# On Radar-Raingage Comparison

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(Manuscript received 15 November 1974, in revised form 16 June 1975)

ABSTRACT

Spacial smoothing by the radar beam as well as post-detection integration reduce the variability of the distribution of rainfall rate in space. It is shown that when radar data are compared with instantaneous point rainfall rate a random error and a bias are introduced by the smoothing. This could account for some of the difficulties in the hydrological use of radars. It is shown that when raingage data are smoothed in time there is an optimum smoothing time interval such that the random error and the bias are reduced to a negligible level. A method is suggested for the optimum comparison of radar and raingage data and the possibility of a determination of *Z-R* relationships from such comparisons is discussed.

## 1. Introduction

In evaluating the capability of radar as a hydrological tool it is customary to compare radar measurements of rainfall with those given by one or more raingages. In effect, raingages are used for calibration of the radar. It is recognized that the compared measurements are of different character: raingages give an almost instantaneous value of point rainfall rate while radar-derived values correspond to a volume-averaged rainfall rate. The averaging volume is the radar measurement cell which is related to the beamwidth and/or the extent of the post-detection integration. In addition, the radar measures rainfall rate at an appreciable height over the ground. Even if this measurement is below cloud base and effects of drifting by low-level winds, evaporation and drop-size sorting are neglected, there still is a time delay between the radar and the raingage measurements. For a rapidly changing rainfall rate this time delay introduces a further discrepancy between the two measurements. Both the effect of smoothing by the radar measurement cell and the time delay depend on the distance from the radar at which the raingage is located.

It is recognized that the radar-raingage comparison is acceptably good when total amounts over several hours of rainfall are considered. Instantaneous comparisons show great discrepancies due to the various problems in the conversion from reflectivity to rainfall rate and also to the different character of the two measurements.

Attempts to establish a *Z-R* relationship from radar-raingage comparisons are successful only in uniform rain. In precipitation varying appreciably in time and space the discrepancies due to the time delay and smoothing of radar data appear to make impossible any improvement over an average *Z-R* relationship. Thus the capability of radar as a hydrological tool is limited to time-integrated rainfall.

Researchers in the field are well aware of this problem and hardly a discussion on radar measurements of rainfall takes place without mention of the subject. The modern computer handling of radar data makes it perhaps possible at the present time to deal with this problem in operational situations.

To see qualitatively the effects of the comparison of smoothed and unsmoothed data let us consider a raingage record of a storm as in Fig. 1. The instantaneous rainfall rate values $R$ were smoothed by taking a running mean over a period of 10 min and the smoothed values, which will be called $R_1$, are indicated by the dashed line. On the left-hand side of Fig. 1 the plot of $R$ vs $R_1$ is shown. The departure of the points from the 45° line is due to the smoothing. Furthermore, the 45° line is not the best fit to the points. Low values of instantaneous rainfall rate tend to be overestimated by $R_1$ and high values underestimated. This systematic discrepancy between values of $R$ and $R_1$ (as opposed to the random scatter) will be called "bias," which is not to be confused with the standard definition in the sense of bias of the mean value.

The discrepancies between space-smoothed data, as those given by the radar, and raingage data are the subject of this paper. The mean square error of the radar-raingage comparison due to smoothing and time delay as well as the correlation coefficient and the biases of the comparison are given in terms of statistical parameters of rainfall rate. The effect on the determination of *Z-R* relationships from radar and raingage measurements is considered. A method to minimize the errors and improve hydrological measurement by radar is suggested.

