# Calculations of Doppler Radar Velocity Spectrum Parameters for a Mixture of Rain and Hail[1]

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ABSTRACT

The radar reflectivity factors, the reflectivity-weighted mean terminal velocities ($\bar{V}_T$), and the standard deviations ($\sigma_v$) of the resulting Doppler spectra, were computed for specified size distributions of rain, dry and wet ice spheres (taken to be hailstones), and rain with hail. Unambigous estimates of the mean velocity and standard deviation can be obtained from a radar measurement of reflectivity for rain alone and for dry ice spheres as a function of maximum sphere size. The results for wet ice spheres are strongly dependent on the thickness of the liquid water coating on the ice core. When rain and hail coexist, large values of reflectivity are associated with large ranges of $\bar{V}_T$ and $\sigma_v$. If the shape of the hail size distribution is known, an independent measurement of the maximum hailstone diameter or a knowledge of the standard deviation of the observed Doppler velocity spectrum can reduce the uncertainty in estimates of $\bar{V}_T$.

## 1. Introduction

A pulsed-Doppler radar having its antenna pointing toward the zenith can measure the power spectrum of the vertical velocities of the hydrometeors in the radar beam. Various authors have proposed procedures for using such Doppler spectra to obtain information on the vertical air velocity and the size distributions of the hydrometeors. Rogers (1964) first proposed that the downdraft speed in rain could be obtained by means of the expression $W_a = \bar{V} - \bar{V}_T$, where $\bar{V}$ is the mean Doppler velocity and $\bar{V}_T$ the mean reflectivity-weighted terminal velocity of the hydrometeors. (All quantities are positive for downward velocities.) This scheme has been used by a number of investigators in the study of the vertical air motions in thunderstorms.

When the Rogers (1964) technique was employed to estimate the air motions in a hailstorm, it underestimated the updraft velocities (Battan and Theiss, 1972). This result is not surprising since the values of $\bar{V}_T$ for hail would be expected to exceed those of rain at the same values of radar reflectivity. Boston and Rogers (1969) and Ulbrich (1974) have calculated the values of $\bar{V}_T$ for various hail size distributions.

Battan and Theiss (1972) investigated a number of methods for estimating the updrafts in hail storms. Subsequently, Ulbrich (1974) developed improved techniques for the same purpose. As noted by Battan and Theiss (1968) and Ulbrich (1974) the Doppler spectra sometimes observed during the fall of hail can best be explained by the coexistence of rain and hail.

The variance $\sigma_v^2$ of the Doppler spectrum, in the case of a radar having a narrow beam, depends on the size spectrum of the hydrometeors and on small-scale turbulence. When the hydrometeors are hailstones and maximum diameters are perhaps 2 cm or greater, it is likely that the small-scale turbulence will make a minor contribution to the variance. At any rate, in this analysis the effects of turbulence are not included.

A number of authors (Donaldson and Wexler, 1969; Boston and Rogers, 1969; Battan and Theiss, 1968; Ulbrich, 1974) have calculated the variances of Doppler spectra which would be produced by aggregates of ice spheres falling in still air. The first two papers employed a hail size spectrum developed by Douglas (1964) from hail observations at the ground. Battan and Theiss (1968) and Ulbrich (1974) used a variety of hail distributions.

In this investigation we have repeated, for the purpose of making comparisons, some of the calculations made by others. The main objective has been to examine the properties of $\bar{V}_T$ and $\sigma_v^2$ *when rain and hail coexist.*

## 2. Analytical treatments

Rogers (1964) derived a simple relationship between the mean reflectivity-weighted terminal velocity $\bar{V}_T$ of raindrops and the radar reflectivity factor $Z$:

$$\bar{V}_T = 3.8 Z^{0.07}, \qquad (1)$$

where the units of $\bar{V}_T$ and $Z$ are in meters per second

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and millimeters to the sixth power per cubic meter, respectively. Somewhat different coefficients and exponents were found by Joss and Waldvogel (1970), and Sekhon and Srivastava (1971) who used measured drop-size distributions. The assumptions in deriving (1) are that the raindrops conform to the Marshall and Palmer (1948) drop-size distribution and that the terminal velocity of each size is in accordance with the Spilhaus (1948) relation.

Starting with the same assumptions, it is possible to derive a simple relation between the standard deviation of the raindrop velocity spectrum and the reflectivity factor, i.e.,

$$\sigma_v = 0.79Z^{0.07}, \quad (2)$$

where $\sigma_v$ is in meters per second.

Eqs. (1) and (2) provide convenient ways for estimating the raindrop velocity spectrum parameters from the measured returned power received at the radar set. However, calculations assuming reasonable size spectra reveal that (2) is somewhat unrealistic when large particles exist.

## 3. Calculations for raindrops alone

A series of calculations were made of $Z$, $\bar{V}_T$ and $\sigma_v$ for various raindrop populations. The raindrops were assumed to be arranged in size according to the Marshall-Palmer distribution, and following Rogers (1964), Rayleigh scattering was assumed. Furthermore, as is done in all the calculations presented in this paper, the hydrometeors were assumed to be spherical and falling in still air. To obtain a specific drop-size distribution the precipitation intensity $R$ was specified and the concentration of raindrops in diameter intervals of 0.02 cm was computed for diameters of $D=0.02$ cm to the appropriate maximum diameter. It was determined by summing the contributions to the rainfall by larger and larger drops until the summation was equal to or slightly greater than the specified $R$. In calculating $R$, the terminal velocities used were those given by Gunn and Kinzer (1949). The extreme drop diameter was taken to be 0.58 cm because drops larger than this are normally unstable in still air. Figs. 1a and 1b present the results of calculations of $\bar{V}_T$ and $\sigma_v$ as functions of $Z$.

