The Two-Dimensional URBMET Urban Boundary Layer Model

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ABSTRACT

A two-dimensional, non-steady model of the flow over an infinitely wide, warm, rough city is presented. The model consists of two layers, a lower analytical constant-flux layer, and an upper finite-difference transition layer, in which the vorticity and heat conduction equations are solved. The atmosphere is assumed to be Boussinesq, hydrostatic and slab symmetric, while all motions are assumed to be adiabatic. Finite-difference solutions are obtained over a variable, interlaced grid, with the use of a time-splitting technique, in conjunction with the donor cell method of differencing the advection terms.

Simulations were carried out reproducing the daytime flow of a neutral atmosphere over a rough city, and the nighttime flow of stable atmosphere over a rough, warm city. Comparisons are presented to show that the model is capable of reproducing many of the observed characteristics of the urban boundary layer.

1. Introduction

Development of numerical models simulating the transport and diffusion of pollutants in urban atmospheres has increased the need for a better understanding of the dynamics of the urban planetary boundary layer. This paper presents a two-dimensional, non-steady model of the flow over an infinitely wide, warm, rough city.

Numerical simulation of the dynamics of thermally induced circulations in the planetary boundary layer started with the sea breeze models of Estoque (1961) and Fisher (1961). Such models were first applied to urban areas by Delage and Taylor (1970), who studied the development, in otherwise calm conditions, of the “urban breeze” which flows into a city during hours when there is a well-developed urban heat island.

The urban circulation model of Vukovich (1971, 1973) assumed no Coriolis effect, and specified a constant linear friction, while the model of Olfe and Lee (1971) obtained steady-state solutions for both the urban wind and temperature fields. The two-dimensional finite-difference calculation by McElroy (1973) of the urban heat island over Columbus, Ohio, compared favorably with observations, but the dynamical computations did not include a vertical motion term.

The present urban boundary layer model, called URBMET for “urban meteorology,” simulates the horizontal and vertical flow over an idealized city. The urban values of the aerodynamic surface roughness parameter and the nighttime surface temperature are specified to be greater than those for the non-urban grid points, which are located in regions upwind and downwind of the city.

Results are presented from simulations of the daytime flows of neutral atmospheres over rough cities, and from the nighttime flows of stable atmospheres over rough, warm cities. Comparisons are presented to show that the URBMET boundary layer model is capable of reproducing many of the observed characteristics of the urban boundary layer.

2. The present model

Finite-difference solutions to the hydrodynamic and thermodynamic equations are obtained in a vertical plane oriented in the direction of a constant geostrophic wind. The vertical fluxes of heat and momentum are assumed to be constant with height in a lower analytical surface boundary layer, while they generally decrease with height in an upper finite-difference transition layer. This approach has been recommended by Clarke (1970a) and Taylor and Delage (1971), because the use of finite differences near the surface in conjunction with eddy mixing coefficients that are proportional to height can be very inaccurate and expensive.

a. Transition layer equations

This layer is assumed to be hydrostatic and Boussinesq, with the latter assumption leading to incompressible flow (Spiegel and Veronis, 1960). All lateral gradients are assumed to be zero, except for that of the undisturbed pressure $p_0$, which corresponds to a
constant geostrophic wind \( u_g \) given by
\[
\frac{1}{\rho_m} \frac{\partial p_t}{\partial y}, \tag{2.1}
\]
where all symbols are defined in Appendix A.

Adiabatic motions are also assumed, while diffusion effects in the direction of the geostrophic wind are assumed to be insignificant as compared to advective effects, although inclusion of horizontal diffusion can have a smoothing effect on predicted fields. Justification for these assumptions can be found in Bornstein (1972a), herein referred to as B. With the additional assumption of initially adiabatic lapse rates, as discussed in Section 4, the transition layer equations can be written as
\[
\frac{\partial u}{\partial t} + \frac{\partial (u' u)}{\partial x} + \frac{\partial (w' u)}{\partial z} = -\frac{1}{\rho_m} \frac{\partial p'}{\partial x} + f v + \frac{\partial}{\partial z} \left( K_H \frac{\partial u}{\partial z} \right), \tag{2.2}
\]
\[
\frac{\partial v}{\partial t} + \frac{\partial (u' v)}{\partial x} + \frac{\partial (w' v)}{\partial z} = f(u_g - u) + \frac{\partial}{\partial z} \left( K_M \frac{\partial v}{\partial z} \right), \tag{2.3}
\]
\[
\frac{\partial \rho'}{\partial z} = \frac{8}{T_m} T', \tag{2.4}
\]
\[
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2.5}
\]
\[
\frac{\partial \theta'}{\partial t} + \frac{\partial (u' \theta')}{\partial x} + \frac{\partial (w' \theta')}{\partial z} = \frac{\partial}{\partial z} \left( K_H \frac{\partial \theta'}{\partial z} \right), \tag{2.6}
\]
where quantities subscripted with an \( m \) are constant space averages, and primed quantities are perturbations resulting from the thermally induced circulations. The advective terms of (2.2), (2.3) and (2.6) are in a form consistent with the "donor cell" numerical scheme, as discussed in Section 3.

The present study is carried out using a vorticity equation formulation, rather than the primitive equations, since the numerical valley breeze study of Thyer (1962) yielded smoother results with such an approach. In deriving the vorticity equation used in the present study, it was assumed that
\[
\theta = T + \Gamma z, \tag{2.7}
\]
where \( \Gamma \) is the dry adiabatic cooling rate. According to Munn (1966) this approximation is valid in the planetary boundary layer.

Taking the vertical derivative of (2.2), and using (2.4) and (2.7), yields
\[
\frac{\partial \zeta}{\partial t} + \frac{\partial (u' \zeta)}{\partial x} + \frac{\partial (w' \zeta)}{\partial z} = \frac{1}{T_m} \frac{\partial \theta'}{\partial x} + f v + \frac{\partial}{\partial z} \left( K_M \frac{\partial \zeta}{\partial z} \right), \tag{2.8}
\]
where \( \zeta \) is the \( y \) component of "vorticity," given by
\[
\zeta = \frac{\partial w}{\partial z}. \tag{2.9}
\]
This form is consistent with the hydrostatic assumption, in that the local rate of change term in the third equation of motion is set to zero.

