Comments on "The Measurement of Charge and Size of Raindrops: Parts I and II"

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1. General comments

Bradley and Stow (1974a, b) have presented some interesting material in their two papers, and there is no doubt that many workers in this field will make use of the theoretical treatments of the various aspects of disdrometer design. However, when it comes to the use of the instrument and the interpretation of the data, it is the present author's opinion that the practice fails to live up to the promise of the theory. Bradley and Stow point out that one of the overriding problems of disdrometer design lies in the fact that it is difficult to sample a sufficient number of raindrops in a short enough time to ensure that both a good statistical sample and reasonable time resolution are obtained. They appear to hold the view that their instrument overcomes this problem to a great extent. Referring to Part II, Fig. 5 presents a drop size distribution showing a total of about 160 drops in a cubic meter. This graph is derived from some 400 data points, and normalized to 1 m^3. This group of data points is one of 23 that were obtained over a period of 21 h 40 min on 24 September 1970, during which time 9520 individual drops were recorded, and the total rainfall was 16.5 mm. The maximum rainfall rate at any time in this period was 7.5 mm h^{-1}, and Fig. 10d shows that rain was falling for about half of the recording period, so that the average collection rate of drops was about 900 per hour during rainfall; 400 drops would have taken just under one-half hour to collect. Since an increase in rainfall rate tends to be marked by an increase in the sizes of drops rather than a large increase in number concentration, these figures would not be very much different even for the most intense rainfall recorded, of about 7.5 mm h^{-1}. The distribution shows about 160 drops m^{-3}, so that 400 drops would derive from a volume of 2.5 m^3. Mason (1971) in considering the validity of rainfall data obtained by various methods notes that Mueller and Sims (1966) state that a 50 m^3 sample is needed to estimate the rainfall intensity to within 10% with 95% confidence. Viewed in this light, a sampled volume of 2.5 m^3 does not seem to constitute a good statistical sample. Furthermore, it is questionable whether or not a collection period of one-half hour can be considered to give good time resolution.

TABLE 1. Values of the factors referred to in the text used to de-normalize the data.

i	ai (mm)	$V(ai)$	Ai	$\beta i=\alpha Ai$	$n(ai)$	$[n(ai)]^{\frac{1}{2}}$	σi
1	0.2	24	1.00	0.74	18	4.2	5.7
2	0.5	28	2.52	1.86	52	7.2	3.9
3	0.7	20	3.24	2.39	48	6.9	2.9
4	0.9	17	3.81	2.81	48	6.9	2.5
5	1.1	16	4.32	3.19	51	7.1	2.2
6	1.3	10	4.74	3.50	35	5.9	1.7
7	1.5	11	5.05	3.73	41	6.4	1.7
8	1.7	14	5.27	3.90	55	7.4	1.9
9	1.9	13	5.45	4.02	52	7.2	1.8

In Part II Section 4, the authors present an expression for the goodness of fit in terms of m values derived from M data points, and p, the number of parameters being fitted. In the case of the data presented in Part II (Fig. 5), $M=400$, $m=9$, $p=2$. The authors then specify that the conditions under which a good fit is obtained are met when p is much less than m, and m is much less than M. While it is reasonable to accept that $9 \ll 400$, it is very much harder to accept that 2 is much less than 9 in such a way as to ensure that the conditions stated are satisfied.

No attempt has been made to include the error in $V(a)$ on the graph showing the drop size distribution (Part II, Fig. 5). On the basis of this graph the authors have shown a preference for certain types of expressions for this function rather than others. In order to assess the errors in $V(a)$ it is necessary to de-normalize the data, thus extracting the original values of the numbers of drops in each radius class, and the corresponding volumes from which those numbers derive. This can be done in the following manner. Tabulating the value of $V(a)$ given by Bradley and Stow for each of the nine radius classes, we assign a number i to each one, where i takes values from 1 to 9, the smallest radius class being 1. Each class derives from a column whose height is proportional to the terminal velocity of that class, $U(ai)$. Taking this height to be 1 for class 1, the ith class will derive from a column

