A Note on the Behavior of the Temperature Structure Parameter in a Convective Layer Capped by a Marine Inversion

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ABSTRACT

Aircraft measurements of $C_T{}^2$ in an unstable marine boundary layer suggest a modification of the surface layer free-convection model. This modification is given by a function of z/z_i, where z is the observation height and z_i the height to the inversion base. This variation of $C_T{}^2$ with height may be expressed as $z^{-4/3}[1+0.84(z/z_i)+4.13(z/z_i)^2]$ for $0\leqslant z/z_i\leqslant 0.8$.

1. Introduction

The optimum design of optical systems for outdoor use requires knowledge of the turbulence and the associated refractive index fluctuations of the atmosphere. The relevant refractive-index structure parameter $C_n{}^2$ (Tatarski, 1961) is related to the temperature structure (in a dry atmosphere) by

$$C_T{}^2=(79\times10^{-6}P/T^2)^{-2}C_n{}^2,$$

where P is the pressure and T the temperature of the

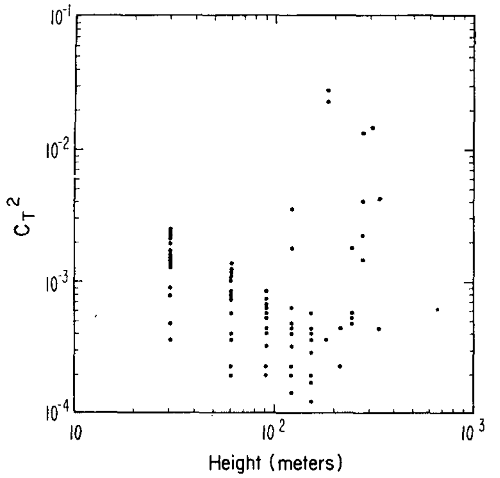

FIG. 1. Variation of $C_T{}^2$ with z/z_i for all measurements.

atmosphere. The temperature structure constant can be determined from temperature-fluctuation measurements. In the presence of water vapor, the description of $C_n{}^2$ is more complicated (Gossard, 1960) and may require consideration of the correlation or anti-correlation of the fluctuations of temperature and water vapor (Friehe *et al.*, 1972).

The behavior of $C_T{}^2$ as a function of other meteorological variables in the atmospheric surface layer, or constant-flux layer (about 10 to 100 m deep), has been studied by Wyngaard *et al.* (1971). Their measurements show that $C_T{}^2$ is a function of the Richardson number, defined by

$$\mathrm{Ri}=(g/\bar{T})(\partial\bar{\theta}/\partial z)/(\partial\bar{u}/\partial z)^2, \tag{1}$$

where g is the acceleration of gravity, $\bar{T}$ the mean temperature, $\partial\bar{\theta}/\partial z$ the mean potential temperature gradient, and $\partial\bar{u}/\partial z$ the mean wind speed gradient.

Ri can be related to the height z above the surface through a nondimensional parameter z/L (Businger *et al.*, 1971), where L (the Monin-Obukhov length) is defined as

$$L\equiv\frac{-u^{*3}\bar{T}}{kgQ}. \tag{2}$$

Here u_* is the frictional velocity scale, k the von Kármán constant, and Q the surface temperature flux.

The purpose of this note is to show that a similar relation can apply to heights above the surface layer. We use measurements of $C_T{}^2$, taken by an instrumented aircraft over the ocean in the vicinity of a floating instrument platform (*Flip*). Simultaneous measurements made on *Flip* determined the Monin-Obukhov length.

The temperature sensor mounted on the aircraft was a platinum resistance-wire thermometer with high-frequency response (Ochs and Lawrence, 1972). The measurements from *Flip*, 100 km off the coast of San Diego, were made by personnel from the University

TABLE 1. Measured values of $C_T{}^2$ [(°C^2 m$^{-2/3}$)$\times10^3$].

Date (1972)	Time	z_i (m)	$C_T{}^2$ 30 m	60 m	80 m	120 m	150 m	180 m	210 m	240 m	270 m	300 m	330 m
23 Feb	1717	360	2.1	1.2	0.84	0.48	0.58						
	1747	360	2.2	1.0	0.63	0.33	0.36						
24 Feb	1452	240	0.78	0.40	0.23	0.23	0.40						
	1539	240	0.90	0.23	0.20	0.20	0.19						
25 Feb	1345	240	0.36	0.20	0.20	0.14	0.12						
	1440	240	0.48	0.23	0.20	0.20	0.17						
27 Feb	0640	270	1.44	0.78	0.68								
	0656	270	1.33	1.2									
28 Feb	0642	330	2.4	1.2	0.58	0.44	0.44						
		330	2.4	0.78	6.9								
	0715	330	2.4	0.84									
28 Feb	1301	510	1.7	0.78	0.53	0.40	0.40						
	1339	510	1.6	1.0	0.4	0.44	0.25						
20 Apr	1224	240	1.5	0.78	0.73	0.40		0.36	0.44		2.2		4.2
	1327	240	1.4	0.73									
20 Apr	1750	210	1.5	1.4		3.5		27.2		0.02			
	1833		1.7					22.5					
21 Apr	1234	270	2.3	0.84		0.44				0.53	4.0	14.4	0.44
	1318		1.5	1.1		0.63				1.8	13.2		
	1800	270	1.4	0.78	0.40		0.19		0.23	0.48	1.4		
	1856		0.73	0.58	0.32	0.32				0.58			

of California at San Diego. Five-minute, level flights past *Flip* were made at various altitudes below and through the marine inversion. The lowest flights were 30 m above the water. The height of the marine inversion was determined by climbing flights.

