An Application of Markov Theory to Spacecraft Launch Planning

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ABSTRACT

To illustrate the effective use of meteorological data in the planning of spacecraft launchings, certain statistical relationships are presented based on Markov theory and empirical counts. The practical results are in terms of conditional probability at Kennedy Space Center, and are based on 15 years of recorded summer weather data which are analyzed under a set of natural environmental launch constraints.

Three specific forecasting problems are treated: 1) the length of record of past weather which is useful to a prediction, 2) the effect of persistence on runs of favorable and unfavorable conditions, 3) the forecasting of future weather in probabilistic terms. The first problem yields the order of the operative Markov chain, the second problem offers an opportunity to compare theoretically derived results on runs with experimental counts, and the third problem permits application of the Chapman-Kolmogorov equations to obtain conditional probabilities for unfavorable launch conditions up to 4 days in the future. A link is provided between such general conditions and the probability that a launch will be delayed at any specific afternoon hour.

1. Introduction

In a planned rocket launching at Kennedy Space Center at 2000 GMT on 15 July, the launch vehicle was subject to certain constraints, and the deadline for a "go" or "no-go" decision for 15 July (or any subsequent date caused by mission postponement) was at the previous midnight, i.e., 15 h before launch.

For mission planning, the effect of persistence in the weather events which cause operational delays is very important, so this study will take persistence into account in its presentation of conditional probabilities for launch conditions. Markov theory will be applied, and the results will be compared with empirical probabilities obtained directly from the data.[3]

It is well known that thunderstorms are a major impediment to summer launch operations at Kennedy Space Center (KSC). Two recent studies of KSC thunderstorms are the statistical investigations by Falls (1969) and Neumann (1971). Falls reaches the conclusion that thunderstorm events at Cape Kennedy are well represented by a negative binomial distribution. Neumann discusses methods for predicting thunderstorms at the Cape and presents prediction equations obtained by nonlinear regression.

Section 2 of this paper defines a launch weather restriction and reveals the distribution of such restrictions over the 15-year data period which is available. Section 2 also presents an analysis of the utility of past weather conditions in advancing a single day with conditional probabilities. Section 3 applies Markov chain theory and, in particular, the Chapman-Kolmogorov equations to the data, comparing the results with experimental counts. Both first- and second-order Markov processes are investigated, and a statistical method is presented which predicts launch conditions on a dichotomous basis for mission planning. Illustrative examples of calculations are added in Section 4. In Section 5, the work is summarized and the conclusions are presented.

2. Launch weather restriction

a. *Definition*

A launch weather restriction is defined as the occurrence of any of the following unfavorable conditions:

Precipitation
Thunderstorm with a cloud ceiling
Cumulus cloud ceiling $<$4000 ft altitude
Wind speed $\geqslant$25 Kt at 30 ft altitude.

The complete record of hourly observations of weather elements at Cape Canaveral, Fla., from 1957–71 is available; additional information includes the peak wind speed each hour and the duration, location and

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[3] Other studies of persistence in meteorological events have been conducted by Feyerherm and Bark (1965), Williams (1952), Hopkins and Robillard (1964), Weiss (1964), Eriksson (1965), Brelsford and Jones (1967), Smith (1974), Gabriel and Neumann (1957, 1962) and Caskey (1963). Additional material on runs is available in Gabriel (1959), Walker and Duncan (1967), von Mises (1964), Feller (1957) and Wilks (1962). A rather complete treatment of hypothesis-testing in regard to Markov chains is found in Anderson and Goodman (1957).

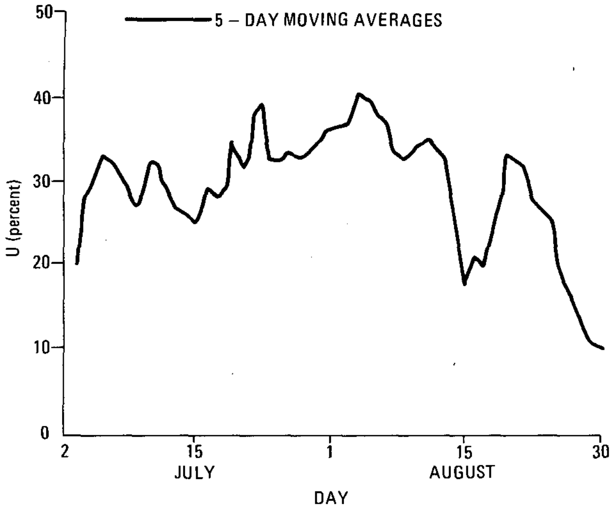

FIG. 1. Frequency of days with unfavorable (U) conditions in July and August, KSC, 1957–71.

other characteristics of each thunderstorm. To treat just the 2000 GMT observations corresponding to the launch hour would ignore a substantial number of cases when restrictive conditions occurred during the afternoon or the weather at 2000 GMT was nearly below limits. Therefore, a broader definition of "unfavorable conditions" has been adopted so that one or more of the four constraints is simply required to be reported at least once in the hourly record from 1700 to 2200 GMT. This makes the concept of "unfavorable" almost synonymous with the common operational weather forecasting term "marginal."

