Charged Droplet Collision Efficiency Measurements

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ABSTRACT

Experimental measurements are presented for collision efficiencies of highly charged droplets. The drop sizes studied range in radius from 10 to 25 \( \mu \)m and carry charges of the order of 10\(^{-4}\) esu. Measured collision efficiencies greater than 100 agreed well with those predicted by Paluch's analytical solution for collision efficiency upper limit. For smaller values, the collision efficiencies predicted from Davis and from Davis and Sartor exceeded those measured.

1. Introduction

The growth of cloud droplets by collision and coalescence is enhanced when attractive electrical forces are present. For some sizes and charges, calculations of collision efficiencies using a computer program developed by Davis (1964, 1969) and Davis and Sartor (1967) predict efficiencies that are orders of magnitude larger than those obtained with neutral drops. This is an important factor in the acceleration of droplet growth in thunderstorms (Sartor, 1973).

Experiments by Telford et al. (1955) and Woods (1965) have shown an increase in the number of collisions observed when the droplets were oppositely charged, although the apparatus used precluded quantitative measurements. Woods (1965), however, noticed no increase in the collision rate for 16 \( \mu \)m radius droplets carrying opposite charges \( \leq 4 \times 10^{-5}\) esu. More recently, Krasnogorskaya and Neizvestnyy (1973), using equal droplets ranging in radius from 5 to 15 \( \mu \)m, obtained good agreement with theory.

Recent technical advances in droplet generation and control have allowed us to measure directly the collision efficiencies of cloud-sized droplets which carry known charges in the range previously investigated by Woods (1965). Since the generator is not restricted to the production of pairs of equal droplets, it was possible to supplement Krasnogorskaya and Neizvestnyy's measurements by investigating interactions between pairs of droplets of unequal size.

2. Method

The collision efficiency between spheres is generally expressed as the ratio of measured capture cross section to the geometric cross section, i.e.,

\[
E = \left( \frac{y_c}{A_1 + A_2} \right)^2,
\]

where \( A_1 \) and \( A_2 \) are the droplet radii and \( y_c \) is the critical radius beyond which the drops do not collide.

We obtained collision efficiencies by a refinement of Woods and Mason's (1965) neutral droplet technique which measured \( y_c \) as a function of the droplet size. Woods and Mason produced freely falling droplets in a still air chamber and recorded their trajectories photographically. They measured the horizontal separation of the droplet trajectories before collision. Since, in these experiments, the droplets entered the chamber in a random manner, many photographs were required to ensure that the collisions were in the field of view and that the maximum separation was measured. We were able to refine this technique and reduce the number of photographs required by using the Abbott and Cannon (1972) droplet generator. With this generator it is possible to adjust electronically the time, size and charge on the droplets and to synchronize them with the camera shutter so that each exposure records an interaction. In addition, the charged droplets could be positioned electrostatically immediately after they were produced so that they would fall with predetermined horizontal spacing thus facilitating the measurement of \( y_c \).

The apparatus is illustrated in Fig. 1. The still air chamber is a sealed plexiglass box 28 cm on a side. The inside walls of the chamber are lined with black foam rubber soaked with water which raises the humidity and reduces droplet evaporation. This foam also acts as a light trap for the dark field illumination. Throughout the experiments the relative humidity was maintained at 98%, the temperature at 23\(^\circ\)C, and the pressure at 620 mm Hg.
Fig. 1. Apparatus for studying the collision efficiency of charged water droplets.

The droplet generator produced pairs of drops which fell through a hole in the upper plate and into the field of view of a 35-mm camera with a 105-mm lens and bellows which gave a 1:1 magnification. Illumination consisted of a continuous beam from a 500 W projection lamp at an angle of 25° to the camera axis and a stroboscopic lamp at the same angle. The droplet paths appeared as streaks upon which bright dots were superimposed. The spacing of these dots was determined by the frequency of the strobe light and the droplet fall velocity.

Fig. 2 is a composite photograph of the three types of interaction observed. In all cases a small droplet is produced first and begins to fall. After an appropriate delay the large droplet is formed and overtakes the small droplet in view of the camera. From photographs like these we were able to determine the size of both droplets and place limits on the value for \( y \). In the collision shown in Fig. 2c, the separation between the streaks at the top of the photograph must be \( \leq y \). In the missed collision shown in 2a, the separation must be \( \geq y \). In Fig. 2b the horizontal separation must be very nearly the critical value \( y \). Here the small droplet was passed and overtook the larger droplet from the rear. While collisions of this type were rare, the information obtained from the collisions shown in 2a and 2c was sufficient to determine the limits of \( y \).