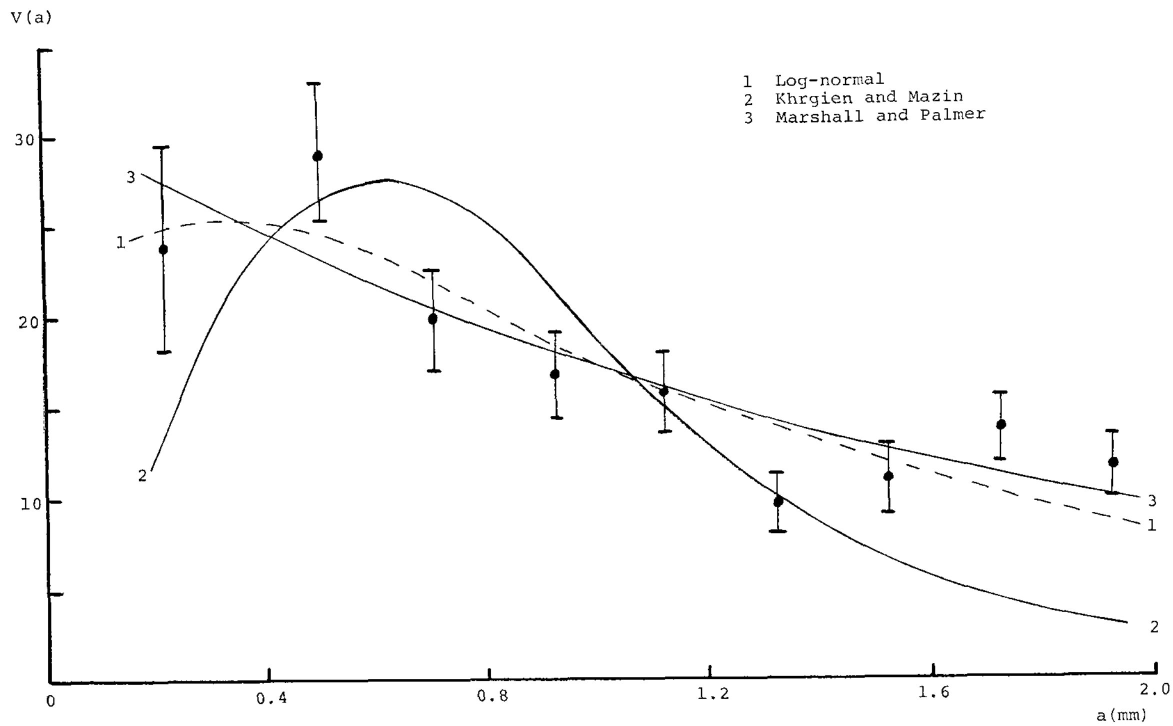

FIG. 1. The drop size distribution given by Bradley and Stow with errors included.

of height proportional to $[U(a1)/U(ai)]=Ai$. Multiplying each value of $V(ai)$ by Ai we obtain a number of drops which expresses the relative number collected in each class. We find that

$$\sum_{i=1}^{9} V(ai)Ai=541$$

in this case, whereas we know that the total number collected was 400. We define a factor $\alpha=400/541$; the number of drops collected in each class range $n(a)$ is then given by $\beta iV(ai)$, where $\beta i=\alpha Ai$. This gives

$$\sum_{i=1}^{9} \beta iV(ai)=400,$$

and each class derives from a column of height proportional to βi. Considering unit collection area cross section, the values of $V(ai)$ carry an error of $\pm\sigma i$, where $\sigma i=[n(ai)]^{\frac{1}{2}}/\beta i$. These values are shown in Table 1, and the error σi is plotted in Fig. 1, which corresponds to Bradley and Stow's Fig. 5 (Part II). It is readily seen that the errors are significant, and the choice of an expression is not so clear as the authors have suggested.

Whatever the effect of the foregoing points may be, a far more serious question arises from the method employed in positioning the disdrometer during data collection. In Part II, Section 2, it is stated that the apparatus could be tilted "so that it was aligned with its axis parallel to the mean rain direction." The meaning of this latter term is not defined by the authors, and as will be seen in the next section, the interpretation of the data will depend on the angle at which the instrument is tilted.

2. Disdrometer collection rates

We consider first the case where there is no cross wind, and the disdrometer entrance, of area $S[m^2]$,

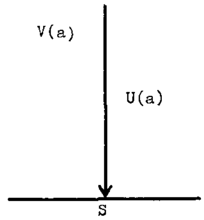

FIG. 2. Schematic view of disdrometer used horizontally with no cross wind.

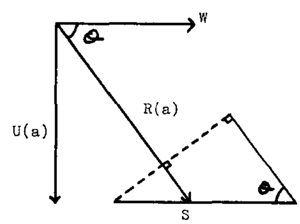

FIG. 3. Schematic view of disdrometer used horizontally with a cross wind.