A link can be provided between this definition and the probability of occurrence of restricted conditions at a particular hour. From all July and August data for the period 1957–71, the relative frequency of restricted conditions at 1800, 1900 or 2000 GMT, is 0.333, 0.390 and 0.434, respectively. The 95% confidence limits on each value are ±0.049.

b. Time distribution of launch unfavorable days

Five-day moving averages of the frequency of unfavorable days, as defined in Section 2a for a launch, are calculated for each July and August day and shown in Fig. 1 for the period 1957–71. A rising trend is apparent in July with a reversal early in August. There are a few irregularities, but only the dip in mid-August is significant at the 1% level by Student's t-test. In view of these results, the data are considered to be sufficiently homogeneous to permit the inclusion of August weather reports. Since the last 15 days of August appear to have fewer unfavorable days, the assumption of homogenity is a conservative approximation. This produces a total of 930 days of observation for the 15-year period under study.

c. Analysis of a 1-day advance

The following question will now be considered: if an advance of 1 day into the future is to be made in a prediction scheme based upon the past weather with its persistent nature, how many days of past weather can be profitably used?

The empirical probabilities are obtained by counting the number of days that fit the class description (r) divided by the total number of days in the data set (n). For example, for a 3-day period from day -1 to day 1,

$$p(F_1|U_0F_{-1})=\frac{r}{n}. \tag{1}$$

Thus, by counting the number of days r in the total data set when a favorable launch day is preceded by an unfavorable day and a previous favorable day, and then dividing r by the total number of days n in the data set, an approximation to $p(F_1|U_0F_{-1})$ is obtained which improves as n increases. Confidence limits are then calculated on p, using Bayes theorem as shown in Pratt *et al.* (1965). There is some prior distribution of p assumed before the present data are utilized. This is called the prior distribution given by $f_I(p)$. Since all that is known is that p has a value between 0 and 1 and any value is equally likely, one can let

$$f_I(p)=1. \tag{2}$$

Then the posterior distribution of p, given the data $D_{r,n}$, i.e., $[f_F(p|D_{r,n})]$, can be obtained from Bayes theorem

$$f_F(p|D_{r,n})=\frac{f(D_{r,n}|p)f_I(p)}{f(D_{r,n})}. \tag{3}$$

The probability of r successes in n trials for a given value of p is given by

$$f(D_{r,n}|p)=p^r(1-p)^{n-r}. \tag{4}$$

As a result, after the data have been utilized, the posterior distribution of p is given by

$$f_F(p|D_{r,n})=\frac{(n+1)p^r(1-p)^{n-r}}{n!(n-r)!\int_0^1 p^r(1-p)^{n-r}dp}. \tag{5}$$

Eq. (5) is the well known beta distribution. The maximum likelihood estimate is given by the maximum of $f_F(p|D_{r,n})$, which corresponds to

$$\frac{df_F(p|D_{r,n})}{dp}=0. \tag{6}$$

This gives the maximum likelihood estimate of p as

$$p=\frac{r}{n}. \tag{7}$$

Then the 95% confidence limits p_u and p_L are given by

$$\int_{p_L}^{p_u} f_F(p|D_{r,n})dp=0.95. \quad (8)$$

It can be shown that the beta distribution for large n can be approximated by a normal distribution with a mean $\mu(p)$ of r/n and a variance $\sigma^2(p)$ of $(r/n)[1-(r/n)]/n$. Thus the normal tables can be used to find the 95% confidence limits p_u and p_L. For the normal approximation this is given by

$$p_u=\frac{r}{n}+1.965\sigma(p), \quad (9a)$$

$$p_L=\frac{r}{n}-1.965\sigma(p). \quad (9b)$$

As shown by Cohen (1968), the zero-order $P(F_0)$, first-order $P(F_1|F_0)$ and second-order $P(F_1|F_0F_{-1})$, probabilities can all be obtained from any four given empirical probability values. For example, Table 1 presents a set of calculated values based upon the empirical values for $P(F_0)$, $P(F_1|F_0)$, $P(F_1|F_0F_{-1})$ and $P(F_1|U_0U_{-1})$.

Some immediately derivable expressions are listed below as Eqs. (10)–(14):

$$P(U_0)=1-P(F_0). \quad (10)$$

TABLE 1. Empirical probabilities and calculated probabilities of conditions for launch.

	Empirical value	Calculated value	Calculated from
$P(F_1)$	0.694±0.026*		
$P(U_1)$		0.306±0.026*	$P(F_1)$
$P(F_1\|F_0)$	0.788±0.028	0.787±0.022*	$P(F_1\|U_0)$
$P(F_1\|U_0)$	0.483±0.051*	0.481±0.064	$P(F_1\|F_0)$
$P(F_1\|F_0F_{1-})$	0.823±0.028	0.807±0.020*	$P(F_1\|F_0U_{-1})$
$P(F_1\|F_0U_{-1})$	0.714±0.070*	0.654±0.103	$P(F_1\|F_0F_{-1})$
$P(F_1\|U_0U_{-1})$	0.493±0.069*	0.492±0.072	$P(F_1\|U_0F_{-1})$
$P(F_1\|U_0F_{-1})$	0.473±0.077	0.472±0.074*	$P(F_1\|U_0U_{-1})$

Note: The subscripts indicate the order of days for a favorable (F) or unfavorable (U) case. The asterisks indicate the values used in subsequent calculations.