Because of (2.5), \( u \) and \( w \) can be defined in terms of a streamfunction \( \psi \) by
\[
u = \frac{\partial \psi}{\partial z}, \tag{2.10}
\]
\[
w = -\frac{\partial \psi}{\partial x}, \tag{2.11}
\]
while from (2.9) and (2.10) it can be seen that
\[
\zeta = \frac{\partial \psi}{\partial z}. \tag{2.12}
\]

The eddy exchange coefficients used in the transition layer in the present study are computed following a method developed by O'Brien (1970). This method assumed that the following are known for both \( K_H \) and \( K_M \), either from solutions to the constant-flux layer equations at \( h \) discussed in the next section, or from assumed boundary conditions at some level \( H^* \), at which \( K \) has decreased to some small value:
\[
K(H^*), \quad \left( \frac{\partial K}{\partial z} \right)_{H^*}, \quad K(h), \quad \left( \frac{\partial K}{\partial z} \right)_h,
\]
where \( H^* \) is not necessarily the top of the transition layer. If the variation with respect to height of \( K \) at \( H^* \) is taken as zero, then
\[
K(z) = K(H^*) + \left( \frac{z - H^*}{H^* - h} \right) \left[ K(h) - K(H^*) + (z - h) \right]
\]
\[
\times \left[ \left( \frac{\partial K}{\partial z} \right)_h + \frac{2(K(h) - K(H^*))}{H^* - h} \right], \quad H^* > z > h, \tag{2.13}
\]
where \( h \) is the top of the surface boundary layer, taken as 50 m.

In the present study, we use
\[
H^* = 1050 \text{ m}, \tag{2.14}
\]
\[
K_H = K_M = 10^3 \text{ cm}^2 \text{ s}^{-1}, \quad z > 1050 \text{ m}, \tag{2.15}
\]
where (2.14) is consistent with the results of Zdunkowski et al. (1971), and reduction of (2.15) to \( 10^3 \) \text{ cm}^2 \text{ s}^{-1} did not significantly alter predicted meteorological distributions.

Deficiencies in the present formulation include a need for an \textit{a priori} knowledge of \( H^* \), and the possibility of stabilities in the transition layer different from those in the constant-flux layer. A better approach would have been to use a dynamic expression for \( H^* \); a more complete discussion of the problems associated with the specification of the eddy exchange coefficients in numerical boundary layer models is given in B.
b. Constant-flux layer equations

In the constant-stress layer, the vertical fluxes of heat $Q_H$ and momentum $Q_M$ are constant with height, and are given by

$$
\frac{Q_M}{\rho_m} = K_M \frac{\partial U}{\partial z} = \frac{\tau}{\rho_m},
$$

(2.16)

$$
\frac{Q_H}{\rho_mc_p} = K_H \frac{\partial \theta'}{\partial z} = \frac{u_a}{\rho_m} \frac{\theta_a}{\rho_m},
$$

(2.17)

where $\tau$ the turbulent shear stress, $u_a$ the friction velocity, and $\theta_a$ the friction potential temperature are constant with height in the constant-flux layer, but vary with atmospheric stability. Note that the total horizontal wind $U$ is given by

$$
U = (u^2 + v^2)^{1/2},
$$

(2.18)

and that the inverse turbulent Prandtl number $r$ is defined as

$$
r = K_H/K_M.
$$

(2.19)

According to Monin (1958), the vertical gradient of the wind in a constant flux layer is given by

$$
\frac{\partial U}{\partial z} = \frac{u_a}{l},
$$

(2.20)

where $\phi$ the nondimensional wind shear is a function of atmospheric stability, and $l$ the Prandtl mixing length is given in the constant flux layer by

$$
l = k_0 (z + z_0).
$$

(2.21)

Atmospheric stability is determined from either the Richardson number $R_i$ or Monin-Obukhov length $L$, defined respectively as

$$
R_i = \frac{g}{\theta_m} \left( \frac{\partial \theta'}{\partial z} \right)^2,
$$

(2.22)

$$
L = \frac{u_a \theta_m}{k_0 r_a},
$$

(2.23)

where $L$ is invariant with height. Using (2.16), (2.17), (2.19), (2.22) and (2.23), it can be shown that

$$
r R_i = \left( \frac{z + z_0}{L} \right) \phi^{-1},
$$

(2.24)

while (2.24), in conjunction with (2.16), (2.17), (2.19) and (2.20), leads to

$$
\frac{\partial \theta'}{\partial z} = \frac{u_a \phi}{k_0 r_a}.
$$

(2.25)

Following Pandolfo (1966), the nondimensional wind shear of (2.20) is specified to be the Monin-Obukhov (1954) form for forced convection, and the Priestley (1959) form for free convection. These are given, respectively, as

$$
\phi = 1 + \alpha \left( \frac{z + z_0}{L} \right), \quad \text{for } R_i \geq R_{i_0},
$$

(2.26)

$$
\phi = (c/3)^{1/4} \left( \frac{z + z_0}{L} \right)^{-1/6}, \quad \text{for } R_i < R_{i_0},
$$

(2.27)

where $\alpha$ and $c$ are constants, and $R_{i_0}$ is the (negative) value of $R_i$ at the transition from forced to free convection.

Using the following two assumptions from Pandolfo (1966)

$$
R_i = \frac{z + z_0}{L}, \quad \text{for } R_i \geq 0,
$$

(2.28)

$$
K_H = K_M, \quad \text{for } R_i \geq 0,
$$

(2.29)

along with (2.26), leads to the following form for (2.24) under stable conditions:

$$
\frac{z + z_0}{L} = \frac{R_i}{1 + \alpha R_i}, \quad \text{for } R_i \geq 0.
$$

(2.30)

In addition, (2.24) and (2.28) yield the following for unstable conditions:

$$
r = \frac{\phi^{-1}}{R_i} \quad \text{for } R_i < 0.
$$

(2.31)

Observations by Clarke (1970b), Oke (1970), Webb (1970) and Businger et al. (1971) have verified (2.28) and (2.29) for the stabilities specified. The expressions for $r$ obtained from (2.31), and used in this model, are given in Appendix B.

The determination of the numerical value of $R_{i_0}$ was carried out by Pandolfo (1966) by assuming that at $R_{i_0}$:

$$
(r)_{forced} = (r)_{free},
$$

(2.32)

$$
\left( \frac{\partial r}{\partial R_i} \right)_{forced} = \left( \frac{\partial r}{\partial R_i} \right)_{free}. \quad r.
$$

(2.33)

Using the expressions for $r$ that are found in Appendix B, in conjunction with (2.32) and (2.33), yields the following:

$$
R_{i_0} = -3^{1/3}(7\alpha)^{-1},
$$

(2.34)

$$
c = 3(6/7)^{1/3}(7\alpha)^{-1}.
$$

(2.35)

The final expressions for the vertical gradients of wind and potential temperature (shown in Appendix B) are obtained by the combination of (2.26) through (2.31). Integration with height of the resulting expressions from 0 to $z + z_0$ yields the profiles given in Appendix B. The use of these profiles requires expressions for $u_a$ and $\theta_a$, which are determined by assuming continuity in the profiles of temperature.
and wind, as well as in their first vertical derivatives, at the top of the constant flux layer, i.e.,

$$\frac{U(h+\Delta z/2) - U(h-\Delta z/2)}{\Delta z} = \left(\frac{\partial U}{\partial z}\right)_h,$$ (2.36)

$$\frac{\theta'(h+\Delta z/2) - \theta'(h-\Delta z/2)}{\Delta z} = \left(\frac{\partial \theta'}{\partial z}\right)_h,$$ (2.37)

where $\Delta z$ is the vertical grid spacing near the surface (discussed in the next section), $h \pm \Delta z/2$ are grid levels for $U$ and $\theta'$, and $h$ is a grid level for their gradients.