It is clear that the values of $\bar{V}_T$ and $\sigma_v$ computed by taking discrete intervals of $D$ in the manner described above are smaller than the corresponding ones obtained from the analytically determined equations [(1) and (2); dashed lines]. The reason for the difference is that the analytic treatments utilized integrations over drop sizes from zero to infinity in the determination of $\bar{V}_T$ and $\sigma_v$, but the computer calculations summed over the

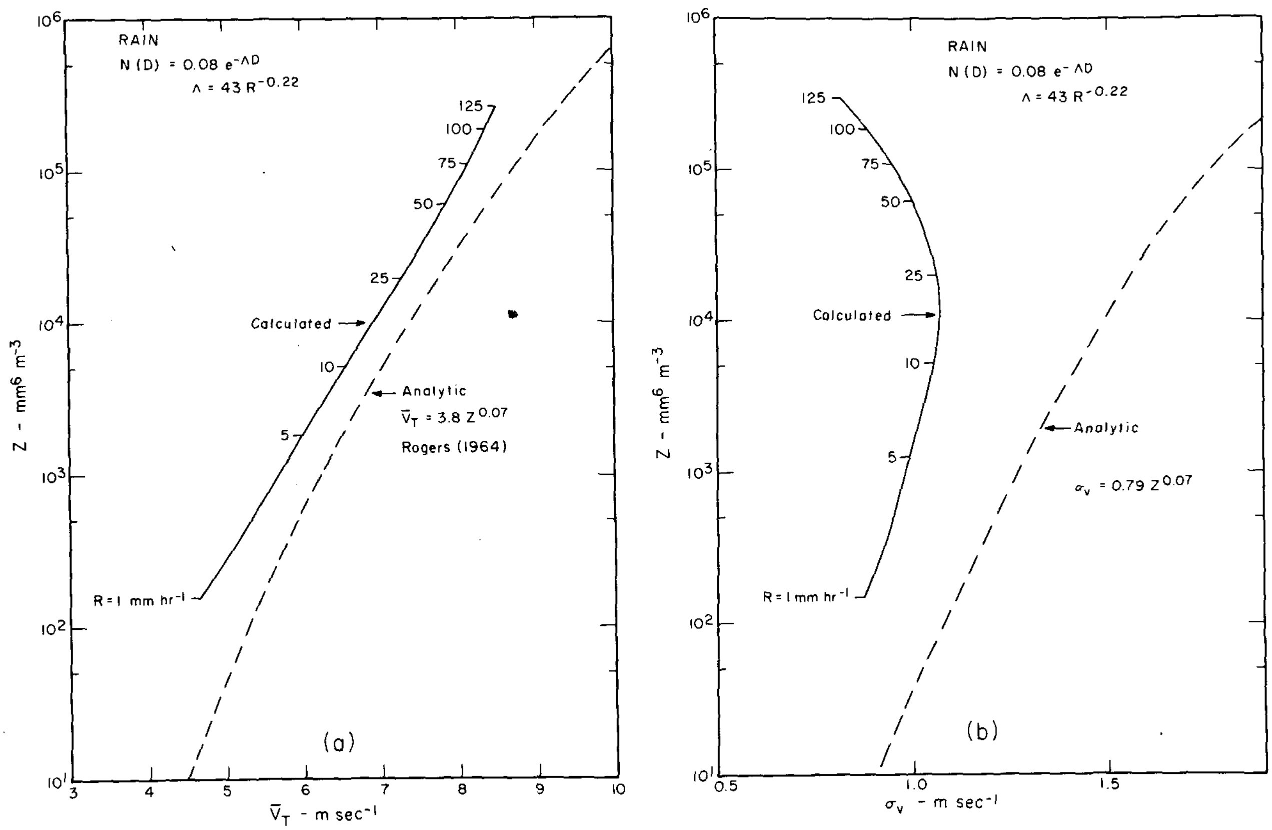



FIG. 1. Reflectivity-weighted mean (a) and standard deviation (b) of the velocity spectrum at sea level as a function of the reflectivity factor for raindrops in the Marshall-Palmer distribution.

finite drop-size range from $D=0.02$ cm to $D\leqslant 0.58$ cm. Since the latter procedure is more realistic, the somewhat smaller values of $\bar{V}_T$ ($\sim$0.5 m s$^{-1}$) determined by the computations are preferable. The solid line of Fig. 1a suggests the relation $\bar{V}_T=3.0Z^{0.08}$.

In Fig. 1b the calculated values of $\sigma_v$ actually begin to decrease as high values of $Z$ are encountered. This widening departure from the analytic values is attributed to the use of the Gunn and Kinzer velocity values in the calculations rather than the simple, but less precise, Spilhaus velocity relation used in the analytic treatment. A comparison of the Spilhaus velocities and the Gunn and Kinzer measurements is shown in Fig. 2.

The initial increase and subsequent decrease of the standard deviation was noted in rain measurements by Hitschfeld and Dennis (1956), but their data do not fit the curves of Fig. 1, apparently because the raindrop size distributions differed from the Marshall and Palmer distributions.

