

## Change in the Transmissivity Parameter with Atmospheric Path Length

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### ABSTRACT

Calculations and measurements show that the transmissivity parameter  $t$  is an increasing function of the optical air mass  $m$ . An equation for estimating  $t$  for several model atmospheres is presented, and the direct beam flux calculated with the estimates is compared with that obtained using a constant value of  $t$ . The equation gives significantly better results for hourly or instantaneous values, and for daily totals on some surfaces.

### 1. Introduction

In climatological studies the atmospheric attenuation of direct beam solar radiation is often described by a transmissivity coefficient  $t^m$ , so that the direct beam flux  $Q$  on a normal surface may be written as

$$Q = Q_s t^m \tag{1}$$

or some equivalent (Buffo *et al.*, 1972; Williams *et al.*, 1972). Here  $Q_s$  is the solar flux outside the atmosphere,  $t$  the transmissivity parameter and  $m$  the optical air-mass.

There is a problem with these expressions:  $t$  is itself an increasing function of  $m$ , or atmospheric path length. The variation of  $t$  with  $m$  is not trivial; calculations by Braslau and Dave (1973) show a 10% increase in  $t$  as  $m$  increases from 1 to 2, corresponding to a change in the solar zenith angle  $Z$  from 0 to 60°, and data cited by Kondratyev (1969) show that the change in  $t$  as  $m$  goes from 1.5 to 3 is of the same magnitude as the variation in monthly means of  $t$ .

The physical basis for the variation of  $t$  with  $m$  is the wavelength dependence of atmospheric attenuation of radiation, so that  $t_\lambda^m$ , the transmissivity at wavelength  $\lambda$ , is a highly irregular function of wavelength over the solar spectrum. Fig. 1 shows a schematic of an absorption band over five wavelength intervals. As the air-mass increases from 1 to 2, the transmissivity is de-

creased only on the "wings" of the absorption band, so the transmissivity for the entire interval decreases less rapidly than  $t$  to the  $m$ th power. The effect is less dramatic with molecular and aerosol scattering, but works in the same way.

The derivation of Eq. (1) proceeds through

$$Q = \int_0^\infty Q_{s\lambda} t_\lambda^m d\lambda, \tag{2}$$

where  $Q_{s\lambda}$  is the solar flux outside the atmosphere at wavelength  $\lambda$ . Eq. (1) results from taking the mean of the  $t_\lambda^m$  weighted by the proportion of the solar flux at each  $\lambda$ , and moving this outside the integral as  $t^m$ . Evaluating  $t$  at  $m=1$  gives the mean-zenith-path transmissivity coefficient of Haltiner and Martin (1957). However, as Fig. 1 makes clear, this is not the same as the  $m$ th root of the mean of  $t_\lambda^m$ . Haltiner and Martin recognized this, but ignored the dependence of  $t$  on  $m$  for simplicity's sake; to account for the dependence we must work with Eq. (2), or a finite interval approximation to it.

### 2. Methods

To approximate Eq. (2), I modified a program for calculating atmospheric transmittance over narrow spectral intervals developed by Selby and McClatchey (1972) to calculate the transmissivity coefficient  $t^m$  over the solar spectrum. This involved assigning an appropriate irradiance to 1630 intervals from 0.28 to 4.0  $\mu\text{m}$  wavelength, summing the transmitted radiation, and dividing by the solar constant of 1353  $\text{W m}^{-2}$  (1.94  $\text{ly min}^{-1}$ ). Irradiance data are from Thekaekara (1971).

The program (LOWTRAN 2) provides different model atmospheres, described in McClatchey *et al.* (1972). These represent tropical, midlatitude and subarctic summer and winter conditions, and the U. S. Standard Atmosphere. Zenith angle, altitude and aerosol concentration, expressed in terms of sea level visibility, can be varied.

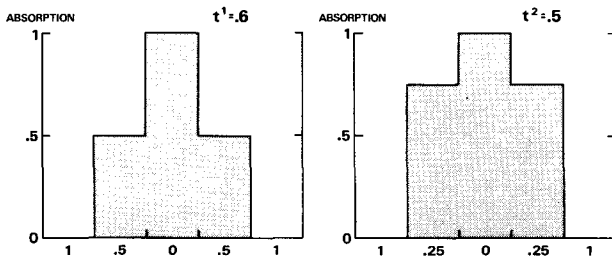


FIG. 1. Schematic of an absorption band over five wavelength intervals;  $t_\lambda^m$  is given below each interval and the transmissivity for the spectral region of the band is given above.