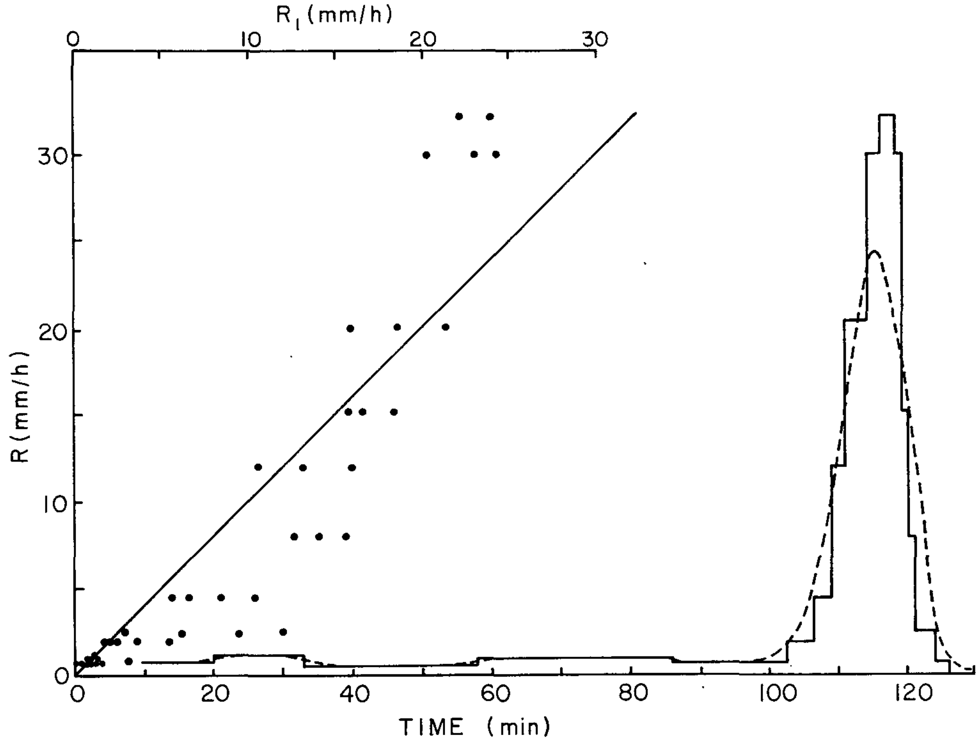


FIG. 1. An illustration of the effect of smoothing rainfall data. The storm shown here occurred in Montreal on 25 May 1970.

## 2. Radar-raingage comparison

Let us envision the distribution of precipitation in space and time as a succession of horizontal rainfall patterns which, starting at the cloud base, fall unchanged toward the ground, so that a number of raingages located along a vertical line will show identical records with only phase differences among them. This assumption is quite reasonable when applied to precipitation under cloud base. Therefore, horizontal distributions at a given height at various times correspond to horizontal distributions at a given time and various heights. Thus if $R(x,y,t)$ is the rainfall rate distribution at the height of the radar measurement (assumed to be made under cloud base) and $R_g(x,y,t)$ the simultaneous distribution at ground, we can write $R_g(x,y,t)=R(x,y,t-\tau)$ with the time delay $\tau$ replacing the vertical variable in the space-time distribution of rainfall rate.

To express the radar-measured average rainfall rate over a volume determined by the radar measurement cell we will assume that (i) the variation of rainfall rate with height within the cell is negligible under cloud base, (ii) the horizontal shape of the cell is square of dimensions $L\times L$ along the $x$ and $y$ axis, and (iii) a faultless radar measures the correct average rainfall rate over the measurement cell.

With these assumptions the distribution of rainfall rate smoothed by the radar measurement cell, $R_0(x,y,t)$, can be expressed as the running average of the actual rainfall rate, $R(x,y,t)$, by

$$R_0(x,y,t)=\frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} R(x-u,\, y-v,\, t)\,du\,dv. \tag{1}$$

Thus, radar sampling at any point of the horizontal pattern at time $t$ will detect a value given by (1). That is, $R_0(x,y,t)$ is the radar measured rainfall rate as a function of the position (within the storm) and time (during the storm's lifetime) of the radar measurement.

Now we will assume a faultless raingage measuring the average rainfall rate over a time interval $\Delta t$ and located below the central point of the area $L\times L$. This time-averaged raingage value is denoted by $R_1(x,y,t-\tau)$ and it can be expressed as a running average of the instantaneous point rainfall rate at ground, $R(x,y,t-\tau)$, by

$$R_1(x,y,t-\tau)=\frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2} R(x,y,t-\tau-w)\,dw. \tag{2}$$

A raingage measurement, simultaneous with the radar measurement, will be given by (2). However, when $R_0(x,y,t)$ is compared with $R_1(x,y,t-\tau)$ there will be a discrepancy between the two due to the difference in smoothing and to the time delay. On the