## 4. Calculations for ice spheres alone

As was done by Boston and Rogers (1969), the average exponential size distribution derived by Douglas (1964) from measurements of Alberta hailstones was utilized in the calculations of radar reflectivity, $\bar{V}_T$ and $\sigma_v$ for the case of ice spheres:

$$N(D)=31\,e^{-3.09D}, \qquad (3)$$

where $D$ is in centimeters and $N(D)\Delta D$ is the number of particles per cubic meter in the $\Delta D$ interval of 0.32 cm. As noted by Boston and Rogers (1969) some of Douglas' hail samples had distributions differing appreciably from the average form. Ulbrich (1974), employing Doppler radar observations of hail, calculated hail distributions having much greater slopes than the 3.09 given in (3). He did find that the hail-size spectrum approached the Douglas distribution during the more mature stages of the storm. It is difficult at this time to specify any exponential hail-size spectrum with confidence. For the purpose of this paper, it seems that Douglas' equation is as good as any other.

Douglas (1964) obtained Eq. (3) by normalizing the hail samples to a solid water content of $M=1$ g m$^{-3}$. He reported that larger values of $M$ were associated with proportionately larger values of $N(D)$ and radar reflectivity without a broadening of the distribution. It does not seem reasonable to expect such a result to be true generally. There is no doubt that the maximum hailstone size varies from one hail shower to the next. This must be accompanied by a change of solid-water content, mean terminal velocity, and precipitation rate. We decided for the sake of this analysis to make calculations for distributions of ice spheres having an exponential form according to (3) and having various upper limits of diameter (DHMAX). It should be noted

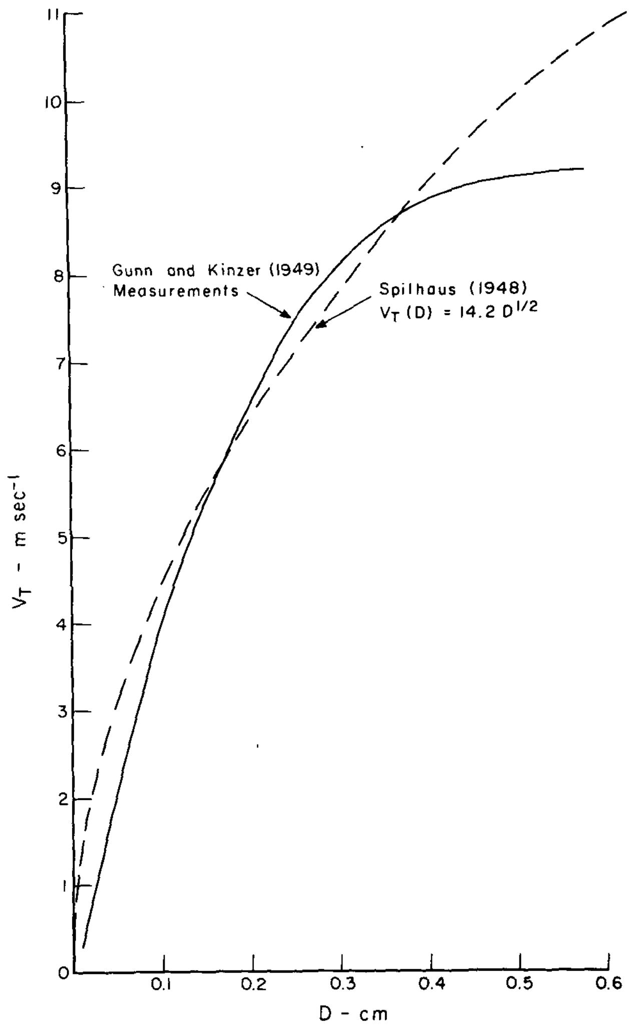



Fig. 2. Terminal velocity of raindrops at sea level.

that the values of $\bar{V}_T$ and $\sigma_v^2$ are independent of the coefficient of (3).

The diameters of the hailstones, except at the smallest size intervals, are too large for the Rayleigh scattering treatment to be valid at the radar wavelengths considered. Therefore, backscattering coefficients determined by Mie scattering theory were utilized. Tabulations of these cross sections for both dry and wet ice spheres are presented in Battan *et al.* (1970).

Terminal velocities of the ice spheres were computed from the expression $V_T(D)=KD^{\frac{1}{2}}$, where the coefficient $K$ depends on the ice density, drag coefficient and properties of air. Calculations were made for values of $K$ of 1300 and 1620 cm$^{\frac{1}{2}}$ s$^{-1}$; they are considered reasonable at altitudes of 0 and 4 km, respectively (Donaldson and Wexler, 1969).

When Mie scattering is assumed, it is appropriate to specify radar backscattering in terms of $Z_e$, the effective reflectivity factor. Values of $Z_e$, $\bar{V}_T$ and $\sigma_v$ of the velocity spectrum produced by ice spheres were calculated by means of the following equations for maximum hail sizes ranging from DHMAX = 0.32 to 4.80 cm for dry and wet hail:

$$Z_e = 3.52\times 10^3 \lambda^4 \sum_{D=0}^{\text{DMHAX}} N(D)\frac{\pi D^2}{4}\sigma_B(D,\lambda)\Delta D \quad (4)$$

