Parameterization of Fractional Cloud Amounts in Climatic Models: The Importance of Modeling Multiple Reflections

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ABSTRACT

The neglect of multiple reflections between clouds and the earth's surface leads to an underestimate of the downward flux of solar radiation reaching the earth's surface. This underestimate is most pronounced in regions of persistent cloud cover and high surface albedos—the snow- and ice-covered regions of the high latitude zone, for example. Since the rate of snow melt (and thus snow albedo) depends upon the downward flux, neglect of multiple reflection is most serious in climatic models that predict snow and ice cover. Two simple algebraic expressions to account for multiple reflections in climate models are given and are shown to be limiting cases of a more general formula. Since this more general formula depends on the spatial distribution of subgrid-scale cloud cover amounts, an unambiguous definition of cloud amount over a GCM-scale grid square cannot be given, even if perfect knowledge of the optical properties of the subgrid-scale clouds were in hand. However, the uncertainties in downward solar flux at the earth's surface (or the albedo of the combined cloudiness-surface system) introduced by lack of knowledge of the two-dimensional geometric distribution of fractional cloud cover are shown to be generally less than 10%, most likely less than the errors in predicting cloud cover amounts or from the neglect of three-dimensional effects of finite-sized clouds.

1. Introduction

Considerable attention has been focused (e.g., SMIC, 1971; Kellogg, 1975) on questions of the stability of polar ice masses to changes in the earth's energy budget. Maykut and Untersteiner (1971) showed with their model for sea ice that about a 20% decrease in the surface albedo of the ice might be able to melt the ice pack in a few years. Manabe and Wetherald (1975), using a three-dimensional general circulation model with snow prediction included, calculated that a doubling of atmospheric CO_2 could cause a surface temperature increase in polar regions several times larger than the global average temperature increase of 2.9 K. This enhanced temperature sensitivity of the polar regions in their model was a result of two factors: 1) snow albedo-temperature feedback and 2) the relatively high static stability of near-surface polar air compared to near-surface air at lower latitudes. The importance of this enhanced polar region sensitivity to the global climate has also been reviewed in Section C3e of Schneider and Dickinson (1974).

It has been known for some time that the solar radiation reaching snow- and ice-covered surfaces is very strongly affected by multiple reflections between the high albedo surface and moderately high albedo clouds. Recently, for example, Catchpole and Moodie (1971) pointed out:

"An extreme effect of multiple reflection in regions of uniformly high surface albedo is the 'whiteout,' which may occur in sunlight or moonlight, when cloud veils overlie snow or ice fields. During whiteout the upward and downward fluxes of light are almost identical, with the result that the horizon cannot be discerned. Three-dimensional snow or ice objects cannot readily be seen when whiteout prevails because they do not cast shadows and all points on their surface are uniformly illuminated . . ."

Holmgren and Weller (1973) measured this phenomenon, noting that "the incoming solar radiation does not decrease very much when a cloud cover forms over an extensive snow-field . . . The average decrease of incoming radiation because of clouds [was] only about 15%" for observations taken over six overcast days in the month of April.

The primary intent of this paper is to emphasize that the effect of multiple reflections can be very important in cloud-covered regions of high surface albedo in determining the solar flux reaching the surface, and hence the rate of snow melt, which, in turn, influences the snow albedo. Therefore it may be essential to include these effects in climate models that predict snow and ice cover but do not yet include multiple reflections (e.g., Manabe and Wetherald, 1975). To that end we give several algebraic expres-

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sions that simply account for multiple reflections in climatic models. One expression gives the effective albedo α_T of a cloud-surface system, another the total cloud absorption for this system, and a third the downward solar flux $Q_s^{\downarrow}$, reaching the surface. The latter has been treated with varying degrees of approximation by different groups' atmospheric general circulation models (GCM's). It is shown that considerable underestimation of $Q_s^{\downarrow}$ and α_T at snow- and cloud-covered grid points is introduced by neglecting multiple reflections, and it is pointed out that inclusion of these multiple reflections involves only a minor modification to existing radiation codes and probably adds negligible extra computational time. However, as shown later, an unambiguous definition of cloud amounts for a GCM size grid square requires not only knowledge of the average optical properties of the subgrid-scale clouds, but also information about the horizontal distribution of these clouds over the grid square. Furthermore, although the greatest inaccuracy in the parameterization of cloudiness-radiation interactions will continue to be the prediction of the clouds themselves, this is no reason to neglect multiple reflections when their important effects can be included simply.

