Relative Diffusion of Tetroon Pairs During Convective Conditions[1]

STEVEN R. HANNA

Atmospheric Turbulence & Diffusion Laboratory, NOAA, Oak Ridge, Tenn. 37830

(Manuscript received 25 August 1975, in revised form 15 April 1976)

ABSTRACT

Observations of the relative diffusion of 13 sets of tetroon pairs in the mixed layer during convective conditions in eastern Tennessee are reported. The root-mean-square separation S is proportional to time t raised to a power of 1 for times from 2 to 30 min and a power of 0.75 for times from 30 to 100 min. On the average, the observations are satisfied by the approximation $dS/dt \approx \sigma_v$, where σ_v is the standard deviation of the lateral wind speed fluctuations, as sensed by the tetroon.

1. Introduction

Observations of mesoscale or regional diffusion are currently important because of our need to make estimates of the environmental impact of various effluents at these scales. The data studied here cover distances from 100 m to 50 km and times from 2 min to 2 h. The observational technique has been developed and applied several times in the past (e.g., Angell *et al.*, 1971, 1974). Two tetroons are released simultaneously and their transponders are tracked by an M-33 radar system. The relative speeds and separations of the balloons are then analyzed, and the results compared with the results of similar experiments in other locations, and with the predictions of theories developed to explain relative diffusion.

2. Experimental technique

The relative diffusion experiment is a small part of a comprehensive transport and diffusion experiment called the Eastern Tennessee Trajectory Experiment (ETTEX), conducted during July and August of 1974. The comprehensive experiment, described by Hanna *et al.* (1974), had as its major goal the evaluation of transport models applied to hilly eastern Tennessee terrain. Nappo (1975) discusses preliminary results in which an observed tetroon trajectory compares favorably with a trajectory calculated using an interpolated wind field from a pibal grid. As another part of ETTEX the fluctuations of pollutant concentration and turbulence in the SO_2 plume from TVA's Bull Run steam plant were measured. The plume experiment was prompted by a theory developed by Briggs (1975) for estimating ground level pollutant concentration during fumigation conditions on convective afternoons, when subsiding air motions between thermals bring the plume to the ground. Most of the SO_2 plume experiments coincided with the relative diffusion experiments described here.

The M-33 radar used for tracking the tetroons was set up at the top of Buffalo Mountain (1030 m MSL), the second highest peak in the Cumberland Mountains. The location of the radar is shown in Fig. 1. In the relative diffusion experiments, instantaneous readings of tetroon range, elevation angle and azimuth angle were taken at the radar console every 2 min for each tetroon.

Tetroons were released from either the ATDL building or the Bull Run steam plant, depending on whether an SO_2 plume experiment was also taking place. These locations are marked on the map in Fig. 1. The two tetroons were inflated so they would fly at the same predetermined level, based on radiosonde observations of the vertical temperature profile. The techniques recommended by Hoecker (1975) were used to determine the necessary amount of inflation. It was planned that the tetroons would fly within the afternoon mixing layer, at heights between about 500 and 1500 m above the ground. Lower flights were not practical because of the presence of 200 m high linear ridges in the Tennessee valley which would interfere with the transponder signal.

The transponders were tuned to slightly different frequencies so that the radar could tell the difference between the two tetroons. Errors in recorded tetroon position are due mainly to the inherent inaccuracy of the radar and to observer errors (Van der Hoven, 1968). We estimate that the expected error in the indicated tetroon position is about 20 m.

Radiosonde observations of vertical profiles of wet and dry bulb temperature were taken during most of

---

[1] Presented at the First AMS Conference on Regional and Mesoscale Modeling, Analysis and Prediction, Las Vegas, Nev., 6–9 May 1975.