Note that $U$ and $\theta'$ at $h+\Delta z/2$ are known from finite-difference prediction, and the other four terms are replaced by the analytical expressions appropriate for each stability class. The resulting expressions for $u_*$ and $\theta_*$ are shown in Appendix B, where $Ri(3h/4)$ is valid at the center of the layer from the surface to $h+\Delta z/2$. Since $h$ is equal to $\Delta z$, this Richardson number is valid at $3h/4$, and is given by

$$Ri(3h/4) = \frac{1.5g h [\theta'(3h/2) - \theta'(0)]}{\theta_m[U(3h/2)]^2}.$$ (2.38)

Using (2.16), (2.17), (2.19), (2.26), (2.27) and (2.29), expressions for the eddy exchange coefficients in the constant flux layer are derived, and the results are given in Appendix B. The vertical variations of $K_M$ and $K_H$ at $h$, needed in (2.13), are obtained from (2.16), (2.17), (2.19), (2.20), (2.21) and (2.25), and are given as

$$\left(\frac{\partial K_M}{\partial z}\right)_h = \frac{K_M(h)}{(h+z_0)} \left[ 1 - \frac{(h+z_0)}{\phi} \left(\frac{\partial \phi}{\partial z}\right)_h \right],$$ (2.39)

$$\left(\frac{\partial K_H}{\partial z}\right)_h = \frac{K_H(h)}{(h+z_0)} \left[ 1 - \frac{2(h+z_0)}{\phi} \left(\frac{\partial \phi}{\partial z}\right)_h \right].$$ (2.40)

Expressions for $\partial \phi/\partial z$ needed in (2.39) and (2.40) are obtained from (2.26) and (2.27), and are also shown in Appendix B. Finally, these expressions, along with (2.28) and (2.30), yield the equations for the vertical variation of $K_M$ and $K_H$ at $h$ found in the Appendix. Note that $Ri(h)$, which appears in these expressions, is given by

$$Ri(h) = \frac{g h [\theta'(3h/2) - \theta'(h/2)]}{\theta_m[U(3h/2) - U(h/2)]^2}.$$ (2.41)

c. Grid and boundary conditions

The interlaced grid of Fromm (1964), shown in Fig. 1 and used in this study, considers velocity components to be the average inflow and outflow rates on horizontal and vertical elements of a cube. All parameters used in the present model are also located in the figure.

Studies by Clarke (1970a) and Taylor and Delage (1971) have discussed the superiority of variable grid spacing in achieving high resolution near the surface and near discontinuities in surface characteristics. Accordingly, the 16 by 16 grid network used in the present study, shown in Table 1, possesses a high resolution at lower levels, and in the vicinity of the city.

The current formulation allows four vertical boundary conditions on the streamfunction, and two each on the lateral velocity and potential temperature. A complete list of the boundary conditions currently

<table>
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<th>s (m)</th>
<th>z (km)</th>
</tr>
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<td>-30</td>
</tr>
<tr>
<td>2</td>
<td>12.5</td>
<td>-20</td>
</tr>
<tr>
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</tr>
<tr>
<td>16</td>
<td>1650</td>
<td>67.5</td>
</tr>
</tbody>
</table>

FIG. 1. Interlaced grid used in the present study, in which the circles represent $\xi$ and $\phi$ grid points, the crosses represent $\theta'$, $u$ and $v$ grid points, and the squares represent $w$ grid points.
used is given below:

1. At $z = 0$

$$u = \frac{\partial \psi}{\partial z} = 0$$  \hspace{1cm} (2.42)

$$w = \frac{\partial \psi}{\partial x} = 0$$  \hspace{1cm} (2.43)

$$v = 0$$  \hspace{1cm} (2.44)

$$\theta' = f(x, t),$$  \hspace{1cm} (2.45)

where $f(x, t)$ is to be specified. Note that (2.42) and (2.43) are consistent with a constant value of $\psi$, which is taken as zero for convenience.

2. At $z = h$

Continuity of $U, \theta', \frac{\partial U}{\partial z}$, \frac{\partial \psi}{\partial z}$.  \hspace{1cm} (2.46)

3. At $z = H$

$$u = \frac{\partial \psi}{\partial z} = 0$$  \hspace{1cm} (2.47)

$$w = \frac{\partial \psi}{\partial x} = 0$$

$$v = 0$$

$$\theta' = 0.$$  \hspace{1cm} (2.50)

4. At the upwind boundary

$$\frac{\partial (u \theta')}{\partial x} = \frac{\partial (w \theta')}{\partial z} = \frac{\partial (u \theta')}{\partial x} = \frac{\partial w}{\partial x} = w = 0.$$  \hspace{1cm} (2.51)

Given the finite-difference scheme associated with the donor cell formulation used for the advection terms, it is not necessary to specify a downwind boundary condition.

Note that (2.5) and (2.47) imply that the vertical gradient of the vertical velocity at $H$ is zero. It has been shown by Vukovich (1975) that an alternative condition to (2.47) is a zero vertical velocity at $H$. However, Delage and Taylor (1970) have stated that “there is no physical necessity for having vertical velocities equal to zero at the top boundary,” and Estoque and Bhumralkar (1970) have shown that non-zero vertical velocities at an upper boundary produce only small effects on the distributions of the other variables.

3. Method of solution

At each time step, following an initialization process described below for the several separate cases considered in the present study, the following procedure is followed: (i) determination of the new values of $\theta'$ at the surface; (ii) construction of the constant flux layer using the latest values of $\theta'$ and $U$ at the surface, and at the lowest finite-difference grid; and (iii) determination of the new $\theta', \xi, \psi, u, w, v$, and $U$ fields, respectively, using the newest available values of all parameters.