2. Results and discussion

The times, dates and measured values of C_T^2 appear in Table 1, and a plot of C_T^2 vs height is shown in Fig. 1.

During the times of measurement, the boundary layer was unstable, with a Monin-Obukhov length of ~ -30 m (Friehe, 1972, personal communication). Above the boundary layer, there was an inversion layer with a height varying from 210 to 510 m. Because of these conditions, we consider it reasonable to adopt the results of Wyngaard *et al.* (1971) near the surface, modifying it empirically in terms of the ratio of height to inversion base height, z/z_i.

When $L \approx -30$ m it was found by Wyngaard *et al.* (1971) that

$$C_T^2 = [4/(3k^{\frac{2}{3}})](\bar{T}/g)^{\frac{2}{3}}(\overline{w'\theta'})^{\frac{4}{3}}z^{-\frac{4}{3}}, \quad \text{for } -7z/L \gg 1, \qquad (3)$$

where $\overline{w'\theta'}$ is the covariance between the fluctuations of the vertical velocity component w' and of temperature θ'. If z_i, the height of the inversion base, is another relevant scaling parameter (see, for example, Deardorff, 1972), then proceeding in a manner similar to that of Wyngaard *et al.*, we may modify his Eq. (3) by using an additional similarity function, $G(z/z_i)$. Thus, Eq. (3) becomes

$$C_T^2 = [4/(3k^{\frac{2}{3}})](\bar{T}/g)^{\frac{2}{3}}(\overline{w'\theta'})^{\frac{4}{3}}z^{-\frac{4}{3}}G(z/z_i), \qquad (4)$$

where $G(z/z_i)=1$ for $z/z_i \ll 1$, because we require that (4) reduces to (3) in the surface layer.

To test the validity of Eq. (4), we normalize each series of measurements to those at the lowest height, 30 m for this series of runs. Using Eq. (4) we have

$$\frac{C_T^2(z/z_i)}{C_T^2(30/z_i)} = \frac{G(z/z_i)}{G(30/z_i)}\left(\frac{z}{30}\right)^{-\frac{4}{3}}. \qquad (5)$$

Because $30/z_i < 0.15$ for all our measurements, we can assume that $G(30/z_i) \approx 1.0$. Thus we can use (5) to find $G(z/z_i)$, i.e.,

$$G(z/z_i) = \frac{C_T^2(z/z_i)}{C_T^2(30/z_i)}(z/30)^{\frac{4}{3}}.$$

A plot of C_T^2 vs z is shown in Fig. 1. In Fig. 2 we plot $G(z/z_i)$ vs z/z_i. By using this transformation, the scatter is reduced significantly over that shown in Fig. 1. For $z/z_i > 0$. with the constraint that $G(0)=1$, a least-squares fit for the data yields

$$G(z/z_i) \approx 1.0 + 0.84(z/z_i) + 4.13(z/z_i)^2, \quad \text{for } 0 \leqslant z/z_i < 0.8. \qquad (6)$$

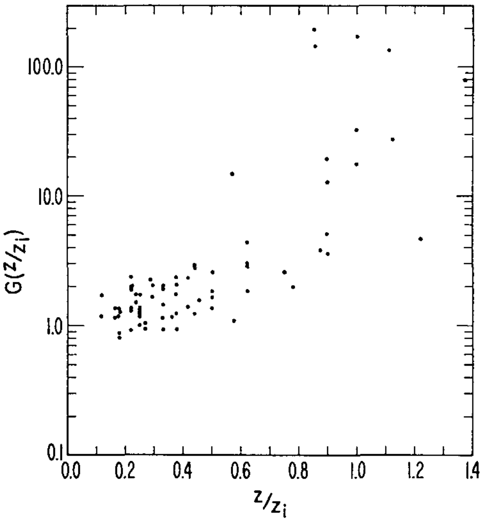

FIG. 2. Variation of $G(z/D)$ with z/D for all measurements.

This would give

$$C_T^2 \approx [4/(3k^{\frac{2}{3}})](\bar{T}/g)^{\frac{2}{3}}(\overline{w'\theta'})^{\frac{4}{3}}z^{-\frac{4}{3}} \times [1.0 + 0.84(z/z_i) + 4.13(z/z_i)^2], \quad \text{for } 0 \leqslant z/z_i \leqslant 0.8. \qquad (7)$$

There is a significant departure from a $z^{-\frac{4}{3}}$ law for values of $z/z_i > 0.1$. This indicates that the atmosphere is no longer in the "free convection" regime, but is affected by the lid on the convective layer at z_i. This effect would restrict the integral scale of the turbulent temperature fluctuations with height compared to the free convection value. In addition, because of the large-scale circulation and mixing, the temperature variance should be greater than the free-convection variance. These effects will increase the value of C_T^2 relative to the free-convection prediction.