2. Simple parameterizations for multiple reflections with fractional cloudiness

Consider a "thin" cloud that covers a fraction C of the area of the sky. The cloud is referred to as "thin" because we neglect radiation reflected or emitted from sides of the cloud. For a practical climate model calculation such an idealized or "effective cloud" is an average of the areal coverage of many clouds over the region in question. The extent to which such an idealized cloud could realistically approximate a three-dimensional geometric distribution of clouds in the atmosphere is not considered here, but such an idealized cloud serves the purpose of illustrating the instances when multiple reflections are important. [We also omit reference in this derivation to selective absorption of solar radiation by optically active gases or zenith angle or wavelength dependence of cloud radiative properties, all of which would have to be accounted for in fully realistic climate simulations.]

Let α be the cloud's albedo (assumed the same for cloud top and base), A its fractional absorptivity, and $t=1-\alpha-A$ its transmissivity. For any of these three optical parameters, the subscript s will denote an optical parameter relative to the direct solar beam for a given solar zenith (direct beam albedo of the clouds, α_s, for example), and the subscript d will denote an optical parameter relative to the diffuse (i.e., reflected or transmitted) beam (diffuse beam albedo of the cloud, α_d, for example). For a homogeneous cloud-covered atmosphere ($C=1$) overlying a surface of albedo α_E (for simplicity assumed the same for direct or diffuse radiation) it is well known (e.g., see Schneider, 1971) that the total combined or effective albedo α_T of the combined cloud-surface system is

$$\alpha_T=\alpha_s+\frac{\alpha_E t_s t_d}{1-\alpha_E\alpha_d}. \tag{1}$$

We now consider various ways of extending Eq. (1) to the case of partial cloud cover, as specified by the fraction C of the sky covered by a layer of clouds. In particular, in problems of climate modeling, variations in cloudiness can often occur on scales small in comparison to the horizontal resolution of the model (typically 100's of kilometers on a side for a GCM grid), and a given value of C may refer to widely differing two-dimensional patterns of cloud cover, varying from a patchy or randomly distributed pattern of fair-weather cumulus to a partly overcast, partly clear situation typical of an advancing frontal system. Furthermore, the cloud optical properties may vary widely over such a grid region. For simplicity, we assume the structure of this subgrid-scale variation is unimportant so that the cloud optical properties used here are thus averages of the optical properties of individual clouds over the grid. Finally, the grid is assumed to be large enough so that the horizontal dimension of the averaging region is large compared with the vertical depth of the clouds.

CLOUDINESS DEFINITION: THE PATCHINESS PARAMETER

a. Unpatchy limit

If we area-weight the albedos of the clear and cloud-covered fractions of the sky, we get the commonly used total combined albedo α_T^0 of the cloudy-cloudless-surface system:

$$\alpha_T^0=C\alpha_T+(1-C)\alpha_E, \tag{2}$$

where α_T comes from Eq. (1). We shall discuss in Section 3 the corresponding expression for total transmission in this same limit.

Physically this expression represents the limiting case where the cloudy parts of the grid region are entirely overcast and the clear parts are fully cloud free (a situation typical of a large-scale cyclone partially covering a grid square). In this limit, the probability that a direct beam that has passed through a cloud will, after reflection at the surface, be incident again upon the bottom of the clouds is unity. Similarly, a beam that passes directly to the surface upon reflection will not further encounter the cloud.

b. Total patchiness limit

In this limit the individual clouds are sufficiently small and so distributed over the grid square that the probability of an upward reflected beam subsequently encountering a cloud is equal to the fraction C of the grid square covered by clouds independent of the previous origin of the beam. Similarly, the probability that a beam will upon upward reflection not encounter a cloud is equal to $1-C$, the fraction of the grid square not covered by clouds, again independent of the previous origin of the beam. The albedo for this limiting case can be derived from Eq. (1) in terms of the average albedo and transmission of the atmosphere, $\hat{\alpha}$ and $\hat{t}$, respectively. Then, allowing for multiple reflections, and for simplicity letting diffuse and direct optical parameters be equal and dropping subscripts s and d, we can rewrite (1) as

$$\alpha_T=\hat{\alpha}+\frac{\alpha_E\hat{t}^2}{1-\hat{\alpha}\alpha_E}. \quad (3)$$

