A Note on the Computation of Atmospheric Surface Layer Fluxes for Use in Numerical Modeling

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ABSTRACT

Empirical formulas which have been derived to describe the profiles of wind, temperature and humidity through the atmospheric surface boundary layer (SBL) are used to derive equations predicting the fluxes of momentum, heat and moisture through the SBL. These formulas can be applied in the computation of lower boundary conditions needed for the diffusion equation in planetary boundary layer models.

1. Introduction

Numerical models which parameterize the effects of turbulence in the planetary boundary layer require special lower boundary conditions. Typically, analytical equations defining the momentum, heat and moisture fluxes through the surface boundary layer (SBL) produce a boundary condition for a finite-difference analog of the diffusion equation in the region above the SBL. In reviewing the literature, however, one can find about as many ways of calculating the lower boundary conditions as there are papers describing the method. Most methods become complicated if non-neutral conditions are allowed. Clarke (1970) proposes that the dimensionless SBL wind shear and temperature gradient profiles be integrated numerically to produce tables from which the heat and momentum fluxes can be determined using a bulk stability parameter measured across the SBL. This method requires that a table of values be stored in the computer and that the fluxes be computed by interpolation. Deardorff (1968) derived formulas for air-sea transfer coefficients which depend on a bulk Richardson number measured across the SBL. While Deardorff's method is convenient for use in numerical models, the formulas were derived from characteristic shear and temperature gradient profiles which have since been revised.

The procedure outlined here is based on the flux-profile relationships derived by Businger *et al.* (1971).

2. Description of procedure

Assuming that the exchange process is the same for moisture as it is for heat, the Monin-Obukhov theory of similitude predicts that the non-dimensional vertical gradients of wind, temperature and moisture are universal functions of a scale length L as follows:

$$\frac{kz}{u_*}\frac{\partial u}{\partial z}=\phi_M\left(\frac{z}{L}\right) \tag{1}$$

$$\frac{kz}{\theta_{v*}}\frac{\partial \theta_v}{\partial z}=\phi_H\left(\frac{z}{L}\right). \tag{2}$$

The scale length L is defined by

$$L=\frac{\bar{\theta}_v u_*^2}{kg\theta_{v*}}. \tag{3}$$

In the above:

- g acceleration of gravity
- k von Kármán's constant (=0.35)
- u horizontal wind speed
- z height above the earth's surface
- θ_v virtual potential temperature; the overbar means averaged through the SBL.

The scale quantities u_* and θ_{v*} are defined by

$$u_*^2\equiv-(\overline{w'u'}), \tag{4}$$

$$u_*\theta_{v*}\equiv-(\overline{w'\theta_v'}), \tag{5}$$

where the prime indicates the perturbation from the mean.

Businger *et al.* (1971) recently analyzed a large number of observations taken on a Kansas plain, and published new formulas for the universal functions. For stable conditions ($z/L>0$)

$$\phi_M=1+\beta\frac{z}{L}, \tag{6}$$

$$\phi_H=R+\beta\frac{z}{L}, \tag{7}$$

while for unstable conditions ($z/L<0$)

$$\phi_M=\left(1-\gamma_M\frac{z}{L}\right)^{-\frac{1}{4}}, \tag{8}$$

$$\phi_H=R\left(1-\gamma_H\frac{z}{L}\right)^{-\frac{1}{2}}. \tag{9}$$

The constants in the above equations are given by $\beta=4.7$, $R=0.74$, $\gamma_M=15$ and $\gamma_H=9$.

Integrating (1) and (2), following Paulson (1970), from the roughness height z_0 to a measurement height z_a, and solving for u_* and θ_{v*} yields

$$u_*=u_a(C_N+\psi_M/k)^{-1}, \tag{10}$$

$$\theta_{v*}=(\theta_a-\theta_0)(C_N+\psi_H/k)^{-1}R^{-1}, \tag{11}$$

where C_N is the drag coefficient for neutral conditions:

$$C_N\equiv\left(\frac{u_a}{u_*}\right)_N=\frac{1}{k}\ln\left(\frac{z_a}{z_0}\right). \tag{12}$$

The functions ψ_M and ψ_H for stable conditions are

$$\psi_M=\beta\frac{z_a}{L}, \tag{13}$$

$$\psi_H=\frac{\beta}{R}\frac{z_a}{L}, \tag{14}$$

and for unstable conditions

$$\psi_M=-\ln\left[\left(\frac{1+\epsilon^2}{2}\right)\left(\frac{1+\epsilon}{2}\right)^2\right]+2\tan^{-1}\epsilon-\frac{\pi}{2}, \tag{15}$$

$$\psi_H=-2\ln\left(\frac{1+\eta}{2}\right), \tag{16}$$

where $\epsilon\equiv[1-\gamma_M(z_a/L)]^{\frac{1}{4}}$ and $\eta\equiv[1-\gamma_H(z_a/L)]^{\frac{1}{2}}$.

The problem which the modeler faces is to calculate values for θ_{v*} and u_* given previously calculated values of wind and virtual potential temperature at the heights z_a and z_0, so that (4) and (5) can be used to compute the turbulent fluxes. Substituting (10) and (11) into (3) gives

$$\frac{z_a}{L}=\left[\frac{g}{\bar{\theta}_v}\frac{z_a(\theta_{va}-\theta_{v0})}{u_a^2}\right]\frac{\left[kC_N+\psi_M\left(\frac{z_a}{L}\right)\right]^2}{kC_N+\psi_H\left(\frac{z_a}{L}\right)}. \tag{17}$$

If the bulk Richardson number is defined as

$$\mathrm{Ri_B} \equiv \frac{g}{\bar{\theta}_v} \frac{z_a(\theta_{va}-\theta_{v0})}{u_a^2}, \tag{18}$$

then (17) can be used to describe z_a/L as a function of $\mathrm{Ri_B}$. In the case of stable conditions, we find

$$\frac{z_a}{L} = \frac{kC_N\left[\mathrm{Ri_B} - (R/2\beta) + \left(\frac{1-R}{\beta}\mathrm{Ri_B} + \frac{R^2}{4\beta^2}\right)^{\frac{1}{2}}\right]}{1-\beta\,\mathrm{Ri_B}}. \tag{19}$$

Although (17) becomes transcendental for unstable conditions, it can be accurately described by an approximation

$$\frac{z_a}{L} = \mathrm{Ri_B} f(C_N), \tag{20}$$

where

$$f(C_N) = 0.471C_N - 1.045, \quad \text{for} \quad \begin{cases} z_a/L \leqslant -0.05 \\ C_N \geqslant 10 \end{cases} \tag{21}$$

The maximum error in the computation of $\bar{u}_a u_*^{-1}$ from (10), (15) and (20) is less than 2%. For larger values of C_N, the percentage error decreases; for example, if $C_N \geqslant 20$, the error is less than 1%. Although (20) does not work out well for $0 > z_a/L > -0.05$, (19) can be used in this interval to calculate z_a/L from $\mathrm{Ri_B}$.

If the bulk Richardson number exceeds a critical value, turbulence is expected to be virtually non-existent. From (19), this critical value can be computed to be β^{-1}.

3. Summary

It has been shown that the Businger *et al.* (1971) gradient and shear profiles for the atmospheric surface layer can be used conveniently in numerical models including effects of the planetary boundary layer. In the procedure outlined above, the values of virtual potential temperature and wind evaluated at two levels in the SBL may be used to calculate the turbulent fluxes of momentum, heat and moisture. The method is non-iterative and gives essentially the same results as the more time-consuming iterative methods, providing the ratio of measurement height (z_a) to surface roughness (z_0) is greater than 30.

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