[where $D$ and $\lambda$ are in centimeters, $N(\Delta D)$ is in per meter cubed, $Z_e$ is in the usual units, and $\sigma_B(D,\lambda)$ is the normalized backscattering cross section]

$$\bar{V}_T = \left\{ \sum_{0}^{\text{DHMAX}} N(D)D^2\sigma_B(D,\lambda)V_T(D)\Delta D \middle/ \sum_{0}^{\text{DHMAX}} N(D)D^2\sigma_B(D,\lambda)\Delta D \right\} \quad (5)$$

$$\sigma_v = \left\{ \sum_{0}^{\text{DHMAX}} [V_T(D)-\bar{V}_T]^2 N(D)D^2\sigma_B(D,\lambda)\Delta D \middle/ \sum_{0}^{\text{DHMAX}} N(D)D^2\sigma_B(D,\lambda)\Delta D \right\}^{\frac{1}{2}}. \quad (6)$$

"Wet hail" in this study means an ice sphere having a thin coating of liquid water. Various water coating thicknesses were used in the calculations. The changes of $\bar{V}_T$ and $\sigma_v$ as functions of $Z_e$ are shown in Figs. 3a and 3b. It is seen that for values of $Z_e \lesssim 10^5$ mm$^6$ m$^{-3}$ the ranges of calculated $\bar{V}_T$ are less than 2 m s$^{-1}$ while the range of $\sigma_v$ is less than 1 m s$^{-1}$. At large values of $Z_e$ the ranges of $\bar{V}_T$ and $\sigma_v$ increase very rapidly.

Reflectivity factor is related uniquely to the maximum hail size DHMAX. In order to make these data comparable to those of Boston and Rogers (1969) and

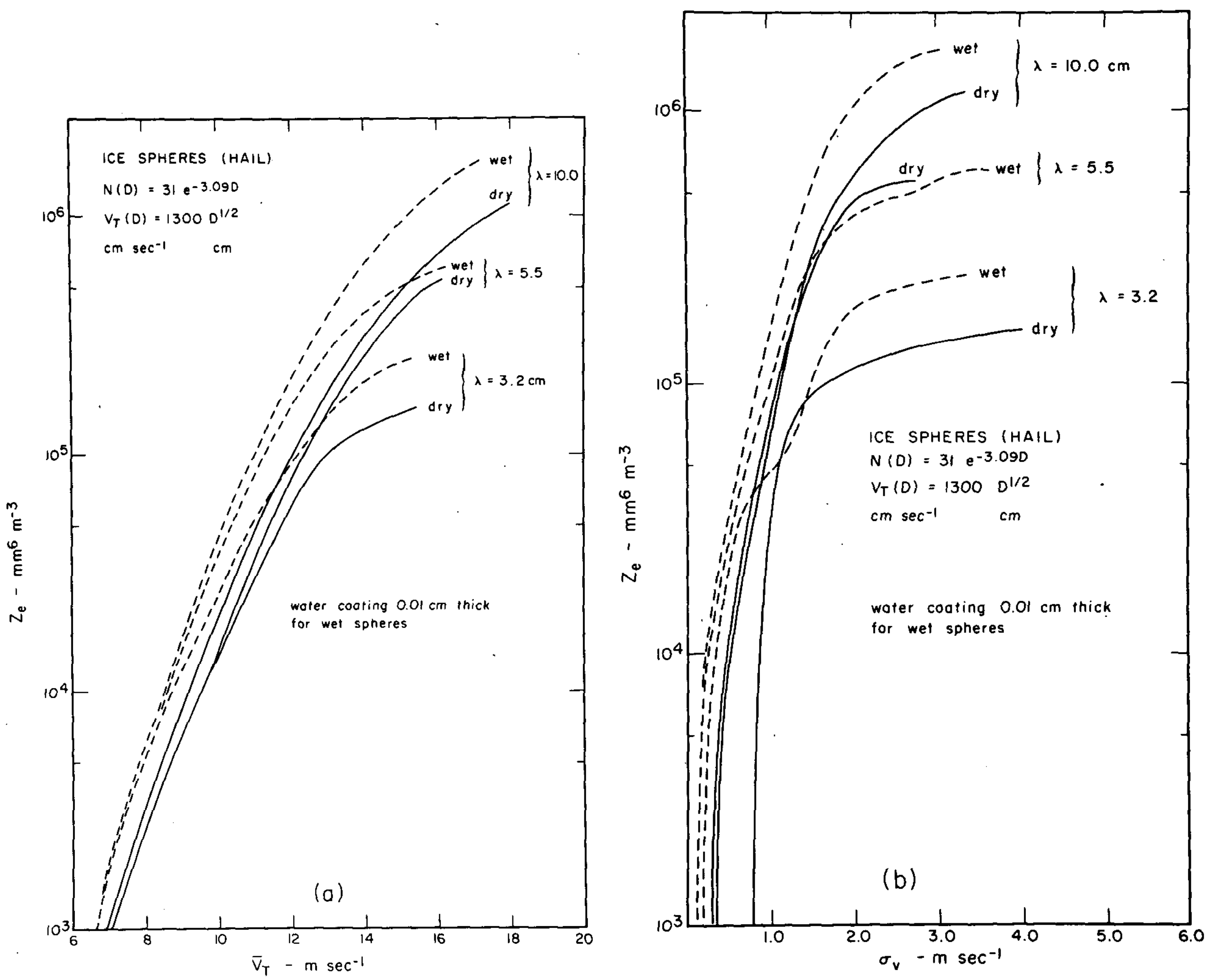



FIG. 3. Mean velocities (a) and standard deviations (b) at sea level as a function of effective reflectivity factor for ice spheres in the Douglas (1964) distribution. Solid lines are for dry hail, dashed lines for wet hail with a water coating 0.01 cm thick.

