

The Effects of Water Vapor, Ozone and Aerosol on Atmospheric Turbidity

WILLIAM C. MALM

Northern Arizona University, Flagstaff 86001

ERIC G. WALTHER

Charles F. Kettering Foundation, Dayton, Ohio 45429

ROBERT A. CUDNEY

Museum of Northern Arizona, Flagstaff 86001

(Manuscript received 16 August 1976, in revised form 2 November 1976)

ABSTRACT

Direct solar radiation intensity was measured with an Eppley sun photometer at 380 and 500 nm near Page, Ariz. The turbidity coefficient B derived from these measurements was anomalous, $B_{380} < B_{500}$. This unusual condition seems to be caused by a combination of an appreciable amount of water vapor in very clean tropospheric air plus an almost monodisperse stratospheric aerosol. We recommend the inclusion of a water vapor term in the basic turbidity equation.

1. Introduction

Turbidity (McCormick and Baulch, 1962; McCormick and Kurfis, 1966), defined as "optical depth" by Herman and Yarger (1965) and Shaw *et al.* (1973), is a measure of the total aerosol in a vertical column of atmosphere and is usually measured from the total attenuation of sunlight at wavelengths of 380 and 500 nm. Turbidity has been measured near Page, Ariz., since February 1973 to establish the "background" level before a nearby coal-fired thermal-electric generating station could possibly change it.

The measurements yielded anomalous extinction (light scattering and absorption more at 500 nm than at 380 nm) nearly 100% of the time. Anomalous extinction has been observed by Porch *et al.* (1971) and occasionally at 8 of the 77 stations scattered around the world (Environmental Data Service, 1974, 1975). On rare occasions this effect may dominate Rayleigh scattering and result in a "blue moon" (Minnaert, 1954). In an attempt to determine the cause of the anomaly, tropospheric aerosol number distributions were measured over an 8-month period at various altitudes. Mie (1908) theory was used to calculate the effects of various aerosol number distributions on turbidity coefficients. In addition, the effect of water vapor and ozone were considered.

2. Theory

Let I_λ be the irradiance at wavelength λ at some observing point and I'_λ the irradiance at wavelength λ

at some nearby observing point further from the sun; Δx and σ will represent the separation between these two points and the extinction per unit length, respectively. If the distance between the two points becomes infinitesimal, then the relation is written $-dI_\lambda = I_\lambda \sigma dx$. The incremental extinction σdx may be rewritten $\sigma_M dM_h$ in terms of the relative optical air mass M_h and the extinction per unit optical air mass σ_M . The relative optical air mass M_h is the amount of optical atmosphere between the sun and the observing point divided by the amount of optical atmosphere vertically above the observing point.

Extinction of sunlight is primarily due to ozone absorption, and aerosol and molecular scattering. Expressed mathematically, $\sigma_M = B_\lambda + b_\lambda + k_\lambda$, where B_λ , b_λ and k_λ are the aerosol scattering or turbidity coefficient as a function of wavelength per unit optical air mass, Rayleigh scattering of air molecules per unit optical air mass and absorption by the ozone layer per unit optical air mass, respectively.

Now $-dI_\lambda/I_\lambda = (B_\lambda + b_\lambda + k_\lambda)dM_h$. Integrating this relation from the earth's surface to the top of the atmosphere we obtain $I_\lambda = (I_{0\lambda}/S) \exp(B_\lambda + b_\lambda + k_\lambda)M_h$, where $I_{0\lambda}$ is the extraterrestrial irradiance of wavelength λ at the mean sun-earth distance, and S a correction factor for the mean sun-earth distance. Because most of the ozone of the atmosphere is above the observing point, regardless of its elevation above sea level, no height or pressure correction is needed for k_λ . Background atmospheric aerosol concentrations depend on altitude. However, local aerosol contribu-

TABLE 1. Turbidity coefficient B .

