots \\
\vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & 1 & \cdots & 1
\end{bmatrix}.
\] (12)

The output energy can be obtained using a matrix equivalent of Eq. (7):
\[
E = YY^T = FXX^T F^T = FRF^T,
\] (13)
where \( R = XX^T \) is the \((N \times N)\) correlation matrix. The output energy can be minimized subject to the constraint set (9) using the Lagrangian method of undetermined multipliers:
\[
\begin{align*}
FRF^T &= -\lambda_0 \left[ f_1(0)+f_2(0) + \cdots + f_N(0) - 1 \right] \\
&\quad -\lambda_1 \left[ f_1(1)+f_2(1) + \cdots + f_N(1) \right] \\
&\quad \vdots \\
&\quad -\lambda_m \left[ f_1(m)+f_2(m) + \cdots + f_N(m) \right] = 0.
\end{align*}
\] (14)

Setting the derivatives of energy with respect to filter-coefficients to zero, we obtain
\[
FR = AH,
\] (15)
where \( A = [\lambda_0, \lambda_1, \ldots, \lambda_m] \) is a \( 1 \times (m+1) \) row matrix.

Solving Eqs. (10) and (15), the matrix representation for the \( N \)-channel distortionless complementary filter is
\[
F = C(HR^{-1}H^T)^{-1}(HR^{-1}).
\] (16)

Eq. (16) is programmable in Fortran and various subroutines have been written to realize an operational digital filter on the CDC 6600/7600 computers.

![Fig. 2. Geometrical relationships in aircraft flight-level measurements.](image)

4. Applications

The method described above has been applied to actual data to verify that data-adaptive methods will give meaningful results for real data. The data set used in Examples I-IV below was acquired by NCAR aircraft utilized in the National Hall Research Experiment during the summers of 1972 and 1973.

a. Example I

The determination of flight level winds is of prime importance in meteorological field experiments using research aircraft. Wind speed and direction are obtained from the velocity vector relationships diagrammed in Fig. 2. Ground speed information is usually obtained quite precisely from an on-board inertial or Doppler navigation system. An airspeed sensor and a heading reference constitute an air data system which is designed to measure the true airspeed vector \( \mathbf{v}_{TAS} \). A second vector diagram of the various quantities involved in the measurements is shown in Fig. 3. The true velocity vector \( \mathbf{v}_p \) is equal to the \( TAS \) vector \( \mathbf{v}_{TAS} \) plus the wind vector \( \mathbf{v}_w \). However, an air data system measures \( \mathbf{v}_m = \mathbf{v}_{TAS} + \mathbf{v}_s \), where \( \mathbf{v}_s \) is attributable to sensor errors. Defining \( \mathbf{v}_e = \mathbf{v}_w - \mathbf{v}_s \) as an error vector, we infer that to an observer, the effects of airspeed and heading reference errors are indistinguishable from "wind."

The NCAR Sabreliner aircraft has an inertial system and a Doppler system to determine ground speed. However, both use the same air data system to compute winds. For this case of two systems sensing the
same kinematic information, we tried to compute the “best estimate” of winds during three different legs using the distortionless complementary constraint. The average value and significant linear trends were removed from the observed time-series (1-sec data samples) so that the input time-series presented to the filter appeared “stationary.” The 2-channel filter coefficients obtained from Eq. (16) were used to determine the filtered output defined by the convolution in Eq. (7). The filtered output was then used to “reconstruct” the data on each of the channels by adding the respective average values and the linear trends that were removed from the original data. In the absence of any prior knowledge about instrumental drifts, the detrended linear slopes should be restored to ensure that gradient information is preserved in the reconstructed data.

The results are collected in Fig. 4. The filter coefficients (the elements of a 1×2 matrix) are shown for various lags. A measure of the variance associated with each data channel is presented as the standard deviation in the data amplitudes after detrending. The corresponding figure for the filtered output is also marked on the diagrams. From the value of the filter coefficients, it is clear that the filter relies most heavily on the inertial input to extract the underlying pattern in the data trace over the given time period. This result also confirms the higher degree of reliability on the inertial system on a short-term basis.

b. Example II

In another case study, we used a similar data set of flight-level winds from two different airplanes, the Buffalo and the Sabreliner flying in close formation. The 1-sec data samples from each aircraft yielded independent measurements of the same meteorological parameter within the limit of measurable variations over small separation distances that are adequate to ensure that aerodynamic interaction between aircraft was minimal. The four channels used were (1) Buffalo Doppler, (2) Buffalo Inertial, (3) Sabreliner Doppler, and (4) Sabreliner Inertial.