For a first-order process

$$P(U_1|F_0)=1-P(F_1|F_0), \quad (11)$$

$$P(F_1|U_0)=\frac{P(F_0)}{P(U_0)}P(U_1|F_0), \quad (12)$$

while for a second-order process

$$P(U_1|F_0F_{-1})=1-P(F_1|F_0F_{-1}), \quad (13)$$

$$P(U_1|U_0U_{-1})=1-P(F_1|U_0U_{-1}). \quad (14)$$

Similarly, it has been shown by Cohen (1968) that

$$P(U_1|F_0U_{-1})=\frac{P(F_0)[1-P(F_0|F_{-1})]-P(F_{-1})P(F_0|F_{-1})[1-P(F_1|F_0F_{-1})]}{P(F_0)-P(F_{-1})P(F_0|F_{-1})}.$$

This can be rewritten as

$$P(U_1|F_0U_{-1})=\frac{P(F_1)P(U_1|F_0)-P(F_1)P(F_1|F_0)P(U_1|F_0F_{-1})}{P(F_1)[1-P(F_1|F_0)]}=\frac{P(U_1|F_0)-P(F_1|F_0)P(U_1|F_0F_{-1})}{P(U_1|F_0)}. \quad (15)$$

This is equivalent to

$$P(F_1|F_0F_{-1})=\frac{P(F_1|F_0)P(U_1|F_0F_{-1})}{P(U_1|F_0)}. \quad (16)$$

By interchanging F and U, one can also obtain

$$P(U_1|U_0F_{-1})=\frac{P(U_1|U_0)P(F_1|U_0U_{-1})}{P(F_1|U_0)}, \quad (17)$$

and Eq. (14) can be used to obtain

$$P(F_1|U_0U_{-1})=1-P(U_1|U_0U_{-1}). \quad (18)$$

Table 1 presents empirical values and calculated values of both favorable and unfavorable conditions for launch the following day. To illustrate the meaning of the symbols in the table, $P(F_1)$ is the probability of the next day being favorable for the launch, i.e., neither restricted nor "marginal," independent of present or previous weather. This is considered a zero-order probability and its complement is $P(U)$. The term $P(F_1|U_0)$ is the probability of the next day being favorable for launch, given that the present day's weather is unfavorable. This is considered a first-order probability. Similarly, $P(F_1|U_0F_{-1})$ is the probability that the next day will be favorable for launch given that the present day is unfavorable and the day before was favorable. This is considered a second-order probability. The results of Table 1 are also plotted in Fig. 2.

The remaining calculated values of Table 1 are complements of values found by Eqs. (15) and (17). Differences between corresponding pairs of numbers in this table are but a few percent and are always within the confidence limits. To illustrate the determination of confidence limits, suppose that $P(F_1|U_0)$

is to be computed from $P(F_1|F_0)$ by Eqs. (11) and (12). Since $P(F_1|F_0)$ is a random variable, its 95% confidence limit p_L is given by Eq. (9), where $p=p(F_1|F_0)$. Using angle braces to denote the averaging process, the confidence limits of the calculated quantity $P(U_1|F_0)$ are found by deriving the variance $\sigma^2[P(U_1|F_0)]$ from

$$\begin{aligned}\sigma^2[P(U_1|F_0)]&=\langle[1-P(F_1|F_0)]^2\rangle-\langle[1-P(F_1|F_0)]\rangle^2\\&=\langle P(F_1|F_0)^2\rangle-\langle P(F_1|F_0)\rangle^2\\&=\sigma^2[P(F_1|F_0)].\end{aligned}\qquad(19)$$

Since the variance is unchanged by subtraction, the confidence limits of the calculated value $P(U_1|F_0)$ is equal to the empirically determined value found for $p(F_1|F_0)$.

Therefore, from Table 1 and Eq. (11)

$$\begin{aligned}P(U_1|F_0)&=(1-0.788)\pm0.028\\&=0.212\pm0.028.\end{aligned}\qquad(20)$$

From Eq. (12)