In constructing the surface boundary layer, both the total momentum flux and the velocity gradient at $h$, given by (2.16) and (2.20), respectively, can be split into $x$ and $y$ components

$$\frac{Q_u}{\rho_m} = \frac{\partial u}{\partial z},$$  \hspace{1cm} (3.1)

$$\frac{Q_v}{\rho_m} = \frac{\partial v}{\partial z},$$  \hspace{1cm} (3.2)

where

$$Q_M = (Q_x^2 + Q_y^2)^{\frac{1}{2}}.$$  \hspace{1cm} (3.3)

Since $U(h/2)$ is assumed to be parallel to the velocity at the lowest finite-difference grid point $U(3h/2)$, the quantities $U(h/2)$ and $(\partial U/\partial z)_h$ can each be broken into $x$ and $y$ components by multiplication by the following two terms, respectively:

$$u(3h/2)/U(3h/2),$$  \hspace{1cm} (3.4)

$$v(3h/2)/U(3h/2).$$  \hspace{1cm} (3.5)

The $\theta'$, $\xi$ and $v$ equations are each solved using the “time splitting” method of Fromm (1964), e.g., for the energy equation, first

$$\frac{\partial \theta'}{\partial t} = - \frac{\partial (u \theta')}{\partial x}$$  \hspace{1cm} (3.6)

is solved, then

$$\frac{\partial \theta'}{\partial t} = - \frac{\partial (w \theta')}{\partial z}$$  \hspace{1cm} (3.7)

is solved, and finally

$$\frac{\partial \theta'}{\partial t} = - \frac{\partial}{\partial z} \left( K \frac{\partial \theta'}{\partial z} \right)$$  \hspace{1cm} (3.8)

is evaluated using the latest values of $\theta'$ at each step.

In the “donor cell” advection scheme, the time variation of a parameter depends on the difference between the fluxes in and out of the sides of a volume centered on the grid location of that parameter. Thus, the finite difference form of (3.6) is

$$\theta'(i,j) = \theta'(i,j) - \Delta t \left[ \frac{(u \theta')_{out} - (u \theta')_{in}}{x_{out} - x_{in}} \right].$$  \hspace{1cm} (3.9)

where $\Delta t$ is the duration of a single time step, taken
as 15 s, and $i$ and $j$ are the horizontal and vertical indices defined in Table 1, respectively.

If $u(i,j)$ is positive, for example, then (3.9) becomes

$$
\theta'(i,j) = \theta'(i,j) - \frac{\Delta t}{\Delta x(i)} [u(i+\frac{1}{2},j) \theta'(i,j) - u(i-\frac{1}{2},j) \theta'(i-1,j)],
$$  \hfill (3.10)

where

$$
\Delta x(i) = x(i + \frac{1}{2}) - x(i - \frac{1}{2}),
$$  \hfill (3.11)

$$
u(i+\frac{1}{2},j) = 0.5[u(i+1,j) + u(i,j)].
$$  \hfill (3.12)

Note that slight modifications in the equations are necessary near the boundaries, and with negative velocities. A similar process is used for (3.7), with details given in $B$.

The advection scheme described above is called the "donor cell method" by Roache (1972), and is an improvement over the "upwind" advection method, as the latter is not mass conservative. Mathematically the two would be identical if an average speed was used in Eq. (3.9), instead of the speeds on either edge of the cube.

The finite-difference analog of (3.8) is

$$
\theta'(i,j) = \theta'(i,j) + \frac{\Delta t}{\Delta z(j)} \left[ \frac{\partial \theta'}{\partial z}(i,j+\frac{1}{2}) - \frac{\partial \theta'}{\partial z}(i,j-\frac{1}{2}) \right],
$$  \hfill (3.13)

where

$$
\Delta z(j) = z(j + \frac{1}{2}) - z(j - \frac{1}{2}),
$$  \hfill (3.14)

and where the caret indicates values at $(i+\Delta l)$, and where $K_H$ is evaluated from (2.13), in conjunction with expressions from Appendix B. The vertical gradient of $\theta'$ is evaluated from

$$
\left( \frac{\partial \theta'}{\partial z} \right)_{i,j+\frac{1}{2}} = \frac{\theta'(i,j+1) - \theta'(i,j)}{\frac{1}{2} \Delta z(j+1) + \Delta z(j)}.
$$  \hfill (3.15)

A similar procedure is used for the vorticity equation, except that the equation analogous to (3.8) has additional terms for the Coriolis and buoyancy effects. The resulting values of $\xi$ are used to solve (2.12) for the streamfunction field using the method of Gaussian elimination, as described by Richtmeyer (1957), but with appropriate modifications for the variable grid spacing. Details can be found in $B$. New values of $u$ and $w$ are then obtained from

$$
\psi(i,j+\frac{1}{2}) = \psi(i, j + \frac{1}{2}) - \frac{\Delta s(j)}{\Delta x(j)},
$$  \hfill (3.16)

$$
\psi(i+1, j+\frac{1}{2}) = \psi(i+1, j + \frac{1}{2}) - \frac{\Delta s(j)}{\frac{1}{2} \Delta x(i+1) + \Delta x(i+1)}.
$$  \hfill (3.17)

respectively. Finally, the finite-difference analogs to (2.3) are solved using the same time splitting technique.

4. Results

Case a. Flow over a rough city

This simulation attempts to reproduce the effects on the structure of the urban boundary layer resulting from the difference in magnitude of $\kappa_0$ at urban and rural sites. The 3 m value of $\kappa_0$ specified for the urban area represents a six-story building, while the 0.5 value used at the other grid points is typical of the one-story houses frequently surrounding central urban areas (Lettau, 1969).

The initial two-dimensional distribution of horizontal wind consists of the equilibrium profiles obtained from simulations reproducing one-dimensional neutral boundary layers over homogeneous terrain. The one-dimensional simulations were carried out using the values of $\kappa_0$ and $u_a$ specified for the two-dimensional simulation. Thus, the initial vertical velocities at both urban edges were non-zero, in accordance with the integral form of (2.5).

Results from the two-dimensional simulation are assumed to be representative of conditions at the beginning of a nocturnal heat island episode, i.e., at about 1800 LST, and are used as the initial conditions for the rough, heated city simulation of Section 4c. Adiabatic lapse rates are maintained throughout the rough city simulation by keeping a zero value of $\theta'$ at the surface. Such lapse rates have been observed in cities during the hours near 1800 LST by DeMarrais (1961), Deland and Binkowski (1966), and Bornstein (1968), as well as at rural O'Neill by Kuo (1968).

The constant value of $\theta$ at the surface allowed a steady-state wind field to be achieved after 3 h of predicted time. Values of the total horizontal wind speed at the urban sites at heights up to several hundred meters are reduced in magnitude below those at corresponding heights at the urban wind boundary, as shown in Fig. 2. The maximum decrease in speed occurred at the first grid point above the surface, and over the downwind half of the city.

The vertical velocities associated with the horizontal winds of Fig. 2 are shown in Fig. 3. The maximum upward and downward values appearing in the figure are associated with the areas of maximum convergence and divergence at the upwind and downwind edges of the city, respectively. In addition, these maxima occur at a height of about 10$^4$ m, and remain approximately constant above this level to the top of the model. This is due to the choice of boundary conditions at $H_r$, and the lack of a return flow mechanism, as is found with a warm city (Case b).