Here, the average albedo $\hat{\alpha}$ equals $C\alpha$ and the average transmission $\hat{t}$ equals $1-C+Ct$. Substitution of these terms into (3) yields the combined cloudy-cloudless-surface system albedo for the totally patchy case:

$$\alpha_T^1=C\alpha+\frac{\alpha_E(1-C+Ct)^2}{1-C\alpha\alpha_E}. \quad (4)$$

It is convenient for interpreting Eq. (4) to write

$$\alpha_T^1=C\alpha+\alpha_E(1-C+Ct)\left[\frac{Ct}{1-C\alpha\alpha_E}+\frac{1-C}{1-C\alpha\alpha_E}\right], \quad (5)$$

where it is evident that the first term in the brackets accounts for the multiple upward transmissions through the cloud and the second term for the multiple upward reflections escaping through the gaps between the clouds.

Similarly, we can compute a closed form expression A_c^1 for cloud absorption fraction of solar flux:

$$A_c^1=\left[AC+\alpha_E(1-C+Ct)\frac{CA}{1-C\alpha\alpha_E}\right], \quad (6)$$

where A is the absorption fraction of the cloud to direct or diffuse radiation.

Likewise, an expression for $Q_S^{1\downarrow}$, the total downward flux at the surface, is

$$Q_S^{1\downarrow}=\frac{Q(1-C+Ct)}{1-C\alpha\alpha_E}, \quad (7)$$

where Q is the solar flux at the cloud top level.

c. Arbitrary patchiness

For a general case, the grid square may be covered by regions of patchy and unpatchy cloud cover. Thus, we can define an albedo for the combined cloudy-cloudless system of arbitrary patchiness

$$\alpha_T^P=P\alpha_T^1+(1-P)\alpha_T^0, \quad (8)$$

where P is referred to as the patchiness parameter. To determine P we need to know (see the Appendix) the probability that a beam incident at the surface went through/did not go through the cloud, the probability that a beam that went through the cloud and reflected at the surface passes through/does not pass through the cloud again, and the probability that a clear sky beam reflected at the surface passes through the clear sky.

Thus, a major problem is apparent: knowledge of the average optical properties of individual clouds (e.g., α, t) over a grid square is insufficient *per se* to provide an unambiguous value of the albedo of that square, and that any parameterization of grid square albedo based on fractional cloud cover also requires information about the horizontal distribution of those clouds over the grid sufficient to determine P. Given the difficulty the state of the art of cloudiness observations already has in determining the fractional cloud cover of a few 100 km's grid square, this ambiguity in the definition of a parameterized grid square cloudiness appears a formidable obstacle to modeling the feedback effect of cloudiness on climate with GCM's (e.g., see Schneider and Dickinson, 1974). Fortunately, as shown later, the numerical differences between the total albedos and between radiation fluxes reaching the surface in the complete or no-patchiness limits, but with the same cloud cover amount in both cases, are always less than 10% and usually of the order of a few percent over a wide range of realistic optical parameters.

3. Comparison of downward flux at the surface and total albedo with different flux parameterizations

a. Downward flux parameterization

The higher order reflections in the downward flux parameterization can be simply accounted for in many climate models by expressions like (2), (4), (6) or (7) and their neglect can be serious, particularly in cases where α_E is greater than a few tenths, as over snow fields. For example, Wiscombe (1975) has noted the great importance of multiple reflections between cloud bottom and snow surface in a study of the solar radiative effects of arctic stratus clouds.