$$P(F_1|U_0)=KP(U_1|F_0),\qquad(21)$$

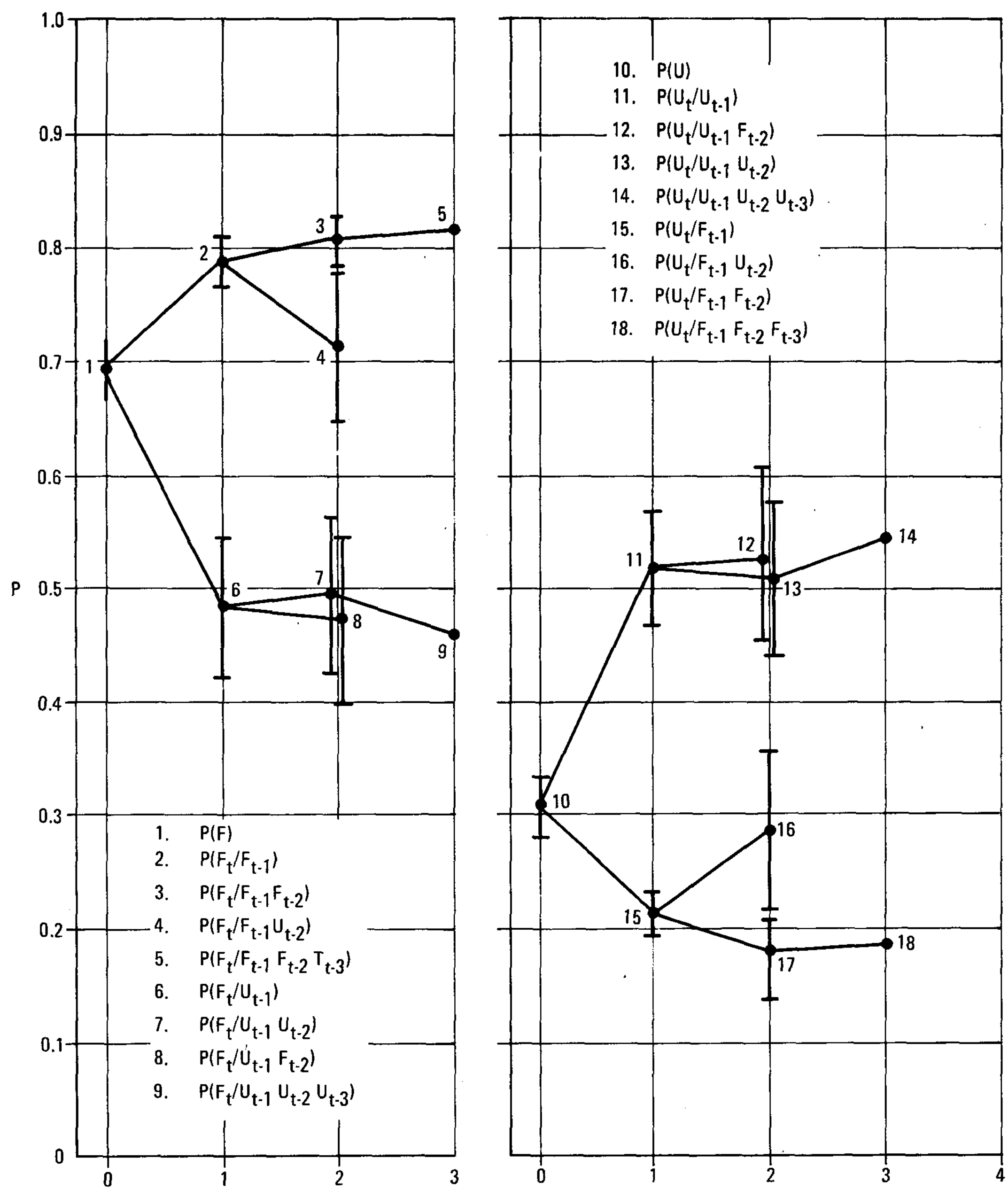

FIG. 2. Probability (P) of favorable (F) or unfavorable (U) conditions the next day, given i days of record. The 95% confidence limits are shown for $i\leqslant 2$.

where

$$K = p(F_1)/p(U_1).$$

The value of K is assumed to have zero confidence bonds for simplicity. The results can be extended to include the confidence bands on K. Following the same procedure as before,

$$\sigma^2[P(F_1|U_0)] = \langle [KP[U_1|F_0)]^2\rangle - \langle [KP(U_1|F_0)]\rangle^2 = K^2\sigma^2[P(U_1|F_0)]. \quad (22)$$

Thus, the confidence limit is changed by a factor of K. Now from Eqs. (20) and (21) and Table 1

$$P(F_1|U_0) = \frac{0.694}{0.306}(0.212 \pm 0.028) = 0.481 \pm 0.064. \quad (23)$$

This result is shown in Table 1. Now, since $p(F_1|U_0)$ and $p(F_1|F_0)$ can be found experimentally, either one can be calculated from the other. In making the choice of an empirical value, it is reasonable to select the value which, together with its corresponding calculated value, has the narrowest confidence limits. Such selections are marked with asterisks in Table 1 and are also plotted in Fig. 2, which shows the probability of a favorable day for given conditions during the previous days. These results were taken from Table 1.

If the present case were described by a zero-order Markov process, the conditional probabilities $P(F_1|F_0)$ or $P(F_1|U_0)$ would be equal to $P(F_1)$. However, as shown in Fig. 2, they are significantly different from the value of $P(F_1)$, so that it is extremely unlikely that a zero-order Markov process would describe the present case. Fig. 2 also shows that the probability of the next day being a favorable day, given that the present day is favorable, will also depend on the previous day—that is, $P(F_1|F_0U_{-1})$ is significantly different from $P(F_1|F_0F_{-1})$. This means that if the present day is favorable, the probability of the next day being favorable would depend on whether the previous day was favorable or unfavorable.