	Wavelength (λ)=380 nm				Wavelength (λ)=500 nm				B_{500}/B_{380}
	Mini- mum	Geo- metric mean	Maxi- mum	Half of 90% con- fidence interval	Mini- mum	Geo- metric mean	Maxi- mum	Half of 90% con- fidence interval	
All data (13 Feb 73- 3 Apr 74)	0	0.039	0.225	0.004	0.015	0.061	0.220	0.004	1.56
Seasons									
Winter ('73)									
Feb	0.013	0.023	0.041	0.010	0.026	0.042	0.061	0.005	1.82
Spring ('73)									
Mar-May	0.015	0.052	0.161	0.005	0.036	0.076	0.200	0.005	1.46
Summer ('73)									
Jun-Aug	0.008	0.084	0.225	0.010	0.041	0.110	0.220	0.008	1.31
Fall ('73)									
Sep-Nov	0.002	0.039	0.121	0.006	0.018	0.059	0.130	0.006	1.51
Winter ('73-'74)									
Dec-Feb	0	0.017	0.081	0.005	0.015	0.034	0.089	0.004	2.00
Spring ('74)									
Mar-Apr	0.002	0.048	0.112	0.011	0.037	0.069	0.136	0.001	1.43
Months									
February 1973	0.013	0.023	0.041	0.004	0.026	0.042	0.061	0.005	1.82
March 1973	0.023	0.048	0.100	0.009	0.045	0.041	0.126	0.008	1.48
April 1973	0.015	0.054	0.161	0.008	0.036	0.079	0.210	0.009	1.46
May 1973	0.026	0.056	0.116	0.008	0.043	0.081	0.126	0.007	1.45
June 1973	0.008	0.054	0.110	0.009	0.041	0.083	0.130	0.008	1.53
July 1973	0.006	0.092	0.143	0.001	0.078	0.108	0.153	0.008	1.17
August 1973	0.069	0.135	0.225	0.018	0.087	0.143	0.221	0.015	1.06
September 1973	0.036	0.064	0.098	0.006	0.056	0.085	0.128	0.007	1.33
October 1973	0.002	0.039	0.121	0.009	0.027	0.057	0.113	0.008	1.46
November 1973	0.003	0.018	0.043	0.006	0.019	0.035	0.039	0.006	1.94
December 1973	0.005	0.014	0.028	0.003	0.017	0.028	0.048	0.005	2.00
January 1974	0.009	0.020	0.081	0.013	0.018	0.035	0.043	0.005	1.75
February 1974	0.001	0.018	0.067	0.008	0.015	0.039	0.089	0.009	2.16
March 1974	0.020	0.053	0.112	0.013	0.038	0.071	0.136	0.014	1.34
April 1974	0.023	0.038	0.050	0.011	0.051	0.061	0.075	0.010	1.61

tions (aerosol due to windblown dust, pollutants from stationary and mobile source, etc.) tend to dominate the turbidity coefficient and B_λ is consequently usually assumed to be altitude independent. The attenuation due to Rayleigh scattering, $b_\lambda M_h$, is corrected for altitude by multiplying by p/p_0 , where p and p_0 are atmospheric pressure at the observation point and sea level, respectively.

The equation for I_λ must be changed from the natural logarithm to the base 10 logarithm in order to compare resulting turbidity coefficients with the data of other authors who use the equation (Robinson, 1969)

$$I_\lambda = (I_{0\lambda}/S)10^{(B_\lambda + k_\lambda + b_\lambda(p/p_0)) M_h} \tag{1}$$

Solving this equation for the turbidity coefficient gives

$$B_\lambda = [\log(I_{0\lambda}/I_\lambda S)]/M_h - b_{h\lambda} - k_\lambda, \tag{2}$$

where $b_{h\lambda} = b_\lambda p/p_0$ is the scattering due to air molecules at height h . The resulting base 10 absorption and extinction coefficients per optical air mass are 0.434 times the base e absorption and extinction coefficients.

3. Measurements

I_λ at 380 and 500 nm and M_h were obtained using Epply Sunphotometer No. 4645. The sunphotometer does not measure the spectral irradiance (I_λ) directly but rather provides a meter deflection which is proportional to the incoming solar radiation. The meter reading corresponding to $I_{0\lambda}$ may be obtained via a Langley plot or calibrated against a working standard. Our instrument was calibrated by the Eppley Laboratory and recalibrated by EPA, Research Triangle Park, N. C., on 11 January 1973 and in December 1974, respectively.