We can see from Eq. (7), for example, that neglect of multiple reflections between an 80% cloud cover with cloud albedo $\alpha=0.5$ and snow albedo $\alpha_E=0.8$ yields an underestimate of $Q_S^{\downarrow}$ by 32%. Since $Q_S^{\downarrow}$ influences the rate of snow melt, which, in turn, can reduce α_E and so, in turn, reduce $Q_S^{\downarrow}$, this feedback between downward flux and α_E appears sufficiently important to require inclusion in climate models that predict snow cover or sea ice as part of the internal dynamics of the model [e.g., in a number of the works of Manabe and his co-workers, as recently surveyed by Smagorinsky (1974) or as discussed in Section C3e of Schneider and Dickinson (1974)].

Generalization of the formulas given earlier to include gaseous absorption and zenith angle dependence is an obvious next step in the parameterization of radiative transfer (see Lacis and Hansen, 1974) in climate models that include prognostic equations for clouds, sea ice and snow.

Those climate models and GCM's that allow for the presence of fractional cloud cover have generally used expressions linear in cloud amount C corresponding to the unpatchy limit, i.e.,

$$Q_S^{\downarrow}=(1-C)F_{\text{clear}}^{\downarrow}+CF_{\text{cloudy}}^{\downarrow}, \tag{9}$$

where $F_{\text{clear}}^{\downarrow}$ is the downward solar flux for a clear sky (and is here simply Q, the direct solar flux above the cloud) and $F_{\text{cloudy}}^{\downarrow}$ is the downward flux for an overcast sky. The simplest formula that might be used for $F_{\text{cloudy}}^{\downarrow}$ neglects multiple reflections and so uses the transmission t of the cloud itself. We denote by $Q_t^{\downarrow}$ the downward flux using this approximation:

$$Q_t^{\downarrow}=(1-C)Q+CtQ. \tag{10}$$

A formula of this type is used in the Geophysical Fluid Dynamics Laboratory GCM and in the Kasahara and Washington (1971) version of the NCAR GCM.

Multiple reflections in the unpatchy limit can be included simply by letting

$$F_{\text{cloudy}}^{\downarrow}=\frac{tQ}{1-\alpha\alpha_E} \tag{11}$$

in Eq. (9), which fully accounts for multiple reflections when $C=1.0$, but otherwise only in the unpatchy limit. Such expressions are used in the UCLA family of GCM's. This treatment of multiple reflections in the unpatchy limit is denoted $Q_S^{0\downarrow}$ and is given by

$$Q_S^{0\downarrow}=(1-C)Q+C\frac{tQ}{1-\alpha\alpha_E}. \tag{12}$$

The difference obtained by using (12) in place of (7) is

$$\Delta Q_S^{\downarrow}\equiv Q_S^{1\downarrow}-Q_S^{0\downarrow},$$

and the errors from the use of (10), which includes no multiple reflection, instead of the completely patchy limit multiple reflections, are given by

$$\Delta Q_t^{\downarrow}\equiv Q_S^{1\downarrow}-Q_t^{\downarrow}.$$

b. Total albedo parameterization

Following the notation in the above subsection for downward flux parameterizations we denote by α_t the total albedo of a grid square neglecting effects of multiple reflections, i.e.,

$$\alpha_t=\alpha_E(1-C)+\alpha C. \tag{13}$$

Likewise, the albedo for an entirely cloud covered sky including multiple reflections is [i.e., Eq. (1) with $\alpha_s=\alpha_d=\alpha$ and $t_s=t_d=t$]

$$\alpha_T=\alpha+\frac{\alpha_E t^2}{1-\alpha\alpha_E}. \tag{14}$$

The albedo for this unpatchy limit case is

$$\alpha_T^0=\alpha_E(1-C)+\alpha_T C \tag{15}$$

and the total patchiness case albedo again is

$$\alpha_T^1=C\alpha+\frac{\alpha_E(1-C+Ct)^2}{1-C\alpha\alpha_E}. \tag{16}$$

Similarly to the downward flux parameterization case, we define the differences

$$\left.\begin{aligned}\Delta\alpha_t&=\alpha_T^1-\alpha_t\\ \Delta\alpha_S&=\alpha_T^1-\alpha_T^0\end{aligned}\right\}.$$

c. Numerical comparisons

Table 1 shows $Q_S^{1\downarrow}$, $\Delta Q_S^{\downarrow}$ and $\Delta Q_t^{\downarrow}$ for unit incident flux ($Q=1$) and non-absorbing clouds ($A=0$) for ranges of surface albedo and cloud albedo and for 20% cloudiness, 50% cloudiness and 80% cloudiness.