Increasing either the geostrophic wind speed, or the urban surface roughness, resulted in larger urban
Fig. 2. Distribution of computed steady-state horizontal wind speed for the flow of a neutral atmosphere over a rough city. Values represent the deviation (cm s⁻¹) from values at the upwind boundary. The geostrophic wind speed is 5 m s⁻¹, and the urban and rural surface roughness are 3 and 0.5 m, respectively.

Fig. 3. Distribution of computed vertical velocity (cm s⁻¹) for the horizontal wind speed field of Fig. 2.
speed deficits and larger vertical velocities, but did not change the location of these maxima.

Case b. Flow over a warm city

This simulation attempts to reproduce the effects of a warm city on the structure of the urban boundary layer during nighttime heat island hours, and resulting from the difference in the value of the surface temperature at urban and rural sites. The initial wind field is obtained in a manner similar to those from Case a, except that all initial vertical velocities are zero, as a uniform value of $z_0$ is specified.

The urban and rural surface cooling rates were estimated from the observations in and around urban Montreal by Oke and East (1971). These observations indicated that the rural atmosphere near the surface cooled rapidly in the evening hours before midnight, while the urban atmosphere near the surface was maintained at a nearly constant temperature during the same period. After midnight both regions cooled at about 50% of the rate found at the rural site before midnight. Thus, the magnitude of the heat island near the surface increased with time in the early evening, and then remained constant until morning. The following rates of temperature change were assumed for the simulations in this section and in the next section, where the flow over a rough, heated city is modeled:

<table>
<thead>
<tr>
<th>Period (LST)</th>
<th>Urban $({}^\circ\text{C h}^{-1})$</th>
<th>Rural $({}^\circ\text{C h}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800-2400</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>2400-0600</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

The prescribed cooling rates caused formation of a surface based stable layer in the rural areas, while advection of cool rural air over the uncooled urban surface during the first 6 h of the simulation caused formation of a slightly unstable layer near the urban surface. During the second 6 h of the simulation, the decreased surface cooling rates slowed the rate of increasing stability at rural sites, while the urban cooling was unable to completely overcome the effects of advection.

The spatial distribution of $\theta'$ after 12 h of simulated time for a typical case is shown in Fig. 4, in which the values represent increases over those at the upwind boundary at the same heights. The surface heat island intensity at this time is 6°C, and the excess warmth of the city has been spread horizontally and vertically by the advection and diffusion process. Note that the heat island above the downwind edge of the city is more intense than that over its center.

The distribution of total horizontal wind speed after 6 h of simulated time is shown in Fig. 5. The region of increased values over the city, as compared to those at the upwind boundary of the model, is produced by the horizontal gradient of $\theta'$ which appears in the buoyancy term of the vorticity equation. This term produces a flow over the upwind half of the warm city which is in the same direction as the mean flow. The wind speed excess was greatest over the downwind half of the city, at a height of 37.5 m, due to the effects of horizontal advection, and it reached a maximum of about 80 cm s$^{-1}$ at 0200 LST.

A small area of decreased speeds is seen near the surface at grid points downwind of the city, although its magnitude decreased with time. This region is also due to the buoyancy term, which produces a flow opposed to the mean flow over the downwind half of the warm city. However, its magnitude is reduced by the effects of the horizontal advection term. The region of decreased speeds located above the region of increased speed grew in magnitude with time, and is analogous to the "return flow" region appearing in simulations of sea breeze circulations arising in otherwise calm conditions (e.g., see Fisher, 1961).

The corresponding vertical velocity field, shown in Fig. 6, indicates maximum values of $w$ occurring at grid points 400 m above the edges of the urban area, i.e., the locations of the maximum gradients in the horizontal speed. The decreases in the magnitude of $w$ above 400 m, due to the return flow region mentioned above, occur even though $w$ was not forced to be zero at $H$. Increasing the geostrophic wind speed led to increases in the urban wind speed excesses and in the maximum vertical velocities.

Case c. Flow over a rough, warm city

This simulation attempts to reproduce the effects on the structure of the urban boundary layer, during the hours from 1800 to 0600 LST, resulting from increases in both the urban $z_0$ and surface temperature. The method of solution for this section is identical with that of the warm city case, except that the wind field is initialized using the steady-state horizontal and vertical winds, which resulted after 12 h of simulated time for the rough city case.

The horizontal wind field resulting after 6 h of simulated time (Fig. 7) shows a region of decreased wind speeds, as compared to those at the upwind rural boundary. This region is found upwind of, and above, a region in which speeds are higher than those at the upwind boundary. A smaller region of decreased speeds is also found near the surface, and downwind of the city, as it was in the warm city case. At 2400 LST both main regions have the same maximum change in speed, i.e., about 30 cm s$^{-1}$, but by 0600 LST (Fig. 8) the region of increased speed enlarged horizontally and vertically, while its maximum value increased to 60 cm s$^{-1}$. In the main region of decreased speeds, values remained almost constant.

When $u_o$ is increased from 3 to 6 m s$^{-1}$, the magnitude of the decrease in speed at the upwind edge of the city remains fairly constant, but the horizontal extent of the region increases, as shown in Fig. 9.
Fig. 4. Distribution of the perturbation potential temperature $\theta'$ (°C) after 12 h of simulation, i.e., at 0600 LST, for the flow over a warm city. Values represent the deviation from values at the upwind boundary, and the surface heat island intensity is 6°C. The geostrophic wind speed is 5 m s$^{-1}$ and the surface roughness is 0.5 m at all sites.

Fig. 5. Distribution of horizontal wind speed (cm s$^{-1}$) after 12 h of simulated time, i.e., 0600 LST, for the flow over a warm city for the parameters listed in Fig. 4. Values represent the deviation from values at the upwind boundary.
Fig. 6. Distribution of the vertical velocity (cm s⁻²) for the horizontal wind field after 12 h of simulated time, i.e., 0600 LST, for the case discussed in Figs. 4 and 5.

Fig. 7. Distribution of horizontal wind speed (cm s⁻¹) after 6 h of simulated time, i.e., 2400 LST, for the flow over a rough, warm city. Values represent the deviation from values at the upwind boundary. The geostrophic wind speed is 3 m s⁻¹, and the urban and rural roughnesses are 3 and 0.5 m, respectively.
Fig. 8. As in Fig. 7, except for 12 h of simulation, i.e., 0600 LST.

Magnitudes in the region of increased speeds are reduced from those found in the 3 m s⁻¹ case, due to the increased spreading of heat by the faster winds. In addition, a shallow layer of decreased speeds develops very close to the surface and downwind of the city. Thus, with the larger value of $u_0$, the surface roughness effects are stronger than the heat island effects in the region adjacent to the urban surface.