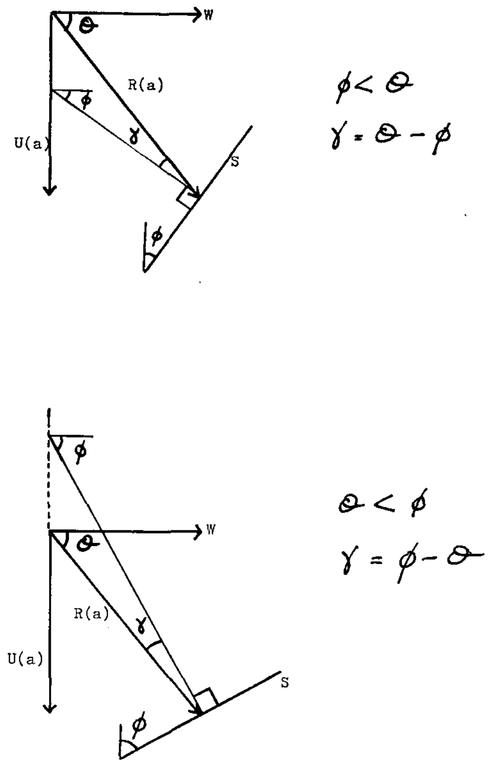

FIG. 4. Schematic view of disdrometer tilted into the cross wind.

is maintained horizontal. Referring to Fig. 2, let the concentration of raindrops of radius a be $V(a)$ [$\mathrm{m^{-3}}$]; the drops fall with vertical velocity $U(a)$ [$\mathrm{m\ s^{-1}}$] through the entrance, and the number $N(a)$ of drops of radius a collected per second is given by

$$N(a)=V(a)U(a)S. \qquad (1)$$

Next, we introduce a horizontal cross wind of W [$\mathrm{m\ s^{-1}}$], and assume that in the steady state this imparts to the drops a horizontal velocity component equal to W. The drops now have a resultant velocity $R(a)$ [$\mathrm{m\ s^{-1}}$] making an angle θ to the horizontal, where $\theta=\tan^{-1}[U(a)/W]$. Referring to Fig. 3, if the disdrometer is left with the entrance hole horizontal, then the component of S normal to $R(a)$ is now $S\sin\theta$, and the collection rate is

$$N(a)=V(a)R(a)S\sin\theta. \qquad (2)$$

However, in this case $\sin\theta=U(a)/R(a)$. Substituting for $\sin\theta$ in Eq. (2) yields $N(a)=V(a)U(a)S$, which is the same as Eq. (1). This means that the collection rate is unaffected by the cross wind if the entrance hole is left horizontal, since the reduction in the cross section of the collection area normal to the drop velocity vector is just compensated for by the increase in the number passing through that area, and the data can be treated as if the drops were falling vertically. This will hold only so long as the disdrometer can be designed in such a way that there is no significant loss of data due to drops colliding with internal parts of the instrument, which may occur in high cross winds, especially for smaller drops.

We now consider the case where the disdrometer is tilted, so that the entrance hole faces the wind. Since $R(a)$ and θ are associated with a particular radius a, the drops will enter the disdrometer at an angle, and, for a given value of W, there will be only one drop size for which the normal to the collection surface S is parallel to $R(a)$. Referring to Fig. 4, let the instrument be tilted so that the normal to the entrance hole makes angle ϕ with the horizontal. Two cases are depicted, one for $\theta<\phi$ and the other for $\phi<\theta$. The angle γ between $R(a)$ and the normal to the collection surface is $\gamma=\theta-\phi$ in the latter case, and $\gamma=\phi-\theta$ in the former. The collection rate $N(a)_\phi$ for the tilted case is then

$$N(a)_\phi=V(a)R(a)S\cos\gamma. \qquad (3)$$

Using the expansion

$$\left.\begin{aligned}\cos\gamma=\cos(\theta-\phi)&=\cos\theta\cos\phi+\sin\theta\sin\phi\\ \text{or}\qquad \cos(\phi-\theta)&=\cos\phi\cos\theta+\sin\phi\sin\theta\end{aligned}\right\}, \qquad (4)$$

and since $\cos\theta=W/R(a)$ and $\sin\theta=U(a)/R(a)$, by substitution we obtain

$$N(a)_\phi=V(a)S[W\cos\phi+U(a)\sin\phi]. \qquad (5)$$

As a check on the validity of this expression, for $\phi=90°$ (entrance hole horizontal), $\cos\phi=0$ and $\sin\phi=1$, whence Eq. (5) reduces to Eq. (1). Additionally, taking the term in brackets in Eq. (5) and equating to K, we find by differentiating that $dK/d\phi=-W\sin\phi+U(a)\cos\phi$. Setting $dK/d\phi$ equal to zero, we obtain $\tan\phi=U(a)/W$, or $\phi=\theta$ for the maximum collection rate for a given radius a, as expected. For convenience we may re-write Eq. (5) in the form

$$N(a)_\phi=V(a)SK. \qquad (6)$$

Suppose the disdrometer is operated for a time T; then the total number of drops of radius a that will be collected is $M(a)_\phi=N(a)_\phi T$ for the tilted disdrometer, and $M(a)=N(a)T$ if it is kept horizontal. In order to calculate $V(a)$ we would evaluate