It is inferred from Table 1 that both Eqs. (10) and (12) give smaller downward fluxes than $Q_S^{1\downarrow}$. However, $Q_S^{0\downarrow}$ is less than $Q_S^{1\downarrow}$ generally by only a few percent and always less than by 10% over all the parameter ranges chosen here. Fortunately, then, the uncertainty $\Delta Q_S^{\downarrow}$ introduced by our lack of knowledge of the patchiness parameter P evidently does cause ambiguities in parameterizations for grid square downward flux greater than 10%, which suggests that uncertainty from the neglect of three-dimensional radiative effects of clouds (e.g., see McKee and Cox, 1974) or the cloud optical properties themselves and their inhomogeneities may well be greater sources of error than the uncertainty in P. On the other hand, use of $Q_t^{\downarrow}$, which neglects multiple reflections altogether, is seen to cause nearly a 50% fractional error for the case of 80% cloud cover, 70% cloud albedo and 80% surface albedo. The errors from neglect of multiple reflection are fairly small when cloud cover and surface albedo are less than a half, but become considerable for overcast skies with high surface albedos—precisely the climatological conditions that often prevail in high latitudes in late spring and early summer. Furthermore, the downward solar flux reaching the earth's surface under these conditions has an important influence on the rate of snow melt, which, in turn, influences the snow albedo.

Thus an important conclusion to be drawn from this simple exercise is that the use of an approximation like Eq. (10), which neglects multiple reflections, yields the greatest underestimate of downward solar flux in regions where the heat budget is most sen-

TABLE 1. Values of grid-averaged downward solar flux at the surface for the total patchiness case (i.e., superscript 1) and differences between this case and the unpatchiness case (subscript S) and one which neglects multiple reflections (subscript t). (Symbols are defined in Section 3 of the text.)

	$t=0.7$	$\alpha=0.3$		$t=\alpha=0.5$			$t=0.3$	$\alpha=0.7$	
α_E	$Q_S^{1\downarrow}$	$\Delta Q_S^{\downarrow}$	$\Delta Q_t^{\downarrow}$	$Q_S^{1\downarrow}$	$\Delta Q_S^{\downarrow}$	$\Delta Q_t^{\downarrow}$	$Q_S^{1\downarrow}$	$\Delta Q_S^{\downarrow}$	$\Delta Q_t^{\downarrow}$
				$C=0.2$					
0.3	0.957	0.003	0.017	0.928	0.010	0.028	0.898	0.022	0.038
0.5	0.969	0.004	0.029	0.947	0.014	0.047	0.925	0.032	0.065
0.8	0.987	0.003	0.047	0.978	0.012	0.078	0.968	0.032	0.108
				$C=0.5$					
0.3	0.890	0.005	0.040	0.811	0.017	0.061	0.726	0.036	0.076
0.5	0.919	0.007	0.069	0.857	0.024	0.107	0.788	0.057	0.138
0.8	0.966	0.005	0.116	0.938	0.021	0.188	0.903	0.062	0.253
				$C=0.8$					
0.3	0.819	0.004	0.059	0.682	0.011	0.082	0.529	0.025	0.089
0.5	0.864	0.005	0.104	0.750	0.017	0.150	0.611	0.042	0.171
0.8	0.941	0.004	0.181	0.882	0.016	0.282	0.797	0.052	0.357

sitive to that flux: the stably stratified high-latitude, snow-covered and cloud-covered regions as occur typically in late spring and early summer. Since the albedo of these high-latitude regions may be an important factor in determining the stability of the climate (e.g., SMIC, 1971; Schneider and Gal-Chen, 1973) and since some of the effects of multiple reflections can easily be accounted for in an algebraic formula such as Eqs. (7) or (12), or appropriate generalizations, it seems reasonable to suggest that climate modeling groups which have not already done so consider the inclusion of the effects of multiple reflections in the radiation parameterizations of their respective models, particularly if snow and ice predictions are already included.