The fact that the droplets were charged permitted us to position them by controlling the duration of the voltage pulse on the generator charge hood (Abbott and Cannon, 1972). With careful adjustment it was possible to set an initial separation of pairs of droplets which resulted in successive pairs either colliding or just missing collision which indicated that the average pair separation was close to \( y \). Photographs of 36 such pairs could be made within 2-5 min.

Droplet size was determined indirectly from determination of the droplet fall velocity assuming Stokes flow. This was obtained by measuring the distance between the strobe dots on the photographs (see Fig. 2) and multiplying by the strobe frequency. The droplet radii so measured were found to be uniform within an error of \( \pm 2\% \), most of which was attributable to the difficulty encountered in measuring the exact distance between the dots.

To complete the experiment, the drop charge was measured before and after each set of 36 photographs was taken. This was done by floating charged droplets in a known field between the parallel plates shown in Fig. 1. As the electric forces must just balance the gravitational forces, the droplet charge may be defined as

\[
q = mghV^{-1},
\]

where \( h \) is the plate separation (3.6 cm), \( V \) the voltage required to float the droplets, and \( m \) the droplet mass as determined from the fall velocity.

To determine \( V \) the plates were grounded until a droplet pair fell through the hole in the upper plate. At this time the voltage was applied to the plates and adjusted until one of the pair floated. It was usually necessary to repeat the sequence several times with successive drop pairs to find the optimum voltage.
Fig. 3. Measured collision efficiencies from set 8 according to
Eq. (1) as a function of the droplet radius ratio. Mean values of
size and charge are given in the legend. The dashed line was
computed from Eq. (3) using the mean values.

The procedure was then repeated with reversed polarity to find the voltage required for the oppositely charged drop. The voltages measured before and after sets of photographs were taken agreed within an error of 5%. The plates and inner surfaces of the chamber were grounded at all times except when these measurements were being made.

3. Comparison with theory

The collision efficiencies of small charged droplets may be computed by numerical integration. The computer program to do this has been developed by Davis (1964, 1969) and Davis and Sartor (1967). Paluch (1970) has derived a simple analytical expression for collision efficiencies in those extreme conditions where the inertial forces of both drops are negligible compared to the electrical forces. As the droplets investigated approach these conditions, we will use the more simple expression from Paluch:

$$E' = \frac{-2q_1q_2}{3\pi\eta A_1A_2(A_1+A_2)g},$$  \hspace{1cm} (3)

where $q_1$ and $q_2$ are droplet charges, $A_1$ and $A_2$ droplet radii, $\Delta g$ the difference in the terminal velocities of the droplets, and $\eta$ the dynamic viscosity of air. This expression gives the upper limit for collision efficiencies between highly charged drops.

Fig. 3 gives the results from one set of paired droplet measurements in which the mean droplet radii were 17.7 and 15.7 $\mu$m. The mean charge on the larger droplets was $+4.3 \times 10^{-8}$ esu and on the smaller $-3.8 \times 10^{-8}$. In this figure the ordinate is computed from Eq. (1), substituting the measured horizontal separation of the streaks on the photographs for $g$. Collisions between pairs are shown as circles, missed collisions as squares. The vertical transition between the circles and squares gives the range of collision efficiencies; in this case, 130–150. The dashed line was computed from Eq. (3) using the mean values for the droplet charges and radii. As can be seen, this gives slightly higher values for the collision efficiency. The horizontal scatter in the data is the result of variation of droplet size in successive pairs and the vertical scatter of the variation in horizontal separation of the paired drops.