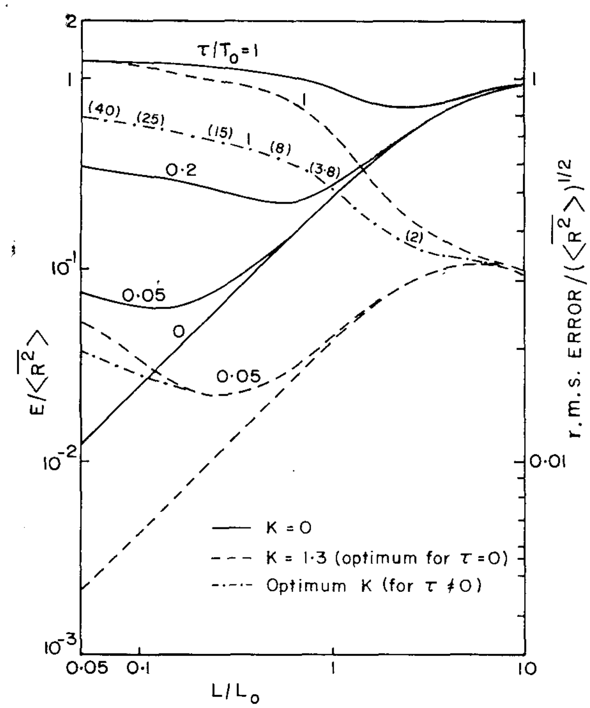


FIG. 2. Mean square error of radar estimation of instantaneous point raingage data (solid lines) and time-integrated raingage data (dashed and dot-dashed lines), for various time delays and as a function of $L/L_0$.

average for a storm this difference can be expressed as the mean square error of the comparison, given by

$$E=\overline{\langle[R_1(x,y,t-\tau)-R_0(x,y,t)]^2\rangle}, \tag{3}$$

where the angle brackets indicate a space average and the overbar a time average. That is, the average is made over all possible positions of the radar measurement cell and the raingage within the storm and during the storm's entire lifetime.

Expanding (3) we have

$$E=\overline{\langle R_1^2\rangle}+\overline{\langle R_0^2\rangle}-2\overline{\langle R_1R_0\rangle}. \tag{4}$$

To investigate the systematic discrepancies between $R_0$ and $R_1$ (as opposed to random differences) we will assume that a linear regression exists between values of $R_0$ and $R_1$. That is, on a plot of $R_0$ vs $R_1$ the radar-determined rainfall rate can be obtained from raingage measurements from

$$(R_0'-\overline{\langle R_0\rangle})=S_1(R_1-\overline{\langle R_1\rangle}), \tag{4a}$$

where $S_1$ is obtained by the least-square method and $R_0'$ are the radar-determined values obtained by the linear regression line. Or inversely, the raingage values $R_1'$ can be obtained from radar measurements by

$$(R_1'-\overline{\langle R_1\rangle})=S_2(R_0-\overline{\langle R_0\rangle}). \tag{4b}$$

If there are no systematic discrepancies, $S_1=S_2=1$ and only a random scatter around the lines will be present. The slopes $S_1$ and $S_2$ and the linear correlation coefficient $r$ between $R_0$ and $R_1$ are given by (see for example Panofsky and Brier, 1965):

$$S_1=\frac{\overline{\langle R_0R_1\rangle}-\overline{\langle R_0\rangle}\,\overline{\langle R_1\rangle}}{\overline{\langle R_1^2\rangle}-\langle\bar{R}_1\rangle^2} \tag{5}$$

$$S_2=\frac{\overline{\langle R_0R_1\rangle}-\overline{\langle R_0\rangle}\,\overline{\langle R_1\rangle}}{\overline{\langle R_0^2\rangle}-\langle\bar{R}_0\rangle^2} \tag{6}$$

$$r=\frac{\overline{\langle R_1R_0\rangle}-\overline{\langle R_1\rangle}\,\overline{\langle R_0\rangle}}{[(\overline{\langle R_1^2\rangle}-\langle\bar{R}_1\rangle^2)(\overline{\langle R_0^2\rangle}-\langle\bar{R}_0\rangle^2)]^{\frac{1}{2}}}. \tag{7}$$

Eqs. (4)–(7) can be expressed in terms of statistics of instantaneous rainfall rate. Taking space and time averages of (1) and (2) we see that neither the radar nor the raingage bias the mean value of rainfall rate. That is

$$\overline{\langle R_0\rangle}=\overline{\langle R_1\rangle}=\overline{\langle R\rangle}. \tag{8}$$