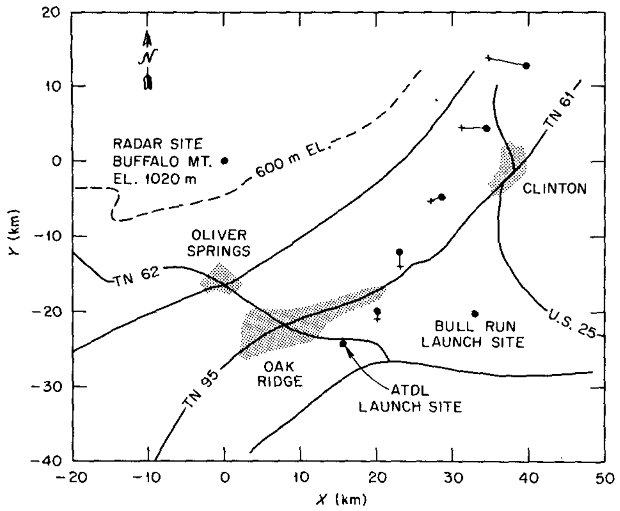

FIG. 1. Map of experiment area, showing radar and launch sites. The separations of tetroons 7293 and 7294 are shown, where the time interval between plotted separations is 15 min. Launch was from the ATDL site.

the experiments. Vertical wind profiles were obtained from either single or double theodolite pibal runs. When an SO_2 plume experiment was also taking place, vertical profiles of temperature, eddy dissipation rate, and SO_2 concentration were obtained from aircraft flights. All of the experiments took place during midday.

3. Observations

Some general characteristics of the data are summarized in Table 1. The month and day are given in the first three digits of the tetroon number. Potential temperature gradient $\partial\theta/\partial z$ is determined by analyzing the radiosonde and aircraft flights. Vertical shear of wind speed, $\partial V/\partial z$, and direction, $\partial\alpha/\partial z$, are obtained from pilot balloon observations, averaged over the layer in which the tetroon flew. Eddy dissipation rate ϵ is measured by an MRI universal turbulence indicator on the aircraft.

Mixing height h is estimated as the average height at which discontinuities in temperature lapse rate, eddy dissipation, SO_2 and haze occur. The average tetroon height and mixing height refer to height above ground level at the launch sites (250 m MSL).

After about 10 min, the tetroons generally reached their equilibrium height, which ranged from 320 to 1220 m above ground for the 26 tetroons. The standard deviation of the vertical tetroon position about the average height ranged from 60 to 430 m. Some tetroons bobbed up and down in convective eddies from nearly ground level to heights of 2000 m during their flight. Due to the buoyant restoring force on the tetroon during such wide excursions, it is not possible for the tetroon to follow the vertical air motions faithfully (Hanna and Hoecker, 1971). For this reason the statistical analysis of vertical motions is not carried very far. The

TABLE 1. General characteristics of tetroon flights.