Thus the condition $P(\cdot|F\cdot)$ can be considered a second-order process. However, in Fig. 2, note that $P(F_1|U_0U_{-1})$ and $P(F_1|U_0F_{-1})$ are very close to $P(F|U)$. This means that $P(\cdot|U)$ can be considered a first-order process. The results for the probability of an unfavorable day are equal to the complement of results for a favorable day and are also shown in Fig. 2.

d. Runs of persistent launch conditions

In counting the number of days in a run, all sequences which originate before 1 July are excluded; and those which extend into September are included, both arbitrarily. The longest run of consecutive unfavorable days after a favorable day is one week. This is shown in Fig. 3, where relative frequencies give the empirical probability of runs of n or more days, with n ranging up to 7 days. Note that $p(n \geqslant 1)$ is the probability of one or more unfavorable days after a favorable day, and its empirical value of 0.212 is equal to $1 - P(F_1|F_0)$, where $P(F_1|F_0)$ is 0.788 (Table 1).

The theoretical probabilities of the same runs are calculable from the basic empirical values found by counting cases. Thus, the probability of n or more unfavorable days after a favorable day is given by

$$P(U^{(n)}F_{-1}) = P(F_{-1})P(U_0|F_{-1})P^{n-1}(U_1|U_0). \quad (24)$$

The equations for a chain of events are discussed more fully in Section 3. Normalizing yields

$$y_n = \frac{P(U^{(n)}F_{-1})}{P(F_{-1})P(U_0|F_{-1})} = P^{n-1}(U_1|U_0). \quad (25)$$

Curve A (Fig. 3) shows the result when the probabilities are evaluated from the data by Eq. (24), and it is in agreement with the empirical information to 0.1%. This graphical relationship provides the probability of unfavorable ASTP launch weather for n days, given the present day is favorable.

Since curve A is straight on semilog paper for $n \geqslant 1$, then

$$y_n = y_0 e^{-k(n-n_0)}. \quad (26)$$

Evaluating k directly from curve A yields $k = 0.658$. Then for a 1-day advance,

$$\frac{y_{n+1}}{y_n} = P(U_1|U_0) = e^{-k(1)} = 0.518. \quad (27)$$

This outcome for $P(U_1|U_0)$ agrees closely with the empirical value (0.517) obtained by counting cases. A similar linear relationship for runs was also examined by Langley (1953) for wet periods in Montreal.

Using the same counting rule, the longest run of favorable conditions after an unfavorable day is found to be 25 days (Fig. 3). Again, the probabilities have been estimated by relative frequencies counted from the data. The theoretical probability of n or more favorable days after an unfavorable day is given by

$$P(F^{(n)}U_{-1}) = P(U_{-1})P(F_0|U_{-1}) \times P(F_1|F_0U_{-1})P^{n-2}(F_2|F_1F_0). \quad (28)$$

Normalizing gives

$$y_n' = \frac{P(F^{(n)}U_{-1})}{P(U_{-1})P(F_0|U_{-1})} = P(F_1|F_0U_{-1})P^{n-2}(F_2|F_1F_0). \quad (29)$$

Curve B (Fig. 3) shows this result when the probabilities in Eq. (28) are evaluated from the data. Agreement with the plotted points is fairly good,