Fig. 9. As in Fig. 7, except for a geostrophic wind speed of 6 m s⁻¹.
The vertical velocity fields at 2400 LST, corresponding to both the 3 and 6 m s\(^{-1}\) cases, are shown in Figs. 10 and 11, respectively. Both show similar features, in that a region of sinking air over the city is sandwiched between two regions of upward motion.

The temperature fields (not shown) resulting from the current rough, warm city simulations are associated with a heat island whose horizontal and vertical dimensions were increased over the previous warm city cases. This is due to the increased mechanical mixing associated with the larger urban values of \(\sigma_0\) used in the current simulations. Lapse rates in the urban transition layer were also less superadiabatic in the present cases for the same reason.

5. Discussion of results

Analysis of the magnitudes of each of the terms of the vorticity equation (2.8) can provide insight into the relative importance of each of the physical processes at selected grid locations of Fig. 7. In addition to these values at five grid locations, Table 2 also
lists the magnitudes of $\xi$ before and after the time (2400 LST) for which Fig. 7 is valid. A positive value of $\xi$ indicates that (positive) $u$'s are increasing in magnitude with height, as they are at each of the grid points in the table at 2400 LST.

At grid point (1, 9), i.e., at the upwind model boundary, $\xi$ is decreasing very slowly with time. This is consistent with a rate of increase of $u$ with time which decreases with height. The increase of $u$ with time is associated with the formation of a weak low-level jet, set up by the inertial oscillation that results from diurnally changing values of $K_M$. The intensity of eddy mixing is decreased with time due to the increasing (with time) stability, which is caused by the diurnal variation of surface temperature.

The region of low speeds over the upwind urban edge [at grid point (5, 7)] results as the flow encounters the urban $z_0$ values, as has been shown in Case a. However, Table 2 demonstrates that $\xi$ in this region is decreasing in magnitude with time, indicative of the winds speeds which are increasing with time at a rate which is decreasing with height. The decrease of $\xi$ with time occurs as buoyancy destroys more $\xi$ that is diffused and advected into the region. Thus the urban heat island effect is slowly overcoming the surface $z_0$ effect associated with the initial conditions of this region.

At the center of the city, i.e., at grid point (7, 9), the horizontal gradients of $\xi$ and $\theta'$ are reduced. Thus, the buoyancy and advection of terms in this region are reduced from their magnitudes at the upwind urban edge, but the flow is still being accelerated by the buoyancy term.

Downwind of the city, but close to the surface [at grid point (11, 3)], speeds are reduced from those at the upwind model boundary, as the creation of $\xi$ by the buoyancy term and the transport of $\xi$ into the region by the horizontal advection term are greater than the diffusion of $\xi$ out of the region by the eddy processes. Note, that both the buoyancy and diffusion terms have reversed roles, as compared to their effects over the upwind part of the city.

Downwind of the city, but further above the surface [at grid point (11, 9)], speeds are increased, as more vorticity is being advected out of the region than is being diffused into the region. If the advection of warm urban air into this downwind region was not present, i.e., as with the case of a zero geostrophic wind, then the buoyancy term would be positive, and perhaps greater in magnitude than the vorticity advection term. Thus, the vorticity would increase with time, due to the establishment of a region of reverse flow into the city, which would be centered at the downward urban edge. In other words, the region of decreased flow downwind of the city (near the surface) would be increased in magnitude, as well as in its horizontal and vertical extents.

Note that the magnitudes of the vertical convection and Coriolis terms are generally only a few percent of the magnitudes of the remaining three terms. An exception is at grid point (11, 3) where the convection term is about 50% of the horizontal advection term.

In summary, this analysis has shown that the urban-induced accelerations near the surface over the upwind half of a city are due to a heat-island buoyancy effect, which strengthens the existing mean flow. Such accelerations in regions downwind of the center of the city are produced by the combination of two advective effects. First, the advection of vorticity out of the region tends to increase its speed with time. Second, the advection of warm air into the region tends to decrease the buoyancy effect, which would have produced a flow counter to the mean flow.

The above analysis indicates that the increased downward flux of vorticity (or momentum) in an urban atmosphere is not the cause of urban-induced wind speed accelerations. The increased flux is due to both increased mechanical turbulence, associated with the large values of $z_0$ in an urban area, and increased thermal turbulence, associated with the slightly unstable lapse rates predicted for the nighttime urban atmosphere.

Additional evidence for this conclusion comes from the results of URBMET simulations of both the non-neutral boundary layer over homogeneous terrain and the neutral boundary layer over a rough city. In the second case, results from Case 4a of the present study showed that the large values of $z_0$ in an urban area increased the values of $K_M$, but decreased the values of the wind speed at all levels.
In the case of a non-neutral boundary layer over homogeneous terrain, corresponding to the conditions at the upwind model boundary for Cases 4b and 4c, the vertical distribution of boundary layer winds during stable conditions showed an enhancement of the small supergeostrophic speed region associated with the equilibrium neutral stability profile. However, during unstable conditions this region was completely eliminated. Thus, as the degree of instability increases, $K_M$ increases, the vertical gradient of wind speed decreases, and wind speed decreases, not increases (except in a very shallow layer near the surface).

The mixing process, however, does act to transport heat upward in the slightly unstable nighttime urban atmosphere, while at the same time, heat is being mixed downward in the stable rural air. Thus, the mixing acts to increase the horizontal temperature gradient at levels above the surface when the value of the urban heat island at the surface is prescribed. Mixing therefore acts to increase the accelerations over the urban half of the city. However, as was pointed out above, the cooling caused by the horizontal advection of air over the city in Cases 4b and 4c was generally larger than the warming caused by the vertical mixing process; thus the air over the city cooled with time at all levels.

The urban heat island model of Yu (1973) simulated the flow over a rough, warm city, for which surface temperature was predicted from an energy balance equation. Results from this model indicated that the increased mechanical turbulence over an urban area could be responsible for the development of a significant urban heat island. The model predicted that the urban boundary layer would have a stable lapse rate, and thus heat would be fluxed to the surface faster in the city than in the country. Hence, a larger fraction of the computed radiative cooling would be encountered in an urban area, which would then cool more slowly than a rural area, thus leading to heat island formation.

However, since the model omitted the downward longwave radiative flux term from the surface energy balance equation, the nighttime radiative cooling of the surface could only be balanced by a turbulent heat flux to the surface and by a conduction of heat from the ground. Hence, the nighttime urban atmosphere had to develop a stable stratification. However, most observations have shown that the urban boundary layer is unstably stratified during conditions of well-developed urban heat islands (Clarke, 1969; Oke and East, 1971).

Differences between the results of the current simulations and those of similar models are consistent within the various assumptions of each model. For example, if the geostrophic wind speed of Fig. 5 were reduced to zero, the regions of increased and decreased speeds near the surface would move to the left. The latter region would then represent a flow from right to left, and not just a decrease in speed. It would also be increased in intensity and horizontal extent, as it no longer would be overwhelmed by advective effects.