From Zawadzki (1973b) we have

$$\overline{\langle R_1^2\rangle}=\frac{1}{\Delta t^2}\int_{-\Delta t}^{\Delta t}(\Delta t-|w|)A(0,0,w)dw, \tag{9}$$

$$\overline{\langle R_0^2\rangle}=\frac{1}{L^4}\int_{-L}^{L}\int_{-L}^{L}(L-|u|)(L-|v|)A(u,v,0)dudv, \tag{10}$$

where $A(u,v,w)$ is the time-space autocorrelation function (ACF) of $R(x,y,t)$ as defined by Zawadzki (1973a), with $u$ and $v$ the space lags and $w$ the time lag. The term $\langle R_0R_1\rangle$ can be transformed using (1) and (2) and changing the order of integration:

$$\overline{\langle R_1(x,y,t-\tau)R_0(x,y,t)\rangle}=\frac{1}{L^2\Delta t}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-\Delta t/2}^{\Delta t/2}\times\overline{\langle R(x,y,t-\tau-w)R(x-u,y-v,t)\rangle}dudvdw.$$

Taking $x'=x=u$, $y'=y-v$, the integrand becomes

$$\overline{\langle R(x,y,t-\tau-w)R(x-u,y-v,t)\rangle}$$
$$=\overline{\langle R(x'+u,y'+v,t-\tau-w)R(x',y',t)\rangle}$$
$$=A(u,v,-\tau-w).$$

Thus we have finally

$$\overline{\langle R_0R_1\rangle}=\frac{1}{L^2\Delta t}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-\Delta t/2}^{\Delta t/2}\times A(u,v,-\tau-w)dudvdw. \tag{11}$$

When (8), (9), (10) and (11) are combined in (4), (5), (6) and (7) we obtain the mean square error of the radar-raingage comparison, the slopes of the regression lines of the two measurements and their correlation coefficient, in terms of the ACF and the mean value of instantaneous point rainfall rate.

## 3. Estimation of errors

For the solution of (9), (10) and (11) the expression for the autocorrelation function is needed. Drufuca and Zawadzki (1975) have calculated ACF's of rainfall rate using raingage data from 10 years of summer seasons. Both time and space ACF's were obtained using storm velocities to convert the time scale to the space scale. This procedure was justified by Zawadzki (1973a) who tested the Taylor hypothesis on radar two-dimensional precipitation patterns. The isotropy of precipitation for scales of the order of 10 km found by Zawadzki also justifies the extension, for small lags, to two dimensions of the one-dimensional ACF obtained from raingage data. The results of these two works show that rainfall rate decorrelates in space and time exponentially and that the decorrelation distance $L_0$ and the decorrelation time $T_0$ (both taken as $1/e$) vary from storm to storm. Their probability distribution was given by Drufuca and Zawadzki (1975).

The Taylor hypothesis implies that a forward lag in time compensates exactly a backward lag in space, taken in the direction of motion. This condition together with the exponential decorrelation and isotropy in space is satisfied by the following form of the ACF:

$$A(u,v,w)=\overline{\langle R^2\rangle}\exp-\left[\left(\frac{u}{L_0}+\frac{w}{T_0}\right)^2+\left(\frac{v}{L_0}\right)^2\right]^{\frac{1}{2}}, \quad (12)$$

where the $x$ axis was taken along the direction of motion.

With this expression for the ACF, the mean square error $E$, the slopes of the regression lines, $S_1$ and $S_2$, and the correlation coefficient between the radar and raingage measured rainfall rate were computed. Fig. 2 shows the values of the mean square error $E$ relative to the mean square rainfall rate as function of the size of the radar measurement cell relative to the decorrelation distance. The solid lines correspond to $\Delta t=0$, i.e., the instantaneous values of rainfall rate, and for various delay times (relative to the decorrelation time) between the radar and the raingage measurements. The line of $\tau/T_0=0$ represents the optimum situation in which the radar measurement cell is at the same height as the raingage, which is approximately the case for a very short range and low elevation of the antenna. It is seen that for small values of $L/L_0$ the time delay is the determinant source of error and its importance decreases with $L/L_0$.