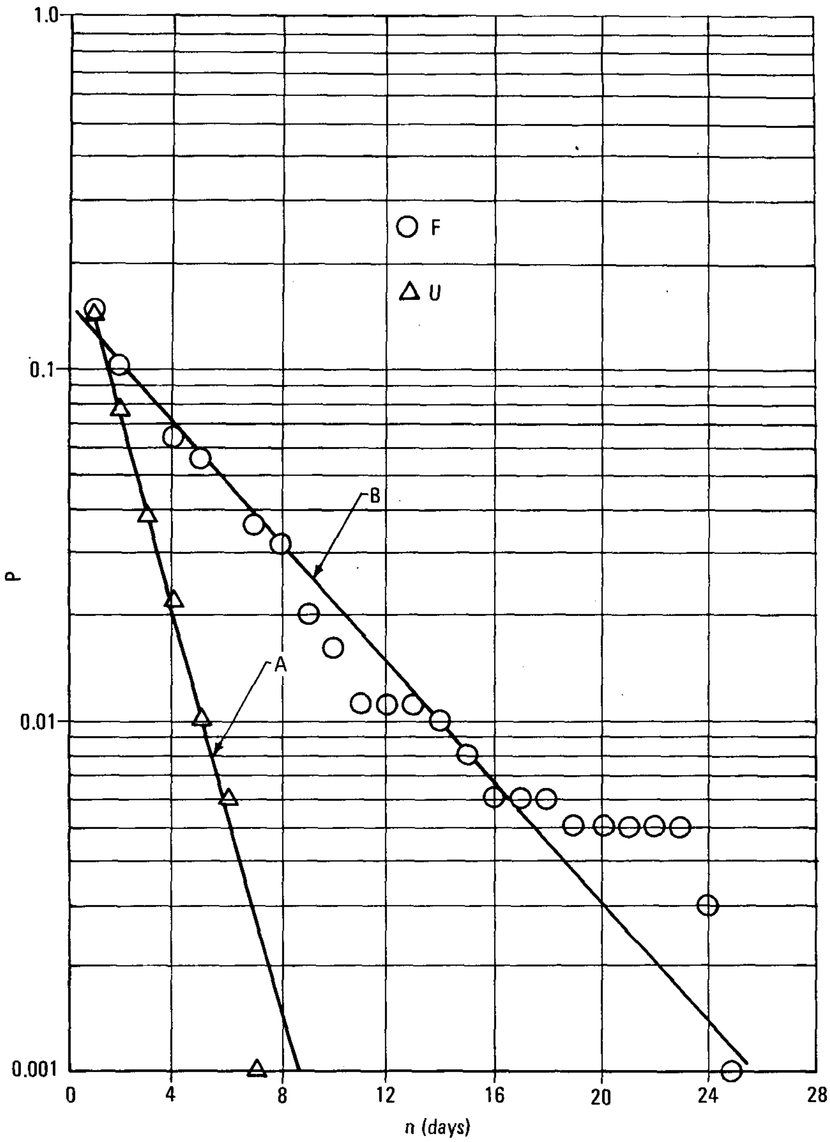

FIG. 3. Cumulative probability (P) of the occurrence of n unfavorable days, given a favorable day, curve A [Eq. (24)], and of n favorable days, given an unfavorable day, curve B [Eq. (28)], based upon 124 reversals in each case out of a total of 826 days. (Empirical values obtained by counting are also shown as individual points.)

although there are departures from the theoretical curve when the probability falls below 0.02. Again, the probability of persistently favorable launch weather for n or more days, given the present day has unfavorable conditions, can be read from the graph.

Estimating k directly from curve B yields a value of 0.203. Then, for a 1-day progression

$$\frac{y'_{n+1}}{y'_n}=P(F_1|F_0F_{-1})=e^{-k(1)}=0.816. \qquad (30)$$

This outcome for $P(F_1|F_0F_{-1})$ is slightly less than the empirical value of 0.825 obtained by counting.

3. Conditional probabilities from the Chapman-Kolmogorov equations

In this section, the applicability of the Markov chain theory will be stressed, and the Chapman-Kolmogorov equations (Breiman, 1969) will be used to obtain conditional probabilities for up to 4 days following a base day with its identifiable favorable or unfavorable condition and available past weather record.

a. Markov chains of order one and order two

The law of multiplication in probability theory states that the probability of a string of events

$E_t, E_{t+1}, \ldots, E_{t+n}$ is the product of n factors, i.e.,

$$P(E_t, E_{t+1}, \ldots E_{t+n}) = P(E_t)P(E_{t+1}|E_t)P(E_{t+2}|E_{t+1}, E_t)\ldots P(E_{t+n}|E_{t+n-1}, ,\ldots E_{t+1}, E_t). \quad (31)$$

A Markov chain of first order also has n factors, but it retains but one previous event as "given" in each factor; thus

$$P(E_t, E_{t+1}, \ldots E_{t+n}) = P(E_t)P(E_{t+1}|E_t)P(E_{t+2}|E_{t+1})\ldots P(E_{t+n}|E_{t+n-1}). \quad (32)$$

Analogously, a Markov chain of second order retains two previous events as "given" in each factor, so that

$$P(E_t, E_{t+1}, \ldots E_{t+n}) = P(E_t)P(E_{t+1}|E_t)P(E_{t+2}|E_{t+1}, E_t)\ldots E(E_{t+n}|E_{t+n-1}, E_{t+n-2}). \quad (33)$$

Many of the papers referenced in Section 1 which treat persistence in precipitation and in dry periods find that a Markov model, especially the first-order chain, is an acceptable device to describe the data.

b. The Chapman-Kolmogorov equations

The probability of passing from the initial state A_i at time n to state A_j at time $n+1$ can be written as $P(A_{j,n+1}|A_{i,n})$. The probability of passing from the initial state A_i at time n to a new state A_k at time $n+2$ is given by the Chapman-Kolmogorov equation for a first-order Markov process as

$$P(A_{k,n+2}|A_{i,n}) = \sum_j P(A_{k,n+2}|A_{j,n+1})P(A_{j,n+1}|A_{i,n}). \quad (34)$$

Thus the probabilities have been summed over all possible intermediate states A_j at time $n+1$. This