The scatter of $G(z/z_i)$ vs z/z_i is most likely caused by uncertainty in z_i, which was obtained from the aircraft sounding. By definition, z_i is the height where $\partial\theta/\partial z = 0$. Acoustic echo sounding observations (Wyckoff *et al.*, 1973) of shallow inversion layers indicate oscillations of about 100 m amplitude. Such oscillations would cause an appreciable error in the determination made from an airplane, especially for the smaller values of z_i ($\sim$210 m).

Since $C_T^2 \propto z^{-\frac{4}{3}}G(z/z_i)$, one would expect $G(z/z_i)$ to decrease as z/z_i approaches 1. The fact that $G(z/z_i)$, shown in Fig. 2, does not become small near $z/z_i = 1$, could be the result of two factors. The abrupt transition of $G(z/z_i)$ at $z/z_i = 0.8$ seen in Fig. 2 could result from a combination of convective mixing in the boundary layer and the large temperature gradient above this transition which should lead to a large temperature variance above the inversion base that diffuses downward. In addition, the spatial variability

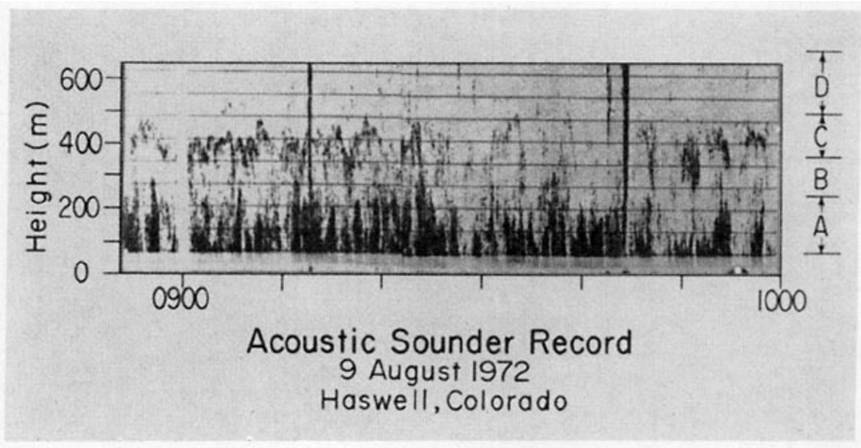

FIG. 3. An example of an acoustic sounding in a convective layer capped by an inversion. The backscattered signal intensity is proportional to the degree of darkening in the record. This intensity is also a function of C_T.

of the inversion base height and the fact that the aircraft traverses a large horizontal distance as it passes through the base would tend to produce a large temperature variance. Fig. 3, taken from Frisch and Clifford (1974), shows an example of an acoustic echo sounding taken in a convective boundary layer capped by a stable inversion. The backscattered intensity is presumably proportional to C_T^2. Thus the dark lower part of the record (region A) corresponds to high values of C_T^2, while the lighter part of the record (region B), which is changing height with time, shows very low values of C_T^2. In B, $\partial\bar{\theta}/\partial z \approx 0$, while the strong echo above occurs in the region of a large positive temperature gradient in the lower part of the inversion base. The height of the weak echo corresponding to the layer adjacent to and just below the inversion base (region B) changes with time. This time-height variability is partially related to some horizontal variability in the height of the inversion base. Hence horizontal measurements of C_T^2 by an instrumented aircraft near an inversion base could be in error. Thus in the neighborhood of $z/z_i = 1.0$, our values of $G(z/z_i)$ may be incorrect. Since the scatter in the data up to $z/z_i \approx 0.8$ is relatively small, we used only the values of $G(z/z_i)$ for $0 \leqslant z/z_i \leqslant 0.8$ for our fit of the data.

Tsvang (1969) reported a $z^{-\frac{4}{3}}$ height dependence of C_T^2 up to 500 m over land. His measurements could be explained if they were made when the inversion was considerably higher than the marine inversion layer off San Diego. Over land, the height of the inversion base capping the mixed layer is typically 1 or 2 km (e.g., Lenschow, 1970).

3. Conclusions

We have extended the surface layer analysis of Wyngaard *et al.* (1971) to 0.8 times the height of the inversion layer by use of a function involving z/z_i where z_i is the height of the base of the inversion layer. The resulting variation of C_T^2 with height was

$$C_T^2 \approx [4/(3k^{\frac{2}{3}})](\bar{T}/g)^{\frac{2}{3}}(\overline{w'\theta'})^{\frac{4}{3}}z^{-\frac{4}{3}} \times [1.0 + 0.84(z/z_i) + 4.13(z/z_i)^2].$$

Because of potential errors in determining z_i, the valid range for this relationship should be restricted to $0 \leqslant z/z_i \leqslant 0.8$.

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