TABLE 1. Statistical parameters of the storm of 25 May 1970.

| $L_0$ (km) | $T_0$ (min) | $\overline{\langle R^2\rangle}$ (mm² h⁻²) | $\langle\bar{R}\rangle$ (mm h⁻¹) | $R_{max}$ (mm) |
|---|---|---|---|---|
| 4.5 | 7.5 | 60 | 2.6 | 32.2 |

To give numerical values to $L$, $\tau$ and $E$ let us consider as an example the storm of Fig. 1, whose statistical parameters are described in Table 1. Thus, for the values of Table 1, $\tau/T_0=0.2$ corresponds to a time delay $\tau=1.5$ min or 90 s. For a fallspeed of 10 m s$^{-1}$ this corresponds to a height difference of 900 m between the location of the raingage and that of the radar measurement cell. If we take $L=5$ km ($L/L_0=1.1$) and for $\tau/T_0=0.2$, we have $E/\overline{\langle R^2\rangle}=0.29$, i.e., a mean square error of 17.4 mm$^2$ h$^{-2}$.

If the raingage data are averaged in time, a smaller mean square error should result since in this case the raingage data will resemble more closely the radar measurements. Moreover, an optimum value of the integration time can be expected. To obtain this optimum value a relationship between the space and time smoothing was established by a non-dimensional factor $K$ defined by

$$\frac{\Delta t}{T_0}=K\frac{L}{L_0}.$$

It was found that for $\tau=0$ a minimum value of the mean square error was obtained for $K=1.3$ (indicated by the dashed lines in Fig. 2). For $\tau\neq 0$ the optimum value of $K$ varies with $L/L_0$ and is indicated by numbers in parentheses. However, for a short delay ($\tau/T_0=0.05$) not much improvement is achieved by taking the optimum $K$ instead of $K=1.3$. For $\tau=0$ an improvement by a factor 5.7 or better in the mean square error is obtained by integrating the raingage data in time over the optimum time interval. For $\tau=0$ and $L/L_0<2$, a nearly linear relationship exists between $E/\overline{\langle R^2\rangle}$ and $L/L_0$; this can be expressed by

$$E_1=0.24\frac{\overline{\langle R^2\rangle}}{L_0}L \quad (13)$$

when instantaneous raingage measurements are considered, and by

$$E_2=0.042\frac{\overline{\langle R^2\rangle}}{L_0}L \quad (13a)$$

for the raingage values integrated in time over the optimum time period ($K=1.3$).

To estimate the importance of the optimization by time integration of raingage data, the percentiles of the cumulative distribution of the ratio $\overline{\langle R^2\rangle}/L_0$ have

TABLE 2. Percentiles of the cumulative distribution of $\langle \overline{R^2} \rangle / L_0$ and of $E$.

| Percentile | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|---|---|---|
| $\langle \overline{R^2} \rangle / L_0$ (mm² h⁻² km⁻¹) | 0.04 | 0.1 | 0.32 | 0.9 | 2.56 | 6.5 | 18.0 | 41 | 120 |
| $E_1$ (mm² h⁻²) | 0.02 | 0.05 | 0.15 | 0.43 | 1.23 | 3.12 | 8.65 | 19.7 | 57.6 |
| $E_2$ (mm² h⁻²) | 0.00 | 0.01 | 0.03 | 0.07 | 0.21 | 0.53 | 1.48 | 3.36 | 9.84 |

been calculated for 527 Montreal storms and are indicated in Table 2, together with the resulting percentiles of $E_1$ and $E_2$ for $L=2$ km. It is seen that when a comparison of radar and instantaneous raingage data is made, a mean square error of 57.6 mm$^2$ h$^{-2}$ or larger will be present in 10% of the storms. For the same storms, this mean square error decreases to 9.84 mm$^2$ h$^{-2}$ when the raingage data are averaged in time for $K=1.3$.

In Fig. 3 the slopes of the regression lines, $S_1$ and $S_2$, are shown as functions of $L/L_0$ and for three values of the ratio $M=\langle \bar{R} \rangle^2 / \langle \overline{R^2} \rangle$. Fifty percent of the 527 storms have values of $M$ in between zero and 0.5. The solid lines correspond to the comparison of radar and instantaneous raingage data.