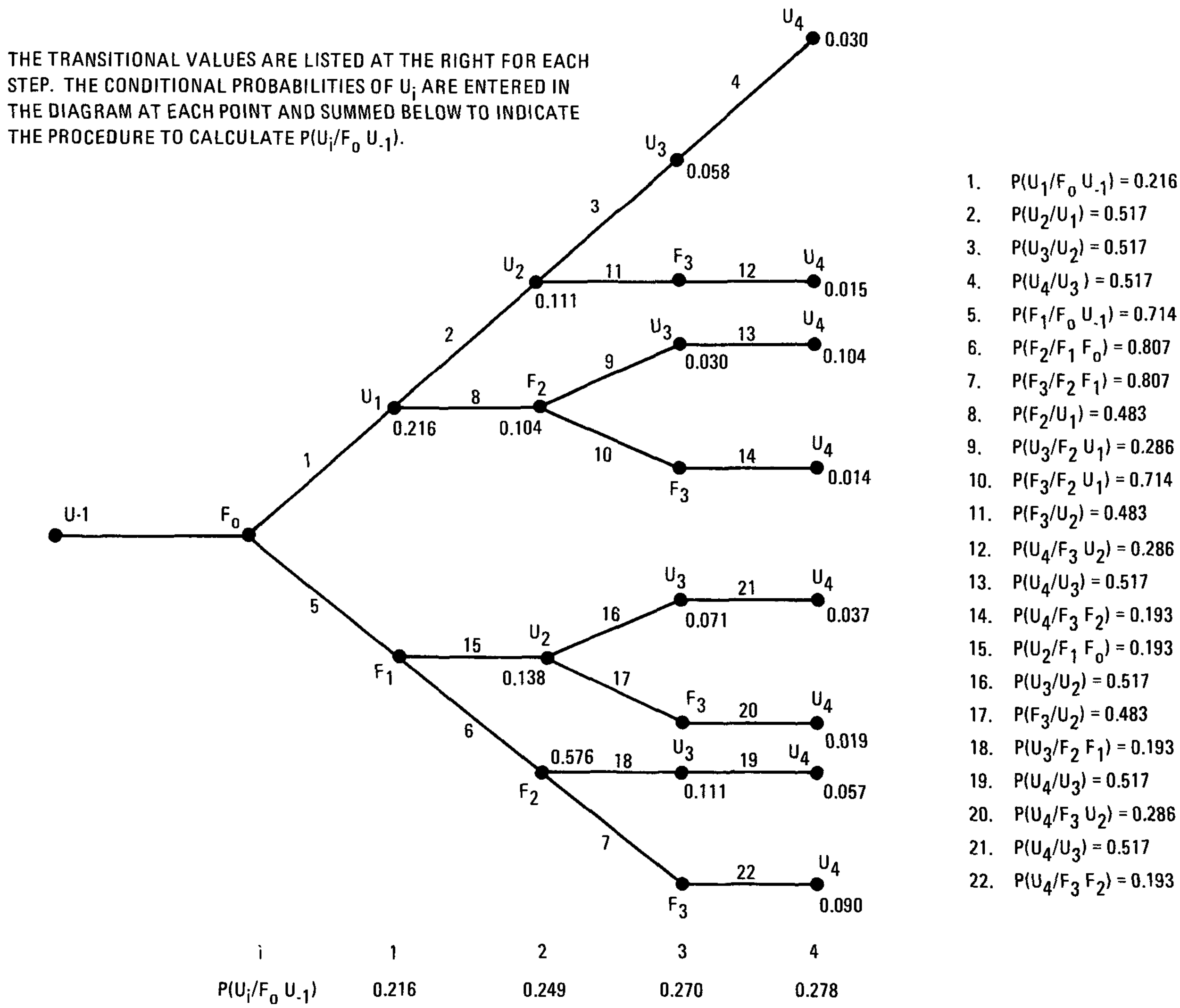

FIG. 4. Branching diagram for the computation of conditional probabilities by the Chapman-Kolmogorov equations.

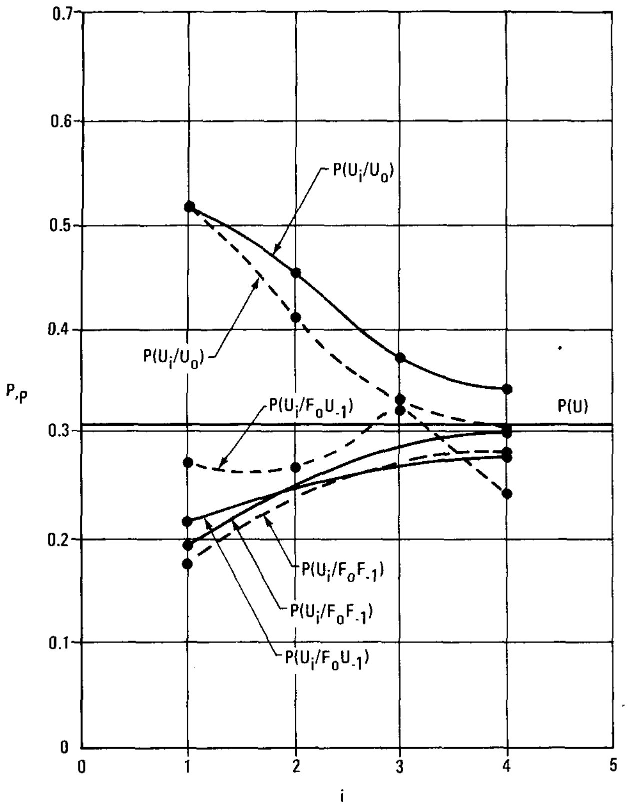

FIG. 5. Conditional probabilities of marginal ASTP launch conditions in July and August at Cape Kennedy based upon the Chapman-Kolmogorov equations (P) and upon empirical counts (p) for i days in the future from day zero.

process can be continued, i.e.,

$$P(A_{l,n+3}|A_{i,n})$$

$$=\sum_k P(A_{l,n+3}|A_{k,n+2})P(A_{k,n+2}|A_{i,n}), \quad (35)$$

etc. The same type of analysis is applicable to second-order or zero-order processes.