TABLE 1. Values of  $a$  for Eq. (2) for various model atmospheres and aerosol concentrations. Aerosol concentrations are expressed in terms of sea-level visibility [see McClatchey *et al.* (1972) for details].

Model atmosphere	Visibility (aerosol)			
	5 km	14 km	23 km	50 km
Tropical	0.075	0.070	0.070	0.070
Midlatitude summer	0.070	0.065	0.065	0.065
Midlatitude winter	0.070	0.060	0.055	0.055
Subarctic summer	0.070	0.060	0.060	0.060

### 3. Results and discussion

With the program, it is a simple matter to obtain  $t$  as a function of  $m$  for a wide variety of conditions. The results can be described simply by adding a correction factor to the zenith value of  $t$ , i.e.,

$$t = t_{m=1} + a \ln m, \quad (3)$$

where  $a$  is a constant depending on atmospheric conditions. The values of  $a$  for different atmospheric conditions are given in Table 1.

Fig. 2a compares the transmissivity coefficient  $t^m$  obtained with Eq. (2) and that obtained using a constant value of  $t$ , taken at  $m=1.3$ , for the midlatitude summer model atmosphere. The constant value coefficient is lower for low sun and higher for high sun. The importance of the difference depends on the application. For the flux to a horizontal surface, the difference is reduced by the cosine correction, as shown in Fig. 2b, and the deviations with low and high sun nearly cancel. Thus the error from using a constant value of

$t$  for calculating daily totals of radiation on a horizontal surface can be very small.

For a vertical surface facing the sun, however, the low sun difference is reduced much less by the sine correction than the high sun difference (Fig. 2c). This would also be true for an east or west facing vertical surface, while a surface facing the equator would have the low sun difference reduced by the azimuth correction.

The results of the more elaborate transmissivity calculations of Braslau and Dave (1973) using the midlatitude summer model atmosphere give slightly smaller values of the constant  $a$ , but are not strictly comparable as they used a narrower spectral interval and different aerosol size distributions and concentrations. As differences in aerosol account for much of the variation in  $a$ , the values given in Table 1 cannot be more realistic than the aerosol concentrations and size distributions used in the simulations, but these are still poorly known. In fact, it is impossible to test the accuracy of models like LOWTRAN 2 directly for space-to-surface transmissivity, because the actual state of the atmosphere, especially the aerosol content, cannot be determined with adequate precision. For a discussion of LOWTRAN 2 and other methods of calculating transmissivity, see LaRocca and Turner (1975).

With these limitations, the values given in Table 1 cannot replace direct measurement of the daily course of solar radiation if high accuracy is needed. However, direct measurement is seldom practical. If an approximation is used, Eq. (2) can fit easily into computer routines and for many purposes can give a significant improvement over the use of a constant value of  $t$ .

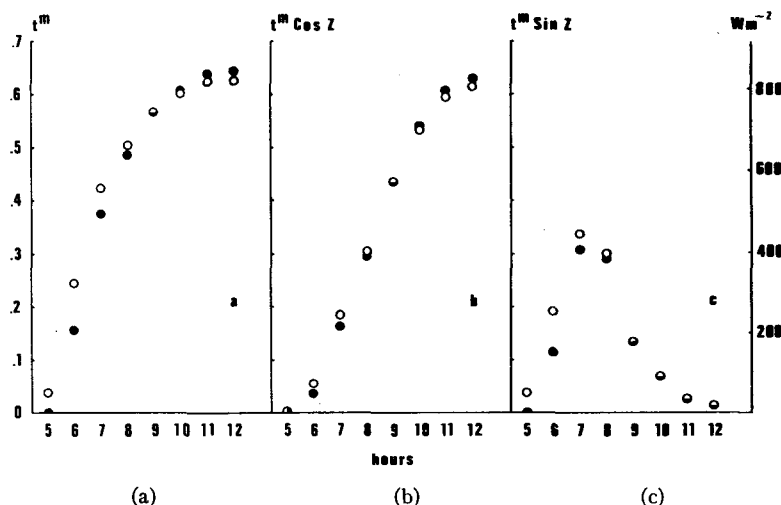


FIG. 2a. Values of the transmissivity coefficient  $t^m$  calculated for the midlatitude summer model atmosphere—23 km visibility, sea-level at 35° north latitude, 21 June— $t$  from Eq. (2), open circles;  $t$  taken at  $m=1.3$  (0900), closed circles. 2b. As above, but adjusted to give the direct beam solar radiation on a horizontal surface. 2c. As above, but adjusted for a vertical surface facing the sun.

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