Tetroon no.	Starting time (EDT)	Duration (min)	Average height $\bar{Z}$ (m)	σ_z (m)	Average speed $\bar{V}$ (m s^{-1})	Average direction	σ_w (m s^{-1})	$\partial\theta/\partial z$ (°C m^{-1})	ϵ ($cm^2\ s^{-3}$)	$\partial V/\partial z$ (s^{-1})	$\partial\alpha/\partial z$ (deg m^{-1})	Mixing height (m)
7221	1015	117	320	70	2.1	SW	0.24	—	—	—	—	—
7222	1015	117	370	120	1.9	SW	0.31	—	—	—	—	—
7223	1303	120	340	120	3.3	WSW	0.59	—	—	—	—	—
7224	1305	120	530	210	3.6	WSW	0.56	—	—	—	—	—
7253	1620	56	660	180	2.9	SSW	0.59	4×10^{-4}	15	0	0	1200
7254	1620	56	720	130	2.5	SSW	0.48	4×10^{-4}	15	0	0	1200
7271	1208	126	890	250	1.8	NE	0.45	2×10^{-4}	20	0	0	3000
7272	1208	126	740	150	1.4	ENE	0.45	2×10^{-4}	20	0	0	3000
7273	1428	94	730	420	2.3	NE	0.54	2×10^{-4}	20	0	−0.1	3000
7274	1428	94	510	250	1.4	NE	0.39	2×10^{-4}	20	0	−0.1	3000
7291	1201	102	380	80	3.3	SSW	0.24	—	—	0.007	0.1	3000
7292	1201	102	400	80	3.2	SSW	0.31	—	—	0.007	0.1	—
7293	1401	82	420	160	4.9	SSW	0.42	—	—	0.006	0	—
7294	1401	82	410	110	4.7	SSW	0.32	—	—	0.006	0	—
7301	1109	100	770	370	2.5	NNW	0.26	0	—	0.005	0	1600
7302	1109	100	710	240	2.2	NNW	0.35	0	—	0.005	0	1600
7303	1320	70	820	430	1.1	NE	1.0	0	—	0	0	1600
7304	1320	70	480	60	0.88	NNE	0.41	0	—	0	0	1600
7311	1307	92	650	380	2.2	NE	1.2	0	60	0	0.06	2200
7312	1307	92	660	270	2.0	NE	0.82	0	60	0	0.06	2200
7313	1511	108	950	350	1.0	E	0.75	0	40	0	0	2200
7314	1511	108	720	290	1.2	ENE	0.73	0	40	0	0	2200
8011	1258	100	1220	320	0.77	E	0.63	10^{-4}	25	0	0	2200
8012	1258	100	520	100	0.60	ESE	0.63	10^{-4}	25	0	0	2200
8021	1311	100	670	86	3.8	SW	0.35	10^{-4}	40	0	0	1000
8023	1311	100	480	152	3.4	SW	0.31	10^{-4}	40	0	0	1000

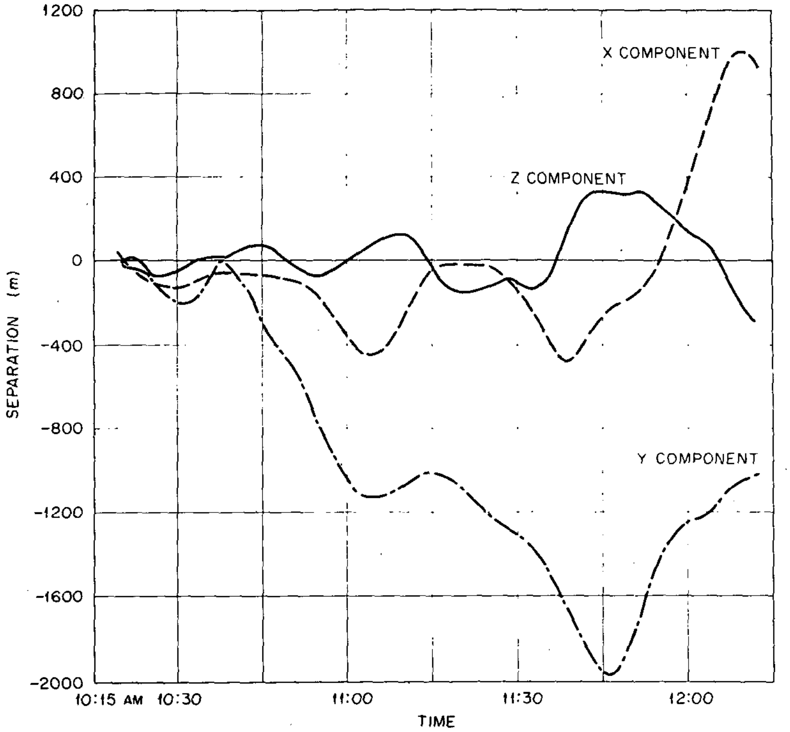

FIG. 2. Components of separation for tetroons 7221 and 7222.

average turbulence intensity $\bar{\sigma}_w/\bar{V}$ of 0.23 indicates a great deal of turbulent convection. The turbulent speed σ_w is not significantly correlated with wind speed, a condition known to be typical of sunny days, when turbulence production is due more to convection than to wind shear. The standard deviations of the turbulent fluctuations of longitudinal or alongwind speed, σ_u, and latitudinal or crosswind speed, σ_v, are also calculated. The average values of the ratios σ_v/σ_w and σ_u/σ_w are 1.3 and 1.9, in rough agreement with values of these ratios reported by Lumley and Panofsky (1964), based on many other independent sets of data.