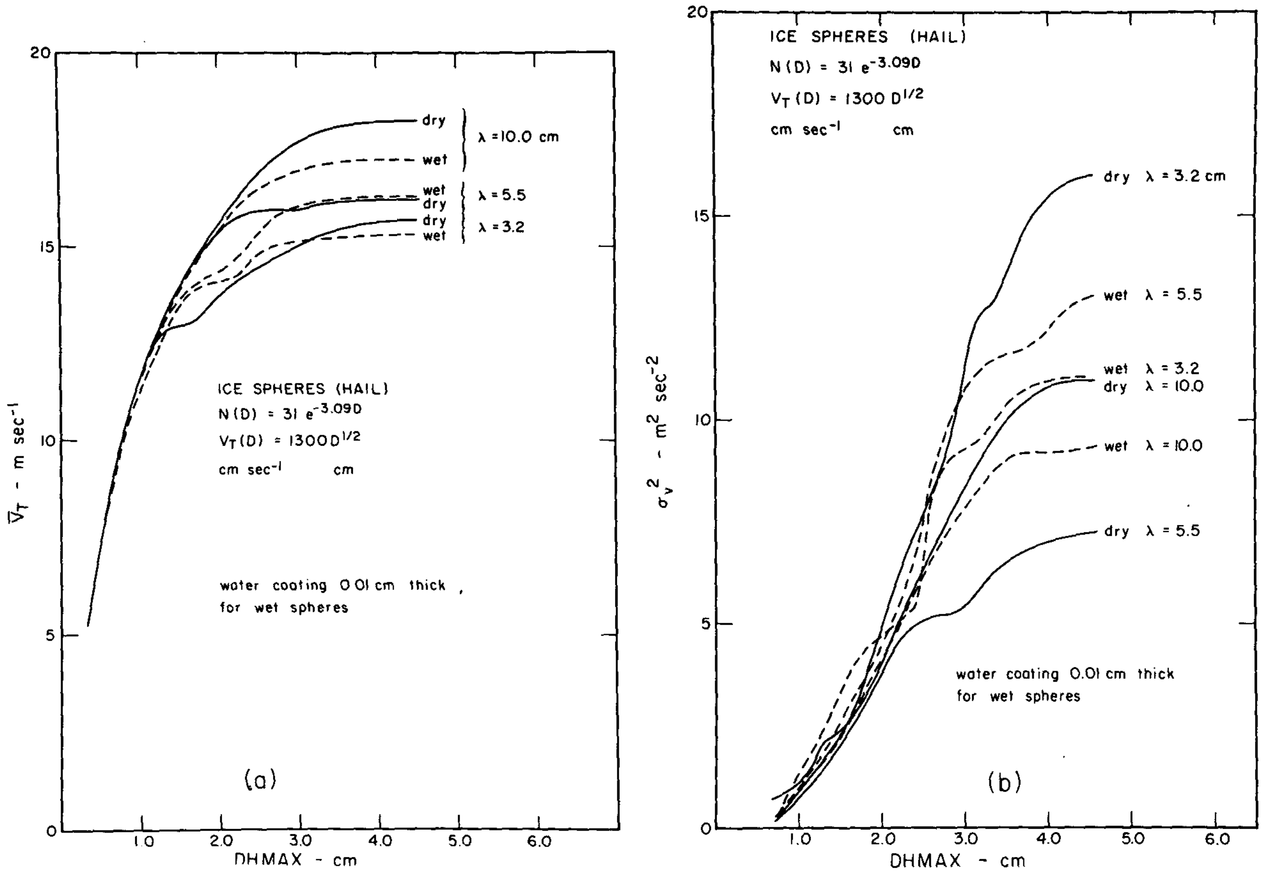



FIG. 4. Mean velocities (a) and variances (b) at sea level for ice spheres, assumed to follow the Douglas (1964) distribution, as a function of maximum size. Solid lines are for dry hail, dashed lines for wet hail with a water coating 0.01 cm thick.

Ulbrich (1974), Figs. 4 and 5 give $\bar{V}_T$ and $\sigma_v^2$ as functions of maximum diameter. In Fig. 5, following earlier authors, we employed $K=1620$ in the fall velocity equation. The resulting data are similar to those found by Ulbrich (1974), for the same size distribution for both dry and wet ice spheres, because we employed the same definition for wetness. On the other hand, there are large differences between the values of $\bar{V}_T$ and $\sigma_v^2$ in Figs. 5a and 5b and those reported by Boston and Rogers (1969) for wet hail. The discrepancies are attributable to different definitions of "wet hail." Boston and Rogers (1969) defined a "wet hailstone" to be a water sphere having the indicated size.[2]

When an ice sphere has a sufficiently thick coating of liquid water, it will backscatter as if it were an all-water sphere, but in the case of microwaves the required thickness is quite large, being 0.5 cm for $\lambda=3.21$ cm according to Herman and Battan (1961). Hence the Boston and Rogers treatment would correspond to ice spheres having liquid-water coatings of about 0.5 cm. It is not known how great the effective water thickness on real hail is likely to be, but it is unreasonable to expect water coatings as thick as 0.5 cm. On the other hand, it is conceivable that some hailstones might be composed of spongy ice consisting of enough liquid to constitute an equivalent water thickness of 0.5 cm. According to Atlas *et al.* (1964) an ice sphere coated with spongy ice can be treated as a solid ice sphere with a water coating having a mass equivalent to that contained in the spongy ice. It is expected that such large quantities of liquid water are not common. The higher values of $\bar{V}_T$ and $\sigma_v$ calculated in this study for thin water coatings are probably more realistic than those of Boston and Rogers. The curves in Figs. 5a and 5b show, as expected, that as the liquid water thickness increases, the results of our calculations approach those of Boston and Rogers.