The boundary between these two regions would be exactly at the center of the warm urban area, and the results would be totally consistent with those of Delage and Taylor (1970), who simulated the flow into a warm, but not rough, city during otherwise calm conditions. The same movement to the left would occur with the regions of upward and downward flow of Fig. 6. In addition, the region of downward flow in the extreme right-hand corner of the figure would probably increase in magnitude, so as to be symmetric with that currently found over the left-hand urban edge. It would also probably move into position at the right of the right-hand urban edge.

The urban heat island model of Vukovich (1971) simulated the flow over a warm (but again not rough) city, and the results demonstrated that a mean flow will displace the locations of areas of wind speed change and areas of vertical motion. Although the results were presented for simulation periods of only 1 h, these centers were advected downstream from the same initial locations as found in Figs. 5 and 6 of the current simulations. It is interesting to note, that since his background velocity was independent of $x$, $z$ and $t$, the lateral boundaries between these regions were generally vertical, as opposed to those of Figs. 5 and 7.

The results of Estequie and Bhumralkar (1969) demonstrate how the region of decreased velocity near the surface in Figs. 5 and 7 of the present results weakens with time as it is advected to a location downwind of their heated island. With a 10 m s$^{-1}$ wind, the region was completely transported past their downwind model boundary (at 35 km beyond the center of a 10 km wide island) after 3 h of simulated time. These effects are reproduced in the current warm and rough, warm city simulations, but no such weakening is seen in the results of Vukovich (1971). This may be due to the limited integration time for which the results are presented, or to the linearized nature of the model.

Urban influences on the nighttime airflow over a city have been studied by Angell et al. (1971) using data from 43 tetoon flights over Columbus, Ohio, during 10 nights in March 1969. The observations are in general agreement with the results of the present study, in that they showed reduced wind speeds over the city during lapse conditions (non-heat-island conditions), and increased speeds during inversion (heat island) conditions. The observed vertical velocities pattern over Columbus revealed an area of downward motion upwind of an area of upward motion during inversion conditions. This is also in agreement with the present results from the warm, rough city simulations.
Observations from a surface anemometer network in and around New York City (Johnson and Bornstein, 1974) show that the average daytime wind speeds near the surface over the city under non-heat-island conditions are reduced in magnitude, as compared to those outside of the city. This decrease is due to the large value of the urban surface roughness parameter, and these observations are in qualitative agreement with the present results from the rough city simulations. The results from the present rough, warm city simulations are in similar agreement with the corresponding nighttime wind observations reported by Bornstein et al. (1972). These observations showed that upwind rural speeds > 3.5 m s\(^{-1}\) were associated with reduced speeds over New York, while speeds below that value were associated with increased speeds over the city.

A similar “critical” nighttime speed of 5 m s\(^{-1}\) was found for London by Chandler (1965). Finally, the highest speeds over New York were found over the downwind half of the city, in agreement with the results of the present warm, rough city simulations, and with those of other theoretical studies, such as that of Estoque and Bhumralkar (1969).

6. Conclusions

The URBMET urban boundary layer model has been shown to be capable of reproducing many of the observed characteristics of daytime and nighttime urban boundary layers under non-heat-island and heat-island conditions, respectively. These include wind speed decreases and increases over a city, as compared to speeds over nearby non-urban areas, which arise from the large value of the urban roughness parameter, and from the heat island induced “urban breezes,” respectively. The magnitude of a predicted nighttime “critical” upwind rural wind speed above and below which speeds near the urban surface were decreased or increased, respectively, compares favorably with those observed over several cities. In addition, vertical velocity patterns corresponding to the various horizontal speed changes were in agreement with tetroon-derived patterns over an urban area.

Simulations of the thermal structure of the nighttime boundary layer revealed that it is the advection of cool rural air over a rough, warm city, during periods of radiative cooling, which produces the near-adiabatic lapse rates which have been observed near the surface over various cities. Simulated temperature profiles predicted a heat island, whose intensity decreased with height at a rate intermediate between that observed over New York City by Bornstein (1968) and Montreal by Oke and East (1971). Elevated inversion layers (frequently observed over urban areas) did not occur in the present simulations, as the rural surface-based radiation inversion was completely eliminated by mechanical turbulence as it was advected over the city. Inclusion, in the energy equation, of the radiative effects of atmospheric pollutants might have produced such an elevated layer, as shown by the results of Atwater (1971a, b) and Bergstrom and Viskanta (1973).

It is hoped that future operational urban boundary layer models will be three-dimensional, and that they would have upper boundary conditions which are based on real-time synoptic forecasts. Lower boundary conditions could be obtained from solutions to surface balance equations for heat and moisture, including furnace heat and moisture production terms. Topography-induced vertical velocities would be included, and horizontal and vertical eddy diffusion coefficients would be derived from solutions to the turbulent transport equations.

The boundary layer model would then be coupled, via the radiative flux divergence term in the energy equation, to a similar finite-difference mesoscale air pollution transport and diffusion model. This model would include photochemical transformations and other natural removal processes. It is hoped that these models would be used to simulate the flow and pollutant patterns over various real mesoscale urban-rural regions.

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APPENDIX A

List of Symbols

Variables and constants: Roman alphabet

\(c\) constant in forced convection profiles
\(c_p\) specific heat of dry air
\(f\) Coriolis parameter
\(g\) acceleration due to gravity
\(h\) top of constant flux layer
\(H\) top of transition layer
\(H^*\) level where \(K(z)\) goes to a constant small value
\(i\) horizontal index for horizontal wind and temperature grid
\(j\) vertical index for the horizontal wind and temperature grid
\(k_0\) von Kármán’s constant

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Constant Flux Layer Equations Used in the Present Model

A. For stable case: \(Ri\geq 0\)

1) \(r = 1\)

2) \(\phi = 1 + \alpha \frac{z + z_0}{L}\)

3) \(\frac{\partial U}{\partial z} = \frac{U_*}{k_0(z+z_0)} \left(1 + \alpha \frac{z + z_0}{L}\right)\)

4) \(\frac{\partial \theta'}{\partial z} = \frac{\theta_*}{k_0(z+z_0)} \left(1 + \alpha \frac{z + z_0}{L}\right)\)

5) \(U(z) = \frac{U_*}{k_0} \left(\ln \frac{z + z_0}{z_0} + \alpha \frac{z}{L}\right)\)

6) \(\theta'(z) = \theta'(0) + \frac{\theta_*}{k_0} \left(\ln \frac{z + z_0}{z_0} + \alpha \frac{z}{L}\right)\)