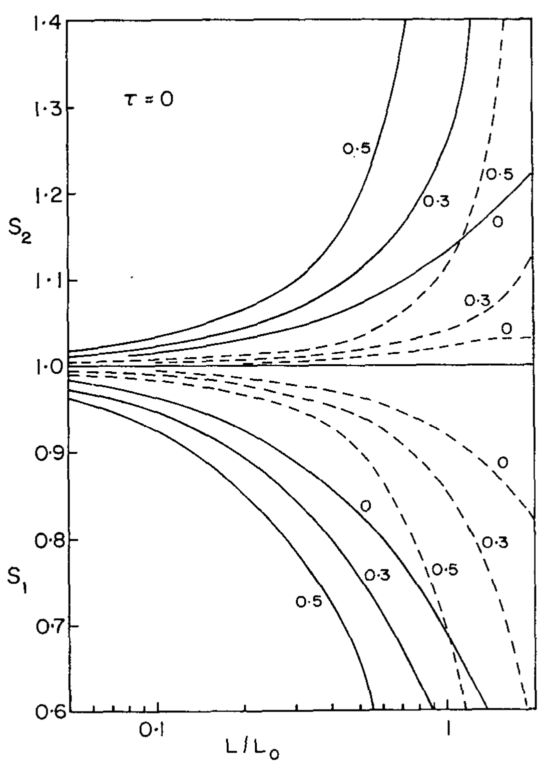


FIG. 3. Slopes of the linear regression lines of the radar estimation of point rainfall rate (upper part) and raingage estimation of radar values (lower part) as a function of $L/L_0$. The solid lines correspond to instantaneous rainfall rate and the dashed to time-integrated rainfall rate, for the optimum integration time. The numbers along the curves indicate values of $M=\langle \bar{R} \rangle^2 / \langle \overline{R^2} \rangle$.

Since the mean values of rainfall rate are, on the average, correctly estimated by both the radar and the raingage, the slope $S_1<1$ indicates that values of point rainfall rate larger than the mean will be overestimations of $R_0$, while values of $R<\langle \bar{R} \rangle$ will be underestimations of $R_0$. The inverse is true for the radar estimation of the raingage measurements. The percentage bias error (relative to the excess over the mean) is

$$e_1 = 100\times\frac{R_1-R_0'}{R_1-\langle \bar{R} \rangle} = 1\left(-\frac{R_0'-\langle \bar{R} \rangle}{R_1-\langle \bar{R} \rangle}\right)\times 100$$

for the point rainfall rate estimation of $R_0$ and

$$e_2 = 100\times\frac{R_0-R_1'}{R_0-\langle \bar{R} \rangle} = \left(1-\frac{R_1'-\langle \bar{R} \rangle}{R_0-\langle \bar{R} \rangle}\right)\times 100$$

for the radar estimation of raingage measurements. Combining (8) in (4a) and (4b) we see that

$$e_1 = (1-S_1)\times 100, \quad e_2 = (1-S_2)\times 100.$$

For the storm of Fig. 1, $M\approx 0.11$ and taking $L=5$ km we obtain from Fig. 3 $S_1\approx 0.7$ and $S_2\approx 1.11$, that is, a bias error of 30% and 11% for $e_1$ and $e_2$. In particular, the maximum rainfall rate $R_{max}$ will be underestimated by the radar by 9.9 mm h$^{-1}$.

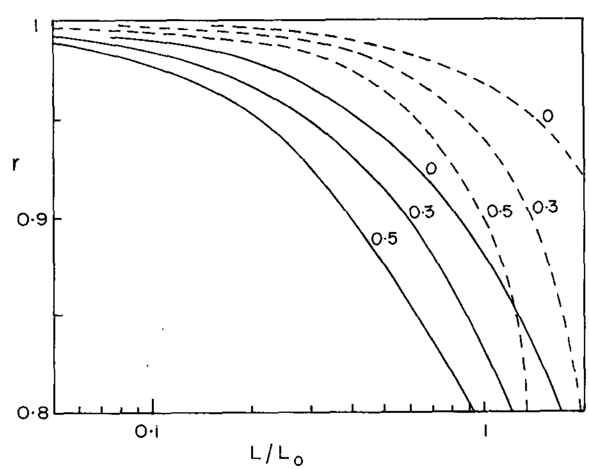


FIG. 4. Correlation coefficient between the rainfall rate smoothed in two dimensions (radar values) and instantaneous point rainfall rate (raingage values), indicated by the solid lines, and time-integrated raingage values (dashed lines), for various values of $M$ as a function of $L/L_0$.