Wind velocities in Table 1 generally have a SW or NE component, corresponding to the alignment of the ridges and valleys in eastern Tennessee. Average speed of the tetroons is 2.3 m s^{-1}, typical of light-wind summertime conditions in this area. Since all of the runs in Table 1 occur during similar climatic regimes, averages of parameters over all the runs can be confidently made.

4. Relative separation of tetroons

As an example of the results, the three components (latitudinal or cross-wind, longitudinal or along-wind, vertical) of tetroon separation for tetroons 7221 and 7222 are plotted as a function of time in Fig. 2. The along-wind direction is defined by the line connecting the tetroon launch position with the averaged position of the tetroons at the time the experiment is concluded. Oscillations with periods of 10–40 min are evident, in agreement with typical periods of convective elements. Clearly there is too much variability in this individual run to permit general conclusions regarding dispersion.

When the 13 runs are averaged together, this variability nearly disappears. For example, the rms total separation is plotted in Fig. 3. At times from 2 min through about 30 min the separation is proportional to time. At times from 30 min to 90 min, the separation S is proportional to time raised to a power somewhat less

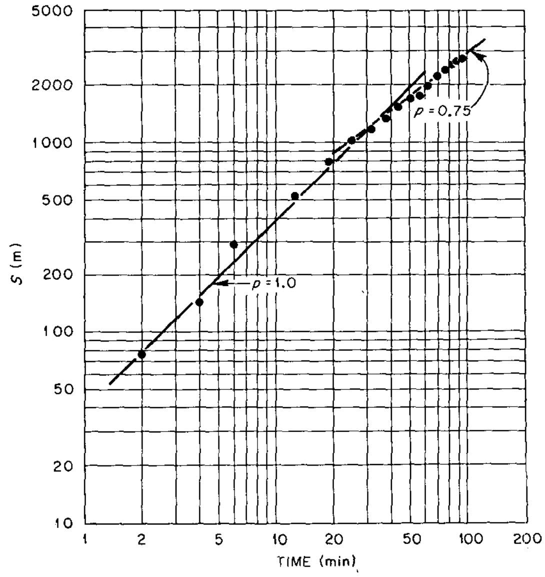

FIG. 3. Root-mean-square total separation S as a function of time.

than 1. From Fig. 3 it is seen that the average rate of separation dS/dt is roughly 0.5–0.7 m s^{-1}. Since the average value of σ_v for the 13 runs is 0.6 m s^{-1}, these observations are roughly satisfied by the relation $dS/dt\approx\sigma_v$.

The rate of change of total separation S with time for these data and for observations from others of the relative dispersion of tetroon pairs are plotted on Fig. 4. The power p for each set of observations is close to unity ($p=1\pm0.3$). It is seen that the magnitude of the separation S during ETTEX is slightly greater than the average magnitude of the separations during several experiments summarized by Islitzer and Slade (1968). This is expected, though, because our experiments were all conducted during highly dispersive summer days. However the dispersion of the tetroons released from Las Vegas (Angell *et al.*, 1971) is greater than the dispersion during ETTEX, presumably due to the greater dispersive power of the atmosphere over rugged desert terrain.