## 5. Raindrops and hailstones together

The coexistence of hailstones and raindrops can lead to Doppler spectra which are significantly different from those observed when only hailstones or raindrops exist. Calculations were made of $\bar{V}_T$ and $\sigma_v$ assuming that rain and hail coexist. As before, the Douglas hail distribution was assumed and terminal velocities were

[2] Private communication.

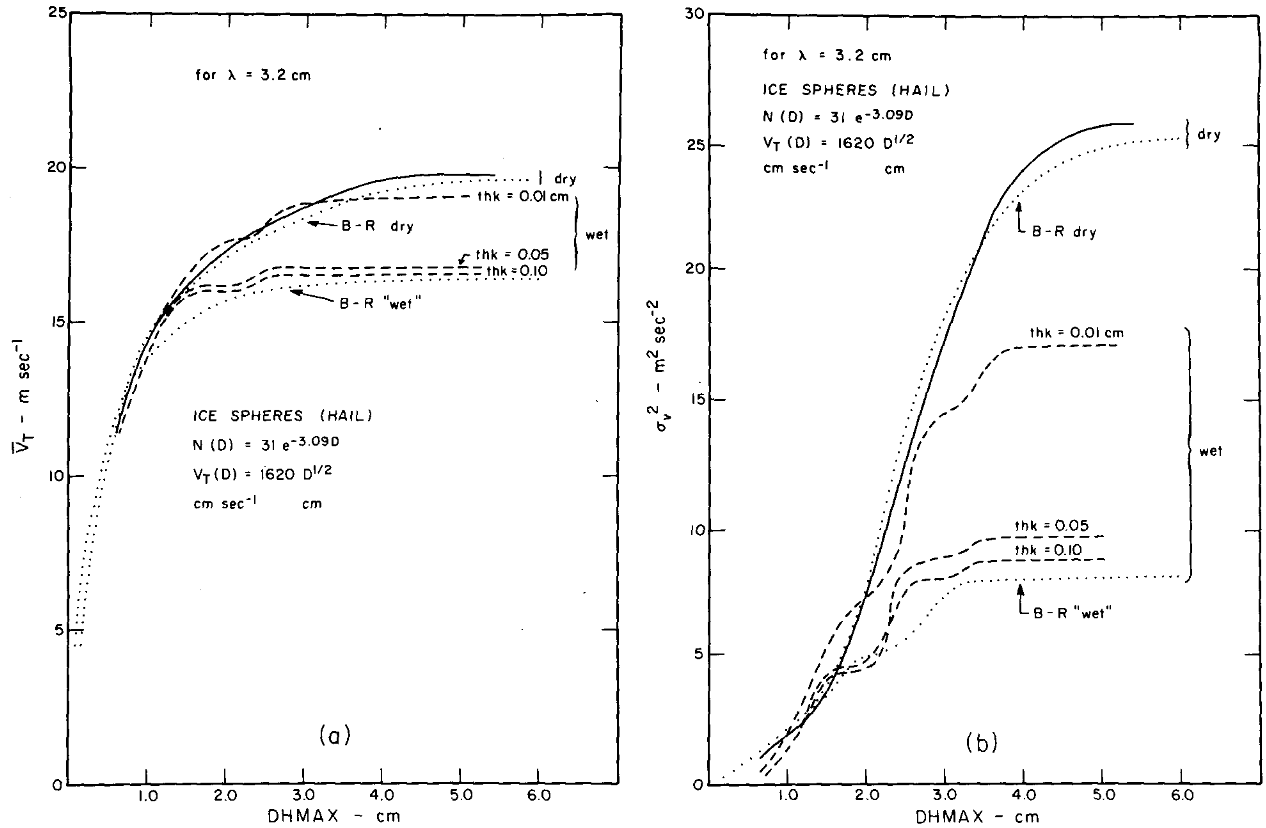



FIG. 5. Mean velocities (a) and variances (b) at about 4 km for ice spheres assumed to follow the Douglas (1964) distribution, as a function of maximum size. Water thickness is designated as "thk"; dotted lines show results of calculations for dry and "wet" hail by Boston and Rogers (1969).

obtained with $K=1300$ cm$^{\frac{1}{2}}$ s$^{-1}$. The Marshall and Palmer distribution with Gunn and Kinzer terminal velocities was taken to specify the rain. The hydrometeor size distribution for one specific case is shown in Fig. 6.

It is recognized that the size distribution of raindrops from thunderstorms is likely to deviate from the Marshall-Palmer distribution, particularly at the large diameter end of the spectrum. On the other hand, for the purpose of this study the precise shape of the raindrop curve is probably not crucial providing it is exponential. Convenience in making the calculations persuaded us to employ the same raindrop spectra used by Rogers (1964). He noted that in rain the value of $\bar{V}_T$ is only "weakly dependent on the exact form of the distribution." Also, since values of $\sigma_v$ in rain usually are less than 1 m s$^{-1}$, the results of this analysis should not depend to an important extent on the precise raindrop spectrum employed. In a subsequent analysis it might be worth assessing the sensitivity of the results to the forms of the distribution curves of raindrops. Uncertainties about the shapes, compositions and distributions of hailstones, and assumptions related to the mixtures of rain and hail and the effects of turbulence on the Doppler spectrum, are probably as important, or more so, than uncertainties about the raindrop spectra.