If raingage data are averaged in time the bias decreases, that is, $S_1$ and $S_2$ approach the value of 1. The dashed curves in Fig. 3 correspond to an averaging time interval given by $K=1.3$. In our example the bias errors decrease to $e_1\approx 6\%$ and $e_2\approx 1.5\%$. It should be noticed here that the regression curve between $R_0$ and $R_1$ is not necessarily linear, as it was assumed, and our calculations should be considered only as the best linear approximation to the problem.

Not all of the mean square error represented in Fig. 2 is due to the bias just discussed. This is demonstrated by the correlation coefficient between $R_0$ and $R_1$ shown in Fig. 4, which measures the scatter around the best linear regression line between $R_0$ and $R_1$.

The averaging time period for the raingage data represented by $K=1.3$ is optimum only for $E$ (for $\tau=0$) as illustrated in Fig. 5, where $E$, $S_1$, $S_2$ and $r$ are shown as functions of $K$ for $L/L_0=1$ and $M=0.5$. The optimum values of $K$ are 0.9 for $r$, 1.3 for $E$, 1.4 for $S_2$, and 2.3 for $S_1$. Also the optimum $K$ for $S_1$, $S_2$ and $r$ changes with $L/L_0$. In any case $K=1.3$ seems to be a good choice since the calculations of $S_1$, $S_2$ and $r$ relay on the untested assumption of a linear correlation between $R_0$ and $R_1$.

## 4. Discussion and conclusions

It is implicit in this work that the post-detection integration be made on $Z$. With radar equipped with logarithmic detectors the integration is sometimes performed on $\log Z$, or $\log R$, and in this case our calculations would apply to the radar beam smoothing only. Since the variability of log $R$ will be much less severe than the variability of $R$, the errors studied here due to post-detection integration will be less important for radar with a logarithmic response. However, other errors are introduced in this case by the variability of the precipitation, as shown by Rogers (1971). On the whole, integration on a logarithmic scale, although with some technical conveniences, introduces extra complications in a problem already complex enough.

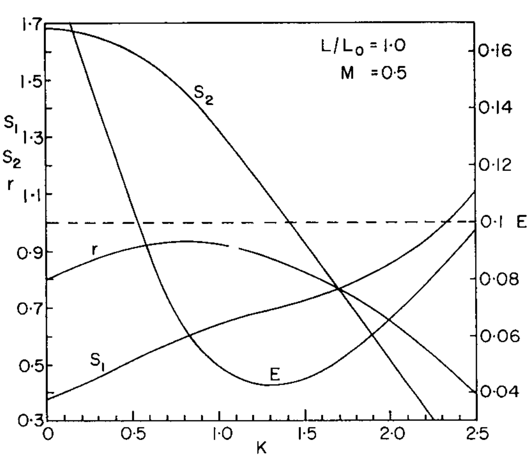


FIG. 5. Effect of the integration time of the raingage values on the curves of Figs. 2, 3 and 4.

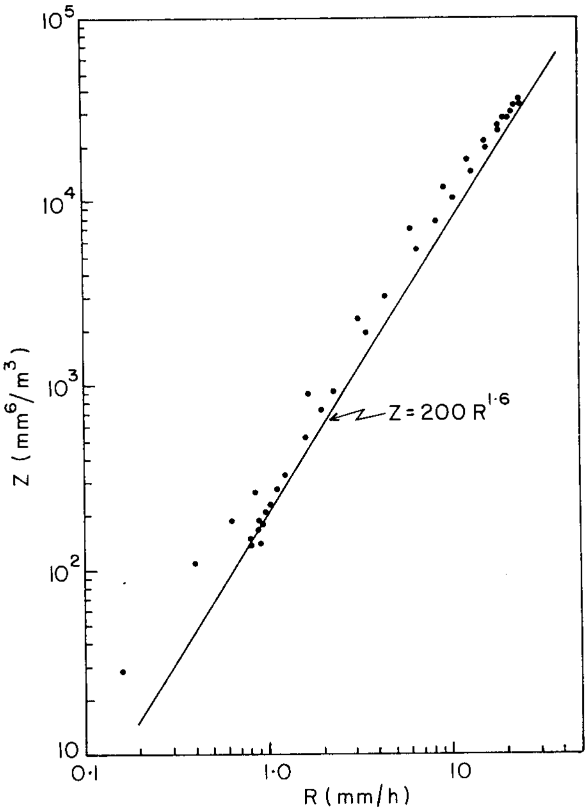


FIG. 6. Effect of smoothing $Z$ and $R$ on the $Z$-$R$ relationship. The points correspond to smoothed $Z$ and $R$ data of the storm of Fig. 1.