The calibration at 500 nm remained unchanged while at 380 nm it changed by less than 1.2%. The values used for k_λ and $b_{h\lambda}$ were obtained from Elterman (1968).

Turbidity measurements were initiated in February 1973 and data taken through April 1974 will be presented here. All measurements were made at the same location, Vermillion Court, Page, Ariz. (a mobile home park in a vast remote region) at 0900, 1200 and 1500 MST. The data for B_{380} and B_{500} are in

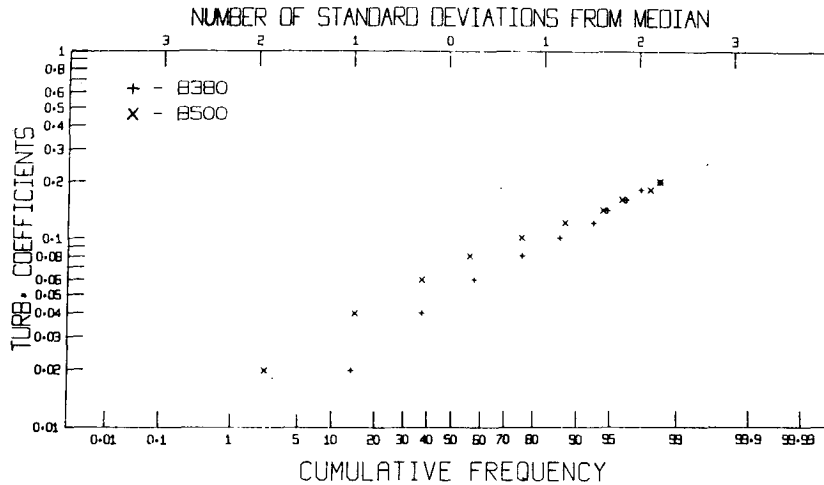


FIG. 1. Logarithmic probability plot of turbidity coefficients recorded at 380 and 500 nm.

Table 1 as seasonal and monthly geometric means along with corresponding maxima, minima and 90% confidence intervals on the means. The difference between weekend and weekday means was not statistically significant (within 90% confidence intervals). The turbidity coefficient correlated with the scattering coefficient, measured by an integrating nephelometer located in Page Airport. The linear correlation coefficient of 0.662 is significant at the 1% level.

The monthly geometric mean turbidity coefficients, B_{380} and B_{500} are presented in Fig. 1 as a logarithmic probability plot. Because the turbidity coefficients measured at 380 nm and 500 nm fall on a straight line, it can be concluded that the distribution of turbidity coefficients is log normal and that geometric rather than arithmetic means should be used in subsequent analysis. The calculated geometric means of $B_{500}=0.061$ and $B_{380}=0.039$ compare quite closely to the geometric means obtained graphically, $B_{500}=0.060$

and $B_{380}=0.43$, again signifying a log-normal distribution.

Fig. 2 shows the monthly variation of B_{500} and B_{380} geometric means. It should be noted that in all cases $B_{380} < B_{500}$. In contrast, 69 of the 77 global stations never reported $B_{380} < B_{500}$ while the other eight stations reported this condition some of the time (Environmental Data Service, 1974, 1975). Of these eight stations, two are located within a few hundred kilometers of Page, Ariz. They are Alamosa, Colo., and Bishop, Calif. According to Porch *et al.* (1971), extinction measurements made at three astronomical observatories (Kitt Peak, Rattlesnake Ridge and Cerro Tololo) have shown that at least 50% of the time the light scattering increased with increasing wavelength. All these locations, including Page, Ariz., have extremely clean atmospheres with extinction coefficients that are typically less than 0.025 km^{-1} at 500 nm.