In this analysis, a maximum hailstone size was specified first. This determined the contribution of hail to a *total* precipitation intensity. Increments of rainfall intensity were then added and that rainfall intensity was used to determine the raindrop size distribution. Treating raindrops as Rayleigh scatterers and hailstones as Mie scatterers, calculations were made of the total reflectivity factor $Z_T$, and the reflectivity-weighted velocity mean and standard deviation.

The results of the calculations at $\lambda=3.2$ cm for rain and dry hailstones are shown in Figs. 7a and 7b. At $\lambda=10$ cm, the curves are shifted toward higher values of $Z_T$. An analogous set of calculations was made for rain and wet hail. As expected, there is a shift of the curves toward higher reflectivity because large, wet ice spheres backscatter more effectively than dry ones.

In Fig. 7 the intersections of the solid curves with the dashed line represent the case of hailstones alone for each maximum hailstone size. Along the dashed line, the contribution $R$ to precipitation by rain is

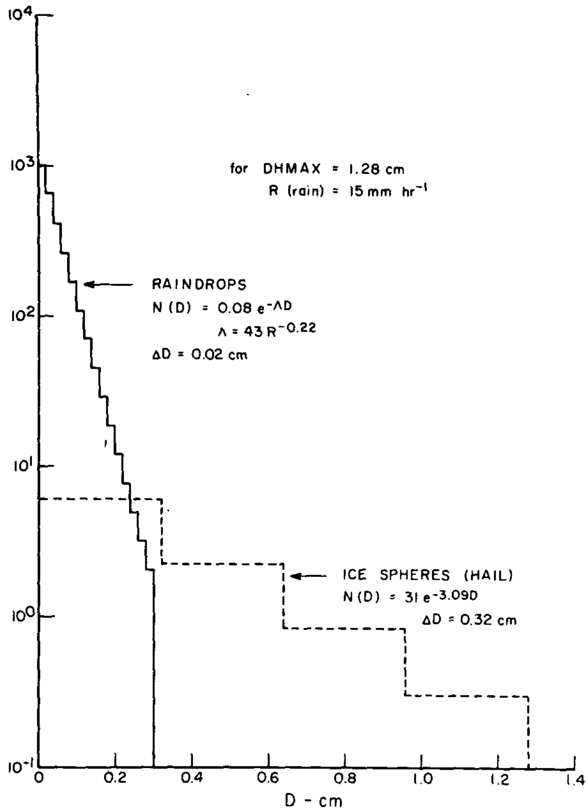



FIG. 6. One particular hydrometeor size distribution used in the calculations. Raindrops follow the Marshall-Palmer distribution, ice spheres follow the Douglas distribution with $M=1.0$ g m$^{-3}$.

zero. Higher points on each solid curve correspond to successively larger contributions of rain to the total precipitation intensity. The results for large values of DHMAX are dominated by the reflectivity of large hailstones and even large amounts of rain cause only small percentage increases in $Z_T$.

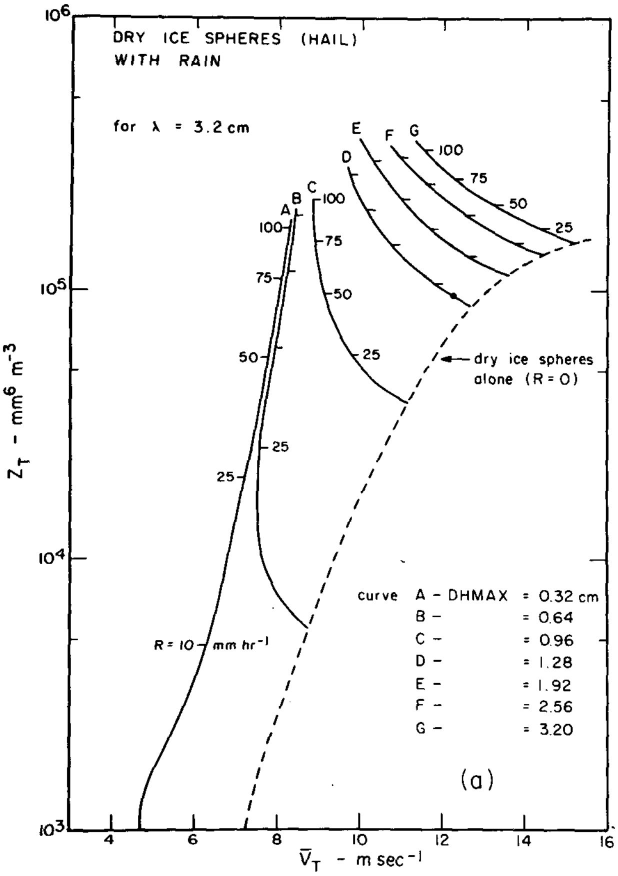
In Fig. 7a, it can be seen that the addition of rain to the hail spectrum increases the mean Doppler velocity for the smallest DHMAX cases and decreases it for the larger cases. As the rain intensity increases, large raindrops are added to the total hydrometeor population. Since these large raindrops have terminal velocities exceeding $\bar{V}_T$ in cases where only small hail exists and the raindrops are highly reflective, the mean can be increased. However, for the larger hail cases, even the largest raindrops have terminal velocities which are less than $\bar{V}_T$ of the hail distribution, and as a result, the mixture of rain and hail has a $\bar{V}_T$ smaller than that of hail alone.