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**Table 1. Summary of the nine sets of droplet sizes and charges studied.**

<table>
<thead>
<tr>
<th>Set</th>
<th>$A_1$ ($\mu$m)</th>
<th>$A_2$ ($\mu$m)</th>
<th>$q_1$ ($\times 10^{-8}$ esu)</th>
<th>$q_3$ ($\times 10^{-8}$ esu)</th>
<th>$E'$</th>
<th>$E''$</th>
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<tr>
<td>1</td>
<td>24.8</td>
<td>19.7</td>
<td>3.4</td>
<td>$-1.9$</td>
<td>1270</td>
<td>1000</td>
</tr>
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<td>2</td>
<td>19.5</td>
<td>16.2</td>
<td>1.6</td>
<td>$-0.98$</td>
<td>1120</td>
<td>1010</td>
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<tr>
<td>3</td>
<td>24.1</td>
<td>19.3</td>
<td>2.8</td>
<td>$-1.6$</td>
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<td>869</td>
</tr>
<tr>
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<td>21.4</td>
<td>16.2</td>
<td>2.1</td>
<td>$-1.1$</td>
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<td>610</td>
</tr>
<tr>
<td>5</td>
<td>19.3</td>
<td>15.7</td>
<td>1.1</td>
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<tr>
<td>6</td>
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<td>11.5</td>
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<td>$-0.29$</td>
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<tr>
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<td>$-0.38$</td>
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<td>0.45</td>
<td>$-0.48$</td>
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<td>68</td>
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</table>

* The last column was computed using a program by Davis and Sartor (1967).

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Fig. 4. Comparison of experimental measurements and Eq. (3). The numbered, dashed rectangles refer to the sets given in Table 1. The asterisks are from Krasnopolskaya and Neizvestny (1973) for 15 $\mu$m droplets.
The data given in Fig. 3 are typical of those obtained from eight other combinations of charges and sizes. The data are presented separately here because they are representative of the case examined by Woods (1965) from which he determined that the collision efficiency was not greater than it would have been had the drops been neutral. From Davis and Sartor (1967), neutral drops in this range would have collision efficiencies ranging from 0.1 to 0.3, or as much as three orders of magnitude smaller than those we have observed.

A summary of the results from all nine cases studied is given in Table 1. Here the values given for the radii are arithmetic means computed from each set of 36 photographs in which the individual values deviated approximately ±2% as is illustrated by the horizontal scatter in Fig. 3 (set 8 in Table 1). Collision efficiencies \(E'\) given in this table are computed from Eq. (3) using mean values for the charges and radii.

Collision efficiencies were also computed using actual values for the radii and relative velocities as measured from the photographs, again using Eq. (3). These results are shown in Fig. 4 with circles denoting collisions and squares misses. In this figure \(E\) was obtained from Eq. (1), substituting the measured horizontal separation between streaks on the photographs for \(y\). \(E'\) is the collision efficiency computed from Eq. (3) using individual sizes rather than means as was done in Table 1. Not all measurements from each set are included here but only those close to the transition between collisions and misses. The numbers accompanying the dashed rectangles refer to the set numbers given in Table 1.

As Fig. 4 illustrates, the agreement between experimental results and the upper limit \(E=E'\) is very good for larger collision efficiencies but becomes a poor approximation as the efficiencies decrease. Since the deviation at smaller values could have resulted from the simplifying assumptions used in Eq. (3), collision efficiencies were computed numerically using Davis and Sartor's computer program. Mean values for charges and sizes were used in the computations and the results are shown in Table 1. Here we see for set 9, the set with the lowest collision efficiency (97), that the computed collision efficiency is 68, 30% lower. We therefore conclude that the deviation at smaller collision efficiencies does not result from the simplifying assumptions.

Fig. 4 also includes the values obtained by Krasnogorskaia and Neizvestnyy. For drops of equal radius (15 \(\mu\)m) their measurements also fall below the predictions of Eq. (3) by more than 30%. They have reported an estimated experimental accuracy of 25% and for similar measurements of 5 and 10 \(\mu\)m droplets they found excellent agreement with those predicted with this equation.

Although our experimental errors are large, estimated at 40%, the trend of the results suggests that the Davis (1964, 1969) and Davis and Sartor (1967) theory overestimates the collision efficiencies of oppositely charged, 15–25 \(\mu\)m drops and the overestimation increases as the charge decreases.

4. Conclusions

This experimental technique permits measurements of the collision efficiencies of oppositely charged drops of unequal radii. Comparison of the results with Paluch's analytic solution for collision efficiency upper limit shows reasonable agreement when droplet charges are high and their relative velocity small. Deviations at lower efficiencies (\(\leq 100\)) cannot be resolved by using the more exact solutions given by Davis and Sartor (1967). The results also support the investigations by Krasnogorskaia and Neizvestnyy (1973) in that the measured collision efficiency of 15 \(\mu\)m radius droplets carrying opposite charges of \(\pm 4 \times 10^{-6}\) esu is much larger than that given by Woods (1965).

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REFERENCES