When observations of the relative dispersion of smoke puffs (e.g., Bauer, 1974) are studied, it is found that their cross-wind spread at small times is often proportional to time raised to the 1.5 power (Gifford, 1957). This power is predicted by Batchelor and Townsend's (1956) similarity theory for relative diffusion at length scales within the inertial subrange of atmospheric turbulence. But as Brier (1950) suggests, observed tetroon separations may not agree with the theoretical dependence of separation on time raised to the 1.5 power, because of the inability of the tetroons to respond to small-scale fluctuations in air motion. The small particles used in the smoke puff experiments respond well to all of the fluctuations in air motion that they encounter.

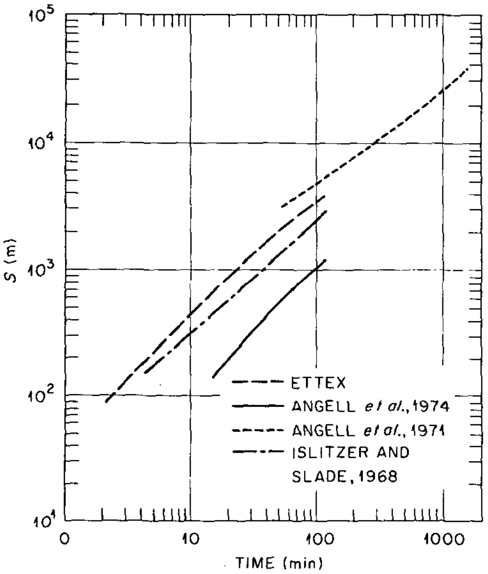

FIG. 4. Comparison of ETTEX root-mean-square total separations S with the results of other experiments.

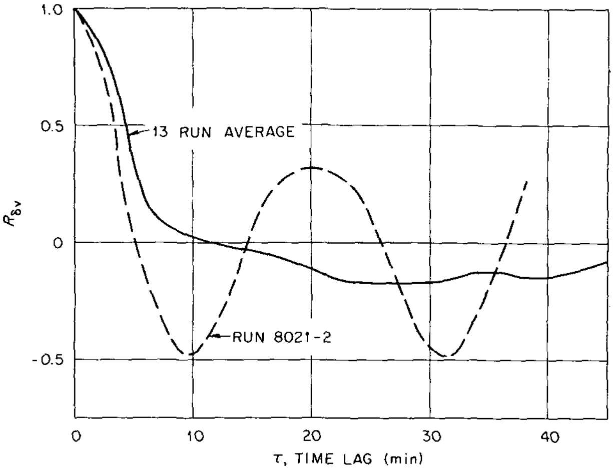

FIG. 5. Correlation coefficient $R_{\delta v}$ for the cross-wind component of the relative wind speeds.

At large times, when large scales of turbulence can take part in dispersion, the separation can be estimated from the relation

$$S=2(Kt)^{\frac{1}{2}}, \tag{1}$$

where K is the eddy diffusivity coefficient (Pasquill, 1974). This formula, however, is valid only at times much greater than the period at which peak energy occurs in the cross-wind turbulent velocity spectrum. It is seen in Fig. 2 that, during the highly convective conditions of ETTEX, significant turbulent fluctuations occur even at periods of 40 min. Consequently it is doubtful that the time regime during which Eq. (1) is valid is included in our observations.

5. Test of the statistical theory of diffusion

According to the statistical theory of relative diffusion, as reviewed by Pasquill (1974), the cross-wind separation Y of two diffusing tetroons is given by

$$\overline{Y^2}=\overline{Y_0^2}+2\sigma_{\delta v}^2\int_0^T\int_0^{t'}R_{\delta v}(\tau)d\tau dt', \tag{2}$$

where $\delta v(t)$ is the relative velocity of the balloons at time t after release, $\sigma_{\delta v}^2$ is the variance of the relative velocity fluctuations, and $R_{\delta v}(t)$ is the correlation coefficient of the relative velocities at time lag τ, i.e.,

$$R_v(\tau)=\overline{\delta v(t)\delta v(t+\tau)}/\sigma_{\delta v}^2. \tag{3}$$