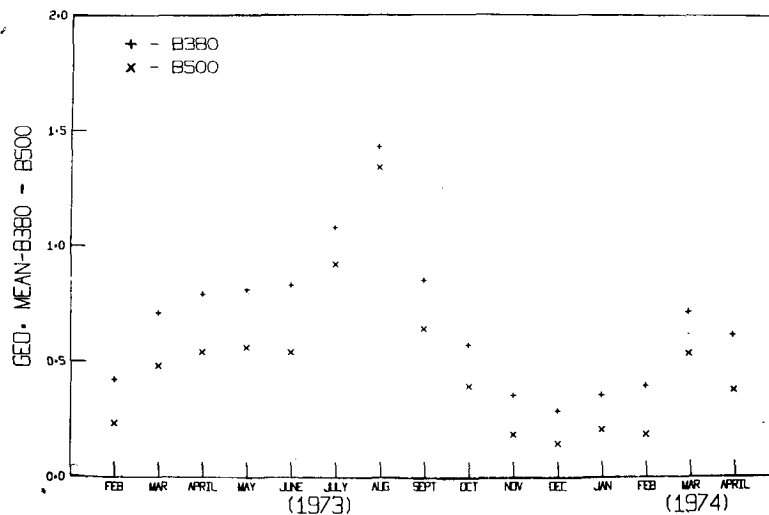


FIG. 2. Monthly geometric mean turbidity coefficients.

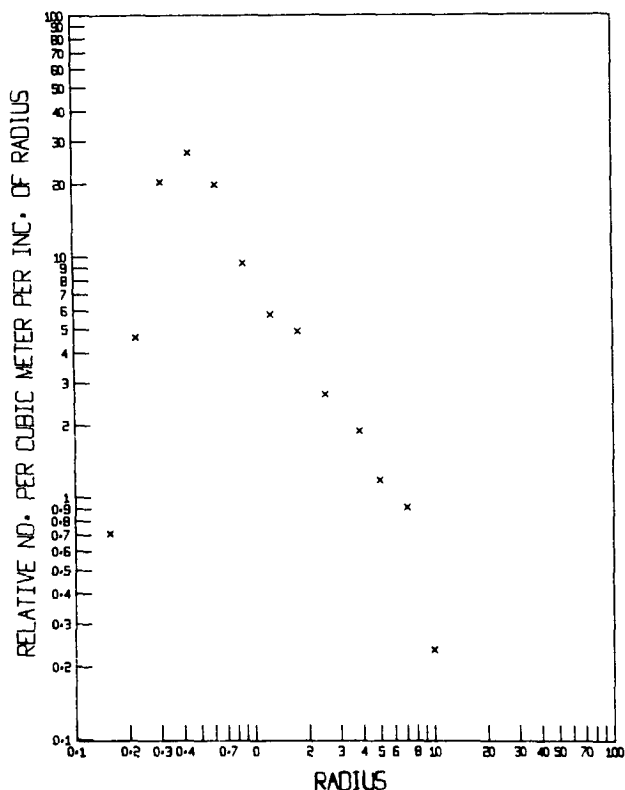


FIG. 3. Junge plot of aerosol number distribution measured at Lake Powell on 25 April 1975.

In order to determine the possible contribution of aerosol size distribution to the turbidity coefficients and anomalous extinction, aerosol number/size dis-

tribution measurements were initiated in November 1974 with a Casella Cascade Impactor Mk IIa.

The number of particles in each size range were plotted as $\ln(dN/d \ln r)$ vs $\ln r$ (following Junge, 1963) as well as on a log-probability plot. A representative distribution made on 26 April 1975 is shown in Figs. 3 and 4.

Ten aerosol number/size distributions, two of which were made in aircraft at 150 and 4000 m, consistently followed the power law proposed by Junge (1963), i.e.,

$$dN/d \ln r = Cr^{-\beta} \tag{3}$$

In this equation, N is the number of particles larger than radius r per unit volume, and C and β are constants determined by experiment; $dN/d \ln r$ is the differential number density per increment in $\ln r$. When $\ln(dN/d \ln r)$ is plotted against $\ln r$, a straight line is obtained with a slope equal to $-\beta$. $|\beta|$ is representative of the relative ratio of small to large particles, the ratio increasing as $|\beta|$ increases. In these measurements, the value of β , typically lying between 2.5 and 4.0 for urban atmospheres, was found to lie between 1.24 and 2.1 with an average of 1.87. β was found to increase as the actual number density was highest. The slope of the curve represented in Fig. 3 is 1.85. Fig. 4, representative of the log-probability plots obtained from our size distributions, indicates that the distribution is not generally log-normal. However, subsections of the distribution do appear to be log-normal, i.e., 0.3 to 1 μ and from 1 to 10 μ with geometric standard deviations of 1.59 and 3.35, respectively.