For further comparison of the three fractional cloud amount treatments, Table 2 shows results for α_T^1, $\Delta\alpha_S$ and $\Delta\alpha_t$ for non-absorbing clouds ($A=0$) and for ranges of α_E, α and t and for 20% cloud cover, 50% cloud cover and 80% cloud cover. We infer from this table that neglect of multiple reflections is most serious for high surface albedo, considerable cloud cover and relatively thin clouds. For example, neglect of multiple reflections underestimates the total patchiness albedo by about 50% for the case $\alpha_E=0.8$, $C=0.8$, $t=0.7$ and $\alpha=0.3$.

On the other hand, the difference $\Delta\alpha_S$ between total patchiness and unpatchiness limit albedo parameterization formulas is no more than a few percent over a wide range of values of parameters α, t, C and α_E. Thus, a similar conclusion can be drawn as for the downward flux parameterization case, namely, neglect of multiple reflections can cause extremely large errors in regions of high surface albedo and extensive cloud cover, but the uncertainty introduced from the ambiguity in choosing patchiness versus unpatchiness formulas is relatively small, probably considerably less than errors from other causes, e.g., errors

TABLE 2. As in Table 1 except for total albedo of the surface-cloudiness system.

	$t=0.7$	$\alpha=0.3$		$t=\alpha=0.5$			$t=0.3$	$\alpha=0.7$	
α_E	α_T^1	$\Delta\alpha_S$	$\Delta\alpha_t$	α_T^1	$\Delta\alpha_S$	$\Delta\alpha_t$	α_T^1	$\Delta\alpha_S$	$\Delta\alpha_t$
				$C=0.2$					
0.3	0.330	−0.002	0.030	0.351	−0.007	0.011	0.372	−0.015	−0.008
0.5	0.515	−0.002	0.055	0.526	−0.007	0.026	0.538	−0.016	−0.002
0.8	0.803	−0.001	0.103	0.804	−0.002	0.064	0.806	−0.006	0.026
				$C=0.5$					
0.3	0.377	−0.004	0.077	0.432	−0.012	0.032	0.492	−0.025	−0.008
0.5	0.541	−0.004	0.141	0.571	−0.012	0.071	0.606	−0.029	0.006
0.8	0.807	−0.001	0.257	0.812	−0.004	0.162	0.819	−0.012	0.069
				$C=0.8$					
0.3	0.427	−0.003	0.127	0.523	−0.008	0.063	0.630	−0.018	0.010
0.5	0.568	−0.002	0.228	0.625	−0.008	0.125	0.694	−0.021	0.034
0.8	0.812	−0.001	0.412	0.824	−0.003	0.264	0.841	−0.010	0.121

in predicting cloud cover amounts, errors from the neglect of the three-dimensional effects of finite sized clouds, or errors due to spatial variations in cloud optical properties.

Note that the above derivations can be trivially modified to include other optically active layers by the usual doubling procedures [e.g., an aerosol layer can be included below the cloud by replacing surface albedo α_E with the aerosol-surface system albedo (see, for example, Schneider, 1971), or a layer of clouds overlying this system included by replacing α_E of Eqs. (4) or (2) with α_T^P, defined by (8), and then using (4) or (2) to calculate a new α_T^1 or α_T^0, respectively, for the new extended system.]

How to establish the patchiness parameter for irregular variations in the cloud cover and wide variations of cloud thickness from nearly transparent cloud wisps to clouds of nearly infinite optical thickness will require further analysis beyond that provided here. The relationship of the present concepts to three-dimensional cloud effects also needs to be explored. However, a reasonable guess for P based on presumed cloud structure may suffice until such an analysis is available.