In Fig. 7b the standard deviation first increases with the introduction of light rainfall and then decreases somewhat as heavy rainfall is added to the hail. Since light rainfall contains few large drops, its addition to the hail distribution places weight at the small end of the size scale far from the central tendency of the hail distribution. Thus the standard deviation increases. As heavier rain is introduced, many large raindrops are included in the total hydrometeor population. These drops reinforce the central tendency of the hail spectrum and the standard deviation decreases, although not to or below the value for hailstones alone.

The data in Fig. 7 clearly illustrate the non-uniqueness of $\bar{V}_T$ and $\sigma_v$ for any value of $Z_e$ even in circumstances where the distribution curves of rain and hail are known. As already noted, they can vary substantially from one time to the next. The uncertainty of the determination of $\bar{V}_T$ is carried over into estimates of the downdraft velocity made by means of the expression $W_a=\bar{V}-\bar{V}_T$. If it were assumed that the Douglas distributions were appropriate aloft, a doubtful assumption according to Ulbrich (1974), independent measurements of the maximum hail size or simultaneous measurements of $Z_e$ and $\sigma_v$ could reduce the degree of uncertainty of estimates of $\bar{V}_T$. They can be used to place limits on the permissible values for DHMAX, hence narrowing the range of estimated $\bar{V}_T$. For example, consider a case where the measured $Z_e=10^5$, and it is suspected that hail and rain coexist. If $\sigma_v=2.4$ m s$^{-1}$, Fig. 7b indicates the presence of ice spheres as large as $D=1.28$ cm (curve D). On Fig. 7a the intersection of $Z_e=10^5$ and curve $D$ yields a $\bar{V}_T=12$ m s$^{-1}$. Clearly this example has ignored the effects of turbulence in broadening the Doppler spectrum and increasing $\sigma_v$. If the evidence, such as observed shear in air motion, indicates the presence of turbulence, another uncertainty factor is introduced into the problem.

## 6. Concluding remarks

It has not been our intent in this study to develop a procedure for estimating the updraft speed in a hailstorm or for estimating the maximum sizes and composition of hailstones. These are important goals which have been addressed recently by Battan and Theiss (1972) and Ulbrich (1974). The many variables in the problem pose some formidable difficulties. The aim of this paper has been to examine certain questions relating to this complex problem. In particular, we analyzed some of the effects of rain and hail on the reflectivity-weighted mean terminal velocities and on the standard deviations of the Doppler spectra which would be observed by means of a vertically-pointing radar. A number of authors have examined the effects of rain alone and hail alone. In this report, similar calculations have been made for the purpose of comparison, but in addition, calculations have been made of certain properties of the Doppler spectra produced when rain and hail coexist. The results indicate that, when such a state of affairs is suspected, estimations of air motion using



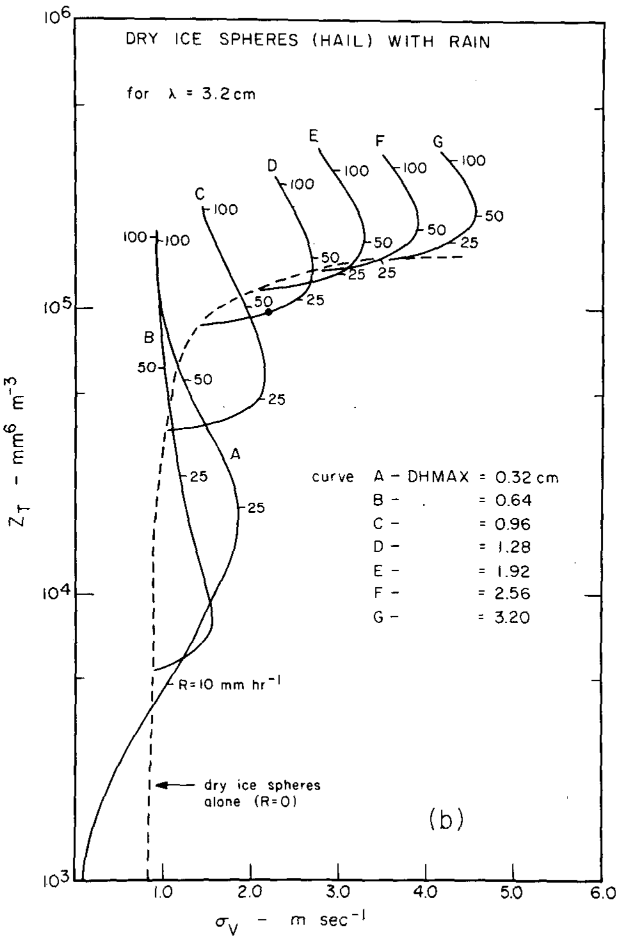




FIG. 7. Mean (a) and standard deviation (b) of the velocity spectrum at sea level for raindrops and dry ice spheres together. Dashed lines are for dry ice spheres alone (as in Fig. 3). The quantity $R$ represents the precipitation rate in the form of rain. The dot on curve D is the point generated from calculations using the distribution shown in Fig. 6.

Rogers' (1964) method and of hailstone sizes must be made with a recognition of the possible errors involved.

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