The results for favorable conditions can be obtained from the following equations:

$$P(F_i|U_0)=1-P(U_i|U_0), \quad (36)$$

$$P(F_i|F_0F_{-1})=1-P(U_i|F_0F_{-1}), \quad (37)$$

$$P(F_i|F_0U_{-1})=1-P(U_i|F_0U_{-1}). \quad (38)$$

These results can be used to predict the behavior in advance, given the present known weather conditions. The method of calculation is illustrated in Fig. 4, and the outcome for U_i, $i=1, 2, 3, 4$, for July and August at KSC is shown in Table 2 and Figs. 4 and 5. Experimental counts are included in the table for comparison with the theoretical predictions, and agreement is to within 5% absolute value in all of the conditional probabilities. This indicates that the Chapman-Kolmogorov equations are indeed applicable to this problem. Note that instead of using a tree diagram a matrix-type equation can be used. Also a chi-square test can be used to test the order of Markovitz.

4. Examples of probability calculations

The values of Table 2 and their complements for unfavorable cases are sufficient to compute the probabilities of runs with no previous conditions. For example, if $P(U_0U_1U_2)$ is desired, Eq. (32) is applied to get

$$P(U_0U_1U_2)=P(U_0)P(U_1|U_0)^2$$
$$=(0.306)(0.517)^2=0.082.$$

If $P(F_0F_1F_2)$ is desired, Eq. (33) is applied to yield

$$P(F_0F_1F_2)=P(F_0)P(F_2|F_1F_0)^2$$
$$=(0.694)(0.823)^2=0.470.$$

In case a probability specified at a particular hour is sought, the appropriate link from Section 2a is inserted. Thus, to find the 2000 GMT probability of a restriction, given a previous day's restriction at 2000, we use

$$P(U_{1500,1}|U_{1500,0})$$
$$=P(U_0|U_{1500,0})P(U_{1500,1}|U_0)$$
$$=P(U_0|U_{1500,0})P(U_1|U_0)P(U_{1500,1}|U_1)$$
$$=(1.00)(0.517)(0.434)=0.224.$$

The unconditional probability of encountering two consecutive restrictions at 2000 is

$$P(U_{1500,1}U_{1500,0})$$
$$=P(U_{1500,0}|U_0)P(U_0)P(U_1|U_0)P(U_{1500,1}|U_1)$$
$$=(0.306)(0.434)(0.517)(0.434)=0.030.$$

5. Summary and conclusions

In a planned rocket launching at Kennedy Space Center, the decision to launch was made some 15 h before launch time. Therefore, mission planning required a knowledge of statistical relationships between the occurrence of weather conditions spaced one, two or a few days apart. Markov theory has been applied

TABLE 2. Conditional probabilities of unfavorable conditions for launch.

	$P(U_i\|U_0)$		$P(U_i\|F_0U_{-1})$		$P(U_i\|F_0F_{-1})$	
i	E*	C**	E*	C**	E*	C**
1	0.517	0.517	0.272	0.216	0.175	0.193
2	0.410	0.454	0.267	0.249	0.242	0.255
3	0.331	0.372	0.319	0.270	0.274	0.284
4	0.306	0.343	0.245	0.278	0.278	0.297

* Empirical value obtained by counting and use of the complementary relationship.
** Calculated value from the Chapman-Kolmogorov Equations.

in this study to elucidate these relationships in terms of conditional probabilities for KSC.

The first forecasting problem investigated was the length of record of past weather which is useful to a prediction. Based upon the historical sequence of hourly reports for July and August from 1957–71, relative frequencies of marginal weather were gleaned from the data and expressed as four empirical conditional probabilities from which other conditional probabilities up to second order were derived. The outcome is contingent upon the nature of the preceding weather. Thus, if afternoon weather for the current day has been unfavorable, the previous afternoon's reports have negligible forecast value. On the other hand, if the afternoon weather for the current day has been favorable, the previous day's reports are important to the prediction. These results signify that first-order and second-order Markov chains, respectively, are operative.

The second forecasting problem studied was the matter of runs of favorable or unfavorable launch conditions. Such runs were found to persist as long as 25 days and 7 days, respectively. In the case of unfavorable present weather, there is a conditional probability slightly greater than 0.50 that inclement conditions will persist another day. On the other hand, the probability of exactly one favorable day, given unfavorable present weather, is only about 0.15. A probability can be read from Fig. 3 for any desired number of days in a sequence of favorable days after an unfavorable day.

Fig. 3 is also useful to an analysis of runs of unfavorable weather. If one assumes the present weather is good, for example, then the probability of a change to inclement weather for exactly one day is also 0.15. Conditional probabilities of runs of unfavorable days with greater minimum length are readily obtained from this graph.

The final forecasting problem investigated was the prediction of launch conditions for a few days ahead, following a base day with known present weather and past weather. Further application of Markov theory in the form of the Chapman-Kolmogorov equations was made, there being evidence of feasible predictions up to 4 days, at which time the empirical value of $p(U_4|U_0)$ generally reaches the unconditional value of $p(U)$. These results are available in tabular form (Table 2) and also graphical form (Figs. 4 and 5). Agreement between theoretical predictions and experimental counts is to within 5%.

Finally, it is noted again that the definition of "unfavorable" used throughout this study is "the occurrence of one or more of the constraints (Section 2a) at any hour between 1700 and 2200 GMT." There is a known probability that restriction will take place at a particular hour on an unfavorable day in July and August, and as was demonstrated in Section 4, this provides a link between the tabular values of this report and calculations of probabilities for specific hours.

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