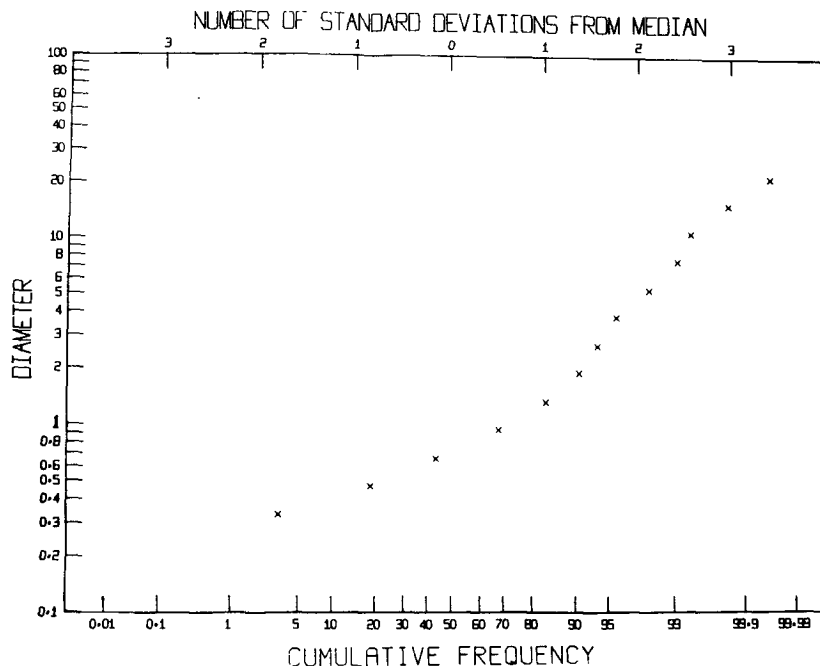


FIG. 4. Log-probability plot of aerosol number distribution measured at Lake Powell on 25 April 1975.

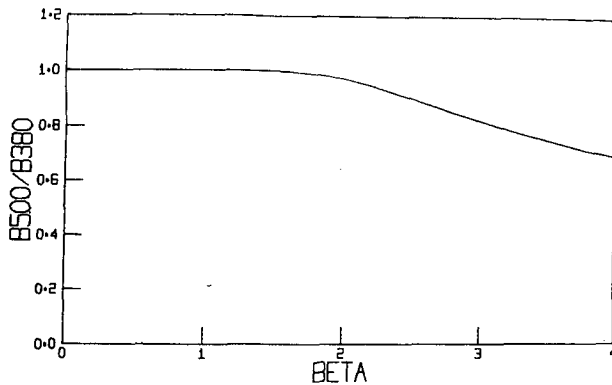


FIG. 5. Ratio of B_{500}/B_{380} for a Junge distribution plotted as a function of β , the slope of the curve represented in Fig. 3.

4. Calculations

Referring to Van de Hulst (1957) for details, the extinction coefficient is given by

$$b_{ext} = \pi \int_{r_1}^{r_2} r^2 Q_{ext}(X, n^*) dN. \quad (4)$$

Using $dN/d \ln r = Cr^{-\beta}$ for the number distribution, the extinction coefficient becomes

$$b_{ext} = \pi C \int_{r_1}^{r_2} Q_{ext}(X, n^*) r^{2-\beta} d \ln r, \quad (5)$$

where Q_{ext} , S , n^* are the scattering efficiency factor, the size parameter and complex index of refraction, respectively.

To determine the effect that β has on the ratio b_{500}/b_{380} ($b_{500} \propto B_{500}$), the index of refraction was assumed equal to 1.5. This value is representative of the minerals and aerosol in the Lake Powell region. The integration limits on r were 0.1 and 10 μm . Although the lower integration limit extends below the instrumental cutoff of about 0.3 μm , the contribution of the 0.1–0.3 μm particles to the total extinction coefficient is negligible when β is less than 2.0. The upper limit of 10 μm was chosen to coincide with an observed average particle size cutoff. The results of these calculations are shown in Fig. 5.