Eq. (2) predicts that the rms separation $(\overline{Y^2})^{\frac{1}{2}}$ is proportional to time, as observed in our experiments, only when the correlation coefficient R is a constant. This can be seen by integrating (2) for R equal to a constant C:

$$\overline{Y^2}=\overline{Y_0^2}+\sigma_{\delta v}^2CT^2. \tag{4}$$

In order to determine if R is indeed a constant for the ETTEX data, values of R were calculated for all of the data. Fig. 5 presents the average correlation coefficient $R(t)$ for the cross-wind component δv of the relative velocities. The average correlation drops to zero after about 10 min and then remains slightly less than zero. The time scale at which $R=1/e$ is about 5 min. This is slightly less than the time scale of 8 min for the tetroon relative diffusion data taken in Idaho by Kao and Wendell (1968). The 13 individual correlation curves used in obtaining the average curve in Fig. 5 have many oscillations. For example, the curve for the combined tetroon run 8021-2 (randomly chosen for illustrative purposes) is shown in Fig. 5. The shapes of the individual curves are similar to this example for all of the observations, exhibiting parabolic form at small times and then sinusoidal form at large times. At any rate, none of the data suggest that R is positive and nearly constant over a large range of time lag τ, as is required by the statistical theory [Eq. (2)] to produce a linear dependence of the rms separation on time.

A requirement of the statistical theory is that both tetroons should remain within the same atmospheric layer. Pasquill (private communication) suggests that the ETTEX observations may not satisfy this requirement, since the tetroons are subject to such great vertical displacements. At times, one tetroon is near the ground and the other near the top of the planetary boundary layer. Twenty minutes later their positions could be reversed.

6. Shear diffusion

Whenever observations of relative diffusion do not agree well with the predictions of similarity or statistical theory, the differences can be blamed on shear diffusion. The tetroons can be separated due to the simple fact that they are flying at different heights and that the wind velocity is different at these heights. Angell *et al.* (1974) explain that much of the erratic behavior of their tetroons in the Los Angeles basin is due to wind shears associated with the sea breeze front. According to Smith's (1965) theory of shear diffusion, the variance of longitudinal separation ($\overline{X^2}$) due to a wind speed shear $\partial V/\partial z$ in unbounded flow is given by

$$\overline{X^2}=0.17(\partial V/\partial z)^2K_zt^3, \tag{5}$$

where K_z is the vertical eddy diffusivity coefficient. Smith (1965) suggests that the eddy diffusivity K_z is equal to $\sigma_w^2t_{wL}$, where t_{wL} is the Lagrangian integral time scale for the vertical motions. Thus Eq. (5) can be written

$$\overline{X^2}=0.17(\partial V/\partial z)^2\sigma_w^2t_{wL}t^3. \tag{6}$$

Table 1 contains the observed wind speeds and direction shears, from pibal data, for the vertical layer $\bar{z}\pm\sigma_z$. During over half of the runs, no shear at all was observed in the layer in which the tetroons flew. At heights between 200 and 1200 m, the average wind speed shear for the 13 runs is about 0.005 s^{-1}. As an example of the application of the technique, the combined run 7301-2 is considered where the shear $\partial V/\partial z=0.005\ s^{-1}$. The observed variance σ_w^2 is 0.1 $m^2\ s^{-2}$ and the Lagrangian integral time scale t_{wL} can be estimated to equal about 10^2 s. For these values, the variance $\overline{X^2}$ at time $t=30$ min has a value from (6) of about 3×10^6 m^2, which is close to the observed separation. Thus it is possible for shear diffusion to account for the observed tetroon separations during some of these experiments. However, there is no significant difference between the observed separations of tetroons on days when there is no wind shear and days when there is wind shear. It can be concluded that wind shear probably plays a role in separating the tetroons, but that the extent of this role is uncertain.