A uniform distribution of power within the radar beam was assumed here. This was justified by Zawadzki (1973b) who has shown that a Gaussian radar beam reduces the variance as much as a uniform one of width equal to the half-power beamwidth. In the same work it was also shown that the square shape of the radar measurement cell conserves more of the variability of the precipitation than a rectangular one, and therefore will introduce smaller errors.

The exponential form of the ACF and the values of $L_0$ and $T_0$ used here need some comments. In fact, the shape of the ACF will depend on the amount of smoothing present in the data. The finer the scale considered the faster is the initial decrease of the ACF. Within scales of the order of 100–200 m, however, there appears to be no appreciable variation of $R$ in space (Joss *et al.*, 1974). The data used by Drufuca and Zawadzki have a resolution of the order of 1 min (or 1 km when time was converted to space) and thus some undesirable smoothing was already present. This tends to increase the decorrelation time and distance and make the results of the present work somewhat conservative.

With the above comments in mind we can conclude that smoothing by the radar beam and post-detection integration introduce a random scatter and a bias with respect to point instantaneous rainfall rate. High values of rainfall rate will be underestimated by the radar and low values overestimated, as found by Desautels and Gunn (1970). The time lag between the radar and raingage measurements increases the mean square error appreciably. An integration of raingage data over an optimum time interval improves the radar-raingage comparison markedly.

With regard to using this comparison to determine a $Z$-$R$ relationship, we should first notice that the results of this work apply to $Z$ as well as $R$. In effect, it was verified that if values of rainfall rate are converted to reflectivity factor by a standard $Z$-$R$ relationship of the form

$$Z=200R^{1.6}, \tag{14}$$

the ACF of $Z$ also has an exponential form although the decorrelation time and distance decrease. Thus, taking $Z$ instead of $Z_0$ will also introduce a random error and a bias which will compensate in part for the effect of taking $R$ instead of $R_0$. That is, the combined effect of going from $R$ to $R_0$ and $Z$ to $Z_0$ should be expected to be less pronounced than the effect of either one separately. Nevertheless, there is a net scatter and bias when a $Z$-$R$ relationship is used instead of $Z_0$-$R_0$. To illustrate this the $R$ values of the storm of Fig. 1 were converted to $Z$ means of (14) and subsequently smoothed over 10 min. In this way the points of Fig. 6 were obtained. Evidently (14) is not the best fit to the points.

Therefore, there are two advantages in determining the $Z$-$R$ relationship from a radar-raingage comparison as opposed to determinations from drop-size distributions: individual relationships should give better results than an average one, and, in principle, the more precise $Z_0$-$R_1$ relationship can be obtained.

From the present work it follows that the procedure for determining the $Z_0$-$R_1$ relationship from radar-raingage comparison should be as follows:

1) $Z_0$ values from radar should be converted to $R_0$ using a standard relationship for different storm types, as suggested by Joss and Waldvogel (1970).

2) The storm velocity, determined from radar scope, should be used to convert time to space in the raingage records. The raingage data, now in the form of $R$ versus distance, should be smoothed over a space interval 1.3 times larger than the dimension of the measurement cell.

3) The smoothed raingage data should be lagged in time with respect to the radar data until the mean square difference between the two is minimum. This will eliminate the time lag effect and also the effect of drifting by low-level winds. In passing it should be mentioned that, for a given beam height over the raingage, the time lag is related to the raindrop size distribution, thus giving additional information on the $Z$-$R$ relationship.

4) Finally a $Z_0$-$R_1$ relationship can be established between the radar and smoothed raingage data.

*Acknowledgments.* This work was supported by a grant from The National Research Council of Canada. Dr. R. R. Roger's comments are very appreciated.

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