$$V(a)=M(a)_\phi/(SKT) \qquad (7)$$

for a tilted disdrometer, and

$$V(a)=M(a)/[SU(a)T] \qquad (8)$$

for a horizontal disdrometer. Clearly the values we obtain for $V(a)$ must be the same in both cases, since this parameter cannot depend on the choice of operating conditions. This requirement will be met if $M(a)=FM(a)_\phi$, where $F=U(a)/K$. From (6) and (1) we can see that this condition is satisfied, since $N(a)=FN(a)_\phi$. Since the instrument measures numbers of drops which are distributed randomly throughout the sampled volume, the accuracy of the calculated values of $V(a)$ will increase as the total number sampled increases. Taking first the horizontal case, the $M(a)$ drops collected derive from a volume of $SU(a)T$ [m^3], so that the error σ in $V(a)$ is given by $\sigma=[M(a)]^{\frac{1}{2}}/[SU(a)T]$. For a tilted disdrometer the error σ_ϕ is given by $\sigma_\phi=[M(a)_\phi]^{\frac{1}{2}}/(SKT)$, since the volume sampled is SKT [m^3]. Thus, $\sigma_\phi=\sigma F^{\frac{1}{2}}$, and we see that the instrument has a different accuracy (usually greater) if operated tilted, although there is the disadvantage that the instantaneous wind speed must be known.

3. Application of this treatment to the data of Bradley and Stow

Bradley and Stow have treated their data as if vertically falling drops were encountering a horizontal collection area, even though they in fact tilted the disdrometer to face "the mean rain direction," since there was a cross wind. If the instrument had been used with its collection surface horizontal, even though there was a cross wind, the results they present would have still held, as shown in the previous section, provided that their instrument did not suffer from data loss due to drop collisions with the interior of the collection chamber. Since the exact dimensions of the instrument are not given, it is difficult to assess its performance in this respect; however, from the diagram they give it appears that the entrance hole radius was considerably smaller than that of the charge measuring cylinder beneath it. The angle of the top shield was 22°, and it seems probable that in the case of the data obtained on 24 September 1970 (from which the various distributions were calculated in full), where the wind speed is given as 10 kt (5.14 m s^{-1}), even the smallest drops would not have suffered greatly from this effect, since, with a terminal velocity of 1.6 m s^{-1}, θ would be 17.5°, and there appears to be some distance available within the instrument for them to slow down. Whether or not this assumption is valid, it is appropriate to ask how the inferred drop size distribution would have differed from that presented if the data had been treated as suggested here. The authors have used Eq. (8), but with the value of $M(a)$ applicable to a tilted disdrometer, i.e., $M(a)_\phi$. This means that they have calculated the quantity $Q(a)=M(a)_\phi/[SU(a)T]$ or $M(a)/[FSU(a)T]$, since $M(a)=FM(a)_\phi$. Since $Q(a)$ is equal to $V(a)/F$, we obtain the real values of $V(a)$ by multiplying those given by F. We can write F in the form

$$F=\{[W/U(a)]\cos\phi+\sin\phi\}^{-1}. \tag{9}$$

TABLE 2. Values of the factor F for four values of ϕ, with a wind speed W of 10 kt (5.14 m s^{-1}), and the values of $U(a)$, θ and $W/U(a)$ corresponding to the nine radius classes.

a (mm)	$U(a)$ (m s^{-1})	θ°	$W/U(a)$	F 15°	30°	45°	60°
0.2	1.60	17.5	3.22	0.30	0.31	0.33	0.40
0.5	4.03	38.0	1.27	0.67	0.62	0.62	0.67
0.7	5.17	45.0	0.99	0.82	0.73	0.71	0.73
0.9	6.09	50.0	0.84	0.93	0.82	0.77	0.78
1.1	6.90	53.5	0.75	1.02	0.88	0.81	0.81
1.3	7.57	56.0	0.68	1.10	0.92	0.84	0.83
1.5	8.06	57.5	0.64	1.15	0.95	0.86	0.84
1.7	8.44	58.5	0.61	1.18	0.97	0.88	0.85
1.9	8.72	60.0	0.59	1.21	0.99	0.89	0.86

Table 2 shows the values of F for $\phi=15°$, 30°, 45° and 60° for the nine drop radii, as well as θ, $U(a)$ and $W/U(a)$ for $W=10$ kt. On the assumption that ϕ lies somewhere between 15° and 60°, we see that the values of $V(a)$ which are obtained are very different from those derived by Bradley and Stow; for the largest drops the value of F may be between 0.86 and 1.21, and between 0.3 and 0.4 for the smallest drops, depending on the value of ϕ chosen.

4. Conclusions

It appears that the effect of tilting the disdrometer ought to be considered in the design and use of the disdrometer, and the results so obtained will be affected by this choice of operating mode. If the instrument is to be used in this way, it would seem to be necessary to bear this factor in mind when designing the disdrometer, in order that useful and reliable data may be obtained. It is further suggested that the results presented by Bradley and Stow suffer from having been derived by a method which ignores this effect.

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