Acknowledgments. This experiment would have been impossible without the help of C. R. Dickson of the ARL Field Office in Idaho Falls, who provided the M-33 radar unit, a mobile radiosonde unit, and an enormous collection of routine but essential equipment. E. White and D. Forsythe of Idaho Falls were in charge of the radar's operation. W. Hoecker of the ARL office in Silver Spring, Md., offered invaluable aid in launching the tetroons. The Coal Creek Mining and Manufacturing Corporation and the Oliver Springs Mining Company (specifically, C. H. Smith and C. Owens) were very cooperative in helping us select and travel to and from the radar site. T. L. Montgomery and J. H. Coleman of the TVA Air Quality Branch loaned us several crucial pieces of equipment for the aircraft. Much of the data analysis was done by S. Perry, J. Pollock and S. Yeh of this office.

REFERENCES

Angell, J. K., P. W. Allen and E. A. Jessup, 1971: Mesoscale relative diffusion estimates from tetroon flights. *J. Appl. Meteor.*, **19**, 43–46.

——, C. R. Dickson and W. H. Hoecker, Jr., 1974: Relative dispersion within the Los Angeles basin as estimated from tetroon triads. NOAA Tech. Memo. ERL ARL 46, 34 pp.

Batchelor, G. K., and A. A. Townsend, 1956: *Turbulent Diffusion. Surveys in Mechanics*, G. K. Batchelor and R. M. Davies, Eds., Cambridge University Press, 352–399.

Bauer, E., 1974: Dispersion of tracers in the atmosphere and ocean: Survey and comparison of experimental data. *J. Geophys. Res.*, **79**, 789–795.

Brier, G. W., 1950: The statistical theory of turbulence and the problem of diffusion in the atmosphere. *J. Meteor.*, **7**, 283.

Briggs, G. A., 1975: Plume rise predictions. *Lectures on Air Pollution and Environmental Impact Analyses*, D. Haugen, coordinator, Amer. Meteor. Soc., 59–111.

Gifford, F. A., 1957: Relative atmospheric diffusion of smoke puffs. *J. Meteor.*, **14**, 410–444.

Hanna, S. R., and W. H. Hoecker, Jr., 1971: The response of constant density balloons to sinusoidal variations of vertical wind speeds. *J. Appl. Meteor.*, **10**, 601–604.

——, C. J. Nappo, R. P. Hosker and G. A. Briggs, 1974: Description of the Eastern Tennessee Trajectory Experiment (ETTEX). ATDL Contrib. File. No. 103, 26 pp., [Available from ATDL, P. O. Box E, Oak Ridge, Tenn., 37830].

Hoecker, W. H., Jr., 1975: A universal procedure for deploying constant-volume balloons and for deriving vertical air speeds from them. *J. Appl. Meteor.*, **14**, 1118–1124.

Islitzer, N., and D. Slade, 1968: Diffusion and transport experiments. *Meteorology and Atomic Energy—1968*, D. Slade, Ed., USAEC Rep. No. TID-24190, 117–188.

Kao, S. K., and L. L. Wendell, 1968: Some characteristics of relative particle dispersion in the atmospheric boundary layer. *Atmos. Environ.*, **2**, 397–407.

Lumley, J. L., and H. A. Panofsky, 1964: *The Structure of Atmospheric Turbulence*. Interscience, 239 pp.

Nappo, C. J., 1975: Time-dependent mesoscale wind flows over complex terrain. Presented at the First AMS Conference on Regional and Mesoscale Modeling, Analysis and Prediction, Las Vegas, 6–9 May.

Pasquill, F., 1974: *Atmospheric Diffusion*, 2nd. ed. Wiley, 429 pp.

Smith, F., 1965: The role of wind shear in horizontal diffusion of ambient particles. *Quart. J. Roy. Meteor. Soc.*, **91**, 318–329.

Van der Hoven, 1968: Atmospheric transport and diffusion in the planetary boundary layer. Tech. Memo. ERL TM-ARL 5, 5–14.