7) \(U_* = \frac{k_0(h+z_0)[1 - \alpha Ri(3h/4)] U(3h/2)}{h + (h+z_0) \ln \left(\frac{0.5h+z_0}{z_0}\right)}\)
\[
\theta_* = \frac{k_0 (h + z_0) \left[ 1 - \alpha \text{Ri} (3h/4) \right] \left[ \theta' (3h/2) - \theta' (0) \right]}{h + (h + z_0) \ln \left( \frac{0.5h + z_0}{z_0} \right)}
\]

\[
K_M (z) = \left[ k_0 (z + z_0) \right]^2 \left[ \frac{\partial U}{\partial z} \right] \left[ 1 - \alpha \text{Ri} (3h/4) \right]^2
\]

\[
K_H (z) = \left[ k_0 (z + z_0) \right]^2 \left[ \frac{\partial U}{\partial z} \right] \left[ 1 - \alpha \text{Ri} (3h/4) \right]^2
\]

\[
\left( \frac{\partial \phi}{\partial z} \right)_h = \frac{\alpha}{L}
\]

\[
\left( \frac{\partial K_M}{\partial z} \right)_h = \frac{K_M (h)}{(h + z_0)} \left[ 1 - \alpha \text{Ri} (h) \right]
\]

\[
\left( \frac{\partial K_H}{\partial z} \right)_h = \frac{K_H (h)}{(h + z_0)} \left[ 1 - \alpha \text{Ri} (h) \right]
\]

B. For lapse, forced convection case: \( \text{Ri}_t \leq \text{Ri} < 0 \)

1) \[
r = \left( 1 + \alpha \frac{z + z_0}{L} \right)^{-1}
\]

2) \[
\phi = 1 + \alpha \frac{z + z_0}{L}
\]

3) \[
\frac{\partial U}{\partial z} = \frac{u_*}{k_0 (z + z_0)} \left( 1 + \alpha \frac{z + z_0}{L} \right)
\]

4) \[
\frac{\partial \theta'}{\partial z} = \frac{\theta_*}{k_0 (z + z_0)} \left( 1 + \alpha \frac{z + z_0}{L} \right)^2
\]

5) \[
U (z) = \frac{u_*}{k_0} \left( \ln \left( \frac{z + z_0}{z_0} \right) \right)
\]

6) \[
\theta' (z) = \theta' (0) + \frac{\theta_*}{k_0} \left( \ln \left( \frac{z + z_0}{z_0} \right) + 2\alpha \frac{z}{L} \right)
\]

7) \[
u_* = \frac{k_0 U (3h/2)}{\left( 1 + \alpha \frac{h + z_0}{L} \right) \left( \frac{h}{z_0 + h} \right) + \ln \left( \frac{0.5h + z_0}{z_0} \right) + \frac{0.5\alpha h}{L}}
\]

8) \[
\theta_* = \frac{k_0 \left[ \theta' (3h/2) - \theta' (0) \right]}{\left( 1 + 2\alpha \frac{h + z_0}{L} \right) \left( \frac{h}{z_0 + h} \right) + \ln \left( \frac{0.5h + z_0}{z_0} \right) + \frac{\alpha h}{L}}
\]

9) \[
K_M (z) = \left[ k_0 (z + z_0) \right]^2 \left[ \frac{\partial U}{\partial z} \right] \left[ 1 + \alpha \text{Ri} (3h/4) \right]^{-2}
\]

10) \[
K_H (z) = \left[ k_0 (z + z_0) \right]^2 \left[ \frac{\partial U}{\partial z} \right] \left[ 1 + \alpha \text{Ri} (3h/4) \right]^{-2}
\]
11) \( \left( \frac{\partial \phi}{\partial z} \right)_h = \frac{\alpha}{L} \)

12) \( \left( \frac{\partial K_M}{\partial z} \right)_h = \frac{K_M(h)}{(h+z_0)} \left[ 1 + \alpha \text{Ri}(h) \right]^{-1} \)

13) \( \left( \frac{\partial K_H}{\partial z} \right)_h = \frac{K_H(h)}{(h+z_0)} \left[ 1 - \alpha \text{Ri}(h) \right] \)

C. For lapse, free convection case: \( \text{Ri} < \text{Ri}_t < 0 \)

1) \( r = \sqrt{3/c} \left| \frac{z+z_0}{L} \right|^{1/6} \)

2) \( \phi = \sqrt{c/3} \left| \frac{z+z_0}{L} \right|^{-1/6} \)

3) \( \frac{\partial U}{\partial z} = \frac{u_*}{k_0(z+z_0)} \sqrt{c/3} \left| \frac{z+z_0}{L} \right|^{-1/6} \)

4) \( \frac{\partial \theta'}{\partial z} = \frac{\theta_*}{k_0(z+z_0)} (c/3) \left| \frac{z+z_0}{L} \right|^{-1/3} \)

5) \( U(z) = \frac{6u_*}{k_0} \sqrt{c/3} \left( \frac{z_0}{L} \right)^{-1/6} \left( \left| \frac{z+z_0}{L} \right|^{-1/6} - \left| \frac{z+z_0}{L} \right|^{-1/6} \right) \)

6) \( \theta'(z) = \theta'(0) + \frac{c\theta_*}{k_0} \left( \frac{z_0}{L} \right)^{-1/3} \left( \left| \frac{z+z_0}{L} \right|^{-1/3} - \left| \frac{z+z_0}{L} \right|^{-1/3} \right) \)

7) \( u_* = \frac{k_0 \sqrt{3/c} U (3h/2)}{6 \left( \left| \frac{z_0}{L} \right|^{-1/6} - \left| \frac{0.5h+z_0}{L} \right|^{-1/6} \right) + \left( \frac{h}{h+z_0} \right) \left| \frac{h+z_0}{L} \right|^{-1/6}} \)

8) \( \theta_* = \frac{3 (k_0/c) \left[ \theta'(3h/2) - \theta'(0) \right]}{3 \left( \left| \frac{z_0}{L} \right|^{-1/3} - \left| \frac{0.5h+z_0}{L} \right|^{-1/3} \right) + \left( \frac{h}{h+z_0} \right) \left| \frac{h+z_0}{L} \right|^{-1/3}} \)

9) \( K_M(z) = \frac{(3/c) [k_0(z+z_0)]^3}{\text{Ri}(3h/4) \left| \frac{\partial U}{\partial z} \right|} \)

10) \( K_H(z) = \frac{(3/c)^{12} [k_0(z+z_0)]^3}{\text{Ri}(3h/4) \left| \frac{\partial U}{\partial z} \right|} \)

11) \( \left( \frac{\partial \phi}{\partial z} \right)_h = -\frac{\phi}{6(h+z_0)} \)

12) \( \left( \frac{\partial K_M}{\partial z} \right)_h = \frac{7}{6} \frac{K_M(h)}{(h+z_0)} \)

13) \( \left( \frac{\partial K_H}{\partial z} \right)_h = \frac{4}{3} \frac{K_H(h)}{(h+z_0)} \)

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REFERENCES