The results are summarized in the graphical plots of Fig. 5. The reconstructed data in this situation were obtained by adding the average values to the filtered output since the sudden jump in level on channel 1 was identified to be a problem with the Doppler system. The filter coefficients for lag 0 were (0.04, 0.32, 0.01, 0.63). It is clear that the filter relies mainly on the Sabreliner inertial and Buffalo inertial measurements. This result is particularly encouraging since the Sabreliner inertial is considered to be the most reliable of all systems by the NCAR aircraft maintenance engineers.

c. Example III

A two-channel intercomparison of pressure sensors on the Buffalo and Sabreliner aircraft was made (Fig. 6). Here, the weighting scheme favored the Buffalo data. This would seem intuitively obvious, particularly since the Sabreliner data show a higher white noise component over and above the underlying pattern. Results of spectral analysis on this data set are also included to illustrate that complementary filtering picks out the coherent part of the signal from the two channels.

d. Example IV

A three-channel intercomparison was conducted between three humidity sensors (manufactured by different companies), all mounted on the same aircraft (Buffalo) for four flight paths. The traces before and after data complementation are presented in Fig. 7. One can judge how much improvement is obtained by matrix filtering by noting that the data records from
Fig. 4. Wind speed data traces from inertial and Doppler systems on the NCAR twin-jet monoplane, the Sabreliner, for three different legs. The observed data are shown by dotted lines and the reconstructed data are superimposed as solid lines. Data complementation is accomplished using a convolutional filter whose characteristics are identified in the form of filter coefficients. The standard deviations on the detrended data traces are shown for each leg.

channels 1 and 3 are well correlated and the filtering incorporates this feature in reconstructing the data on channel 2 that has a higher noise content.

e. Example V

A six-channel case was studied by using simulated data (Yerg, 1973) used to represent observations from
Fig. 5. A four-channel intercomparison of wind speed derived from Doppler and inertial systems mounted on the NCAR Sabreliner and Buffalo aircraft. The reconstructed data (shown on the right) were obtained by adding the respective average values to the filtered output.
Fig. 6. Graphical display of digitized pressure data (1-sec samples) from pressure sensors mounted on the NCAR Sabreliner and Buffalo are shown along with the output from the data-adaptive complementary filter. The spectral characteristics of the three traces are shown as spectral density (mb² Hz⁻¹) vs frequency (Hz) curves. 99% confidence limits on the autospectra are indicated by dotted lines.
Fig. 7. Three humidity sensors making independent measurements of dew point (°C) on four different legs flown by the NCAR Buffalo aircraft on 4 June 1973. The dotted lines represent the observed traces and the solid lines the improved measurements using multi-channel complementary filtering.
Fig. 8. A case study using simulated data on six channels. The input traces were simulated using truncated sine waves of unit amplitude and adding various amounts of white noise to them. The filter coefficients for lag 0 are (0.11, 0.08, 0.35, 0.33, 0.01, 0.11). This weighting scheme determines the output from the filter. The correlation coefficients are tabulated for comparison among various channels. The filter picks out the stationary random component described by the correlation function and provides a minimum variance estimate of the input data.

5. Discussion

This paper has described a technique for optimally complementing simultaneous measurements of a given physical parameter from several data sources. The complementary filter adapts to the spectral characteristics of the observed time series; there is no need to assume any model for either the noise or the signal and there is no need to make any unrealistic assumptions about the extension of the data outside the known interval. The properties of signal plus noise are preserved through the correlation matrix of the input data channels. The correlation matrix and the constraint conditions determine the data-mixing software algorithm.

From the examples worked to demonstrate the use of data-adaptive complementary filtering, it is readily obvious that the method is ideal for integration of several data sources measuring the same physical event on a given measuring platform like an airplane. Although one data source may have a significant short-term noise, its long-term stability can be used to make the "best estimate" of the measured variable by
combining this information with all the other data sets. The optimum complementation is of a dynamic nature because the rule for combining inputs depends on the ability of the filter to "learn" about the errors it is attempting to eliminate. Therefore, we should be able to obtain better estimates of inter-instrument bias [Eq. (2)] and random variability [Eq. (5)] from the reconstructed data since this corrected version accounts for phase delays due to small variations in instrumental responses. Although it seems to be a reasonably fail-safe method during intercomparison phase, it is not clear whether the results can be extrapolated to the data set gathered in the main field program involving aircraft separated by large distances. If physical separation does not introduce appreciable spatial covariances, the knowledge of inter-instrument bias and noise characteristics gathered in the intercomparison phase can be used effectively in the post-processing of data collected in the main meteorological experiment.

Extension of the techniques reviewed in this paper with additional constraint equations to model situational restraints in a multi-element system seems possible and efforts in this direction should be pursued.

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