It is clear from Fig. 5 that the ratio of B_{500}/B_{380} for values of $\beta < 2$ is approximately 1, i.e., the scattering coefficient is independent of wavelength. In fact, the ratio varies from 1.002 to 0.97 when β varies from 0.0 to 2.0. Because aerosol size distributions have been measured through various seasons and altitudes up to 5 km and because all distributions obey a power law with $\beta \lesssim 2$, it must be concluded that the aerosol distribution causing B_{500}/B_{380} considerably larger than 1 is shaped something other than this power law and that this aerosol must be at altitudes greater than 5 km. Fig. 2 indicates a constant background turbidity with $B_{500} \approx B_{380} + 0.019$. The changes observed on a monthly basis appear to be due to the addition or depletion of troposphere aerosol, thus adding approximately equal amounts of turbidity to B_{500} and B_{380} .

As stated previously, the increase in the aerosol number concentration corresponds to an increase in β . An increase in β and aerosol concentration would imply an increased turbidity coefficient along with a

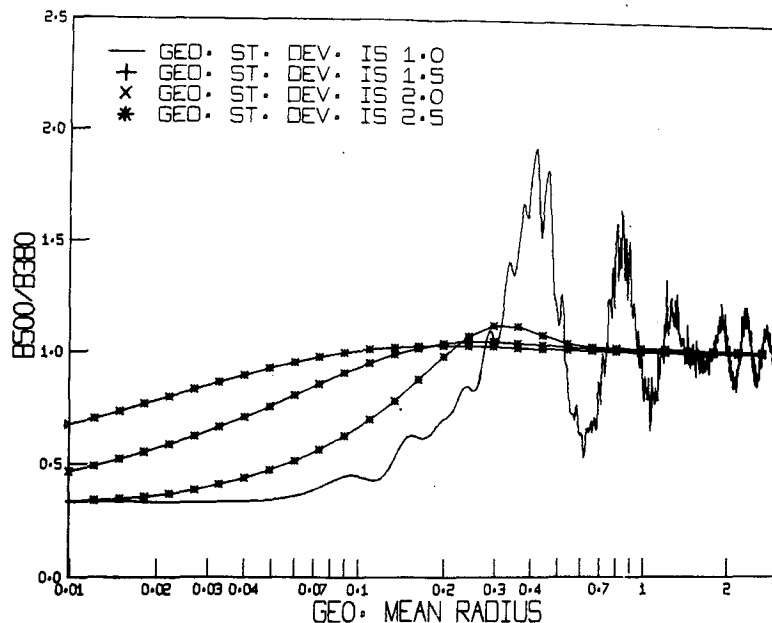


FIG. 6. Ratio of B_{500}/B_{380} for a log-normal aerosol distribution, plotted as a function of the geometric mean radius for various values of the geometric standard deviation.

decrease in B_{500}/B_{380} . Fig. 5 shows just this effect. As the turbidity increases, B_{500} and B_{380} approach the same value.

To obtain ratios of $B_{500}/B_{380} > 1$ an aerosol must have a number distribution tightly centered about some mean radius. Junge and Manson (1961) reported a spatially stable monodisperse sulfate aerosol layer ranging from 0.1 to 2 μm in size at about 20 km. To investigate the effects of such a distribution a log-normal size frequency was used, i.e.,

$$M(r) = (2\pi)^{-1} (\ln \sigma_g)^{-\frac{1}{2}} \times \exp\left\{-\left(\frac{1}{2}\right)\left[\frac{\ln r - \ln r_g}{\ln \sigma_g}\right]^2\right\}, \quad (6)$$

where r_g , σ_g and r are the geometric mean radius, geometric standard deviation and radius, respectively.

This size distribution was substituted into (4) with $n^* = 1.5$. The geometric mean radius was varied from 0.01 to 3 μm while the geometric standard deviation was allowed to take on values of $\sigma_g = 1.0, 1.5, 2.0$ and 2.5. The ratio of b_{500}/b_{380} was calculated and the results are shown in Fig. 6. With a geometric standard deviation equal to one the ratio $b_{500}/b_{380} \approx 1.9$ at a geometric mean radius of approximately 0.4 μm . As the standard deviation is increased