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APPENDIX

General Formulation for Flat Fractional Clouds

Specification of the mean bulk optical properties of a fractional cloud distribution, for flat clouds and a given solar beam, requires knowledge of two probability terms. These are $P_{c,c}$ the probability that a beam having either passed through a cloud or having been reflected downward from a cloud and then reflected upward at the surface again is incident on a cloud, and $P_{nc,nc}$ the probability that a clear sky beam reflected at the surface passes again upward through clear sky. The probabilities of the converse of these events are denoted, respectively, by

$$P_{c,nc}=1-P_{c,c} \quad \text{and} \quad P_{nc,c}=1-P_{nc,nc}.$$

The total albedo of the system, α_T^P, consists in general of six partial albedo terms α_i^P

$$\alpha_T^P(P_{c,c}, P_{nc,nc})=\sum_{i=1}^{6}\alpha_i^P. \tag{A1}$$

Here (see Fig. A1 for a visualization)

$$\alpha_1^P=C\alpha_S$$

is the reflection of the direct beam from the cloud,

$$\alpha_2^P=(1-C)\alpha_E P_{nc,nc}$$

is the reflection of the direct beam from the earth then passing through clear sky,

$$\alpha_3^P=(1-C)\alpha_E P_{nc,c}\alpha_d\alpha_E P_{c,nc}/(1-P_{c,c}\alpha_E\alpha_d)$$

is the reflection of the direct beam from the earth and then through clear sky after one or multiple reflections between cloud and earth,

$$\alpha_4^P=Ct_s\alpha_E P_{c,nc}/(1-P_{c,c}\alpha_E\alpha_d)$$

is the reflection of the diffuse beam through the cloud passing through the clear sky to space after any number of multiple reflections,

$$\alpha_5^P=Ct_s\alpha_E P_{c,c}t_d/(1-P_{c,c}\alpha_E\alpha_d)$$

is the net reflection of the beam that passes through the cloud, reflects at the surface and again passes through

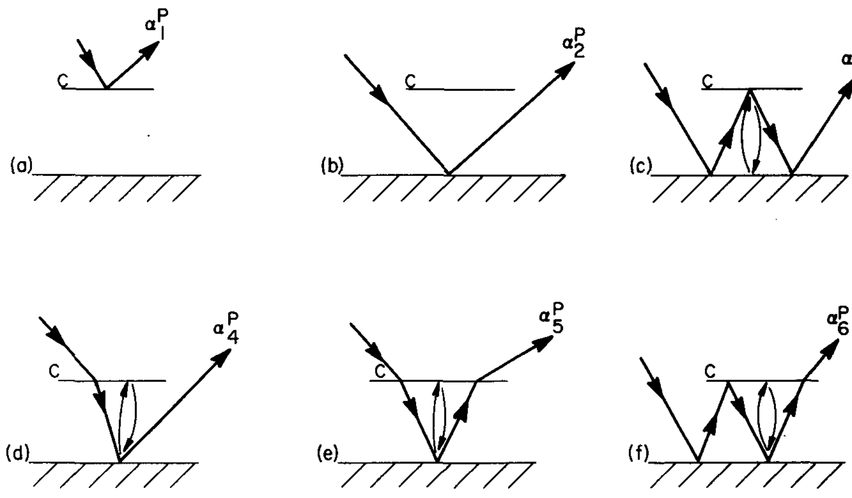

Fig. A1. A visualization of the partial albedos α_i^P ($i=1, 6$) defined by Eq. (A1), where (a) is for α_1^P, etc., and for unit incident direct beam.

the cloud after any number of multiple reflections, and

$$\alpha_6^P=(1-C)\alpha_E P_{nc,c} t_d/(1-P_{c,c}\alpha_E\alpha_d)$$

is the net reflection of the beam that passes through clear sky, reflects at the surface and then passes through the cloud after any number of multiple reflections at the surface.

The patchy limit is defined as the condition where the likelihood of a beam passing through a cloud is independent of its past history but depends simply on the fractional cloud cover, i.e.,

$$P_{c,c}=C,$$

$$P_{nc,nc}=1-C.$$

The unpatchy limit is defined as the condition where clear sky beams upon reflection always remain in clear sky and diffuse cloud beams upon reflection are always again incident on the cloud, i.e.,

$$P_{c,c}=P_{nc,nc}=1.$$

The patchiness parameter P in Eq. (8) expressing the relative contribution of the two limiting cases albedos α_T^1 and α_T^0 can be obtained in terms of the actual α_T^P from (A1) by

$$P=(\alpha_T^P-\alpha_T^0)/(\alpha_T^1-\alpha_T^0). \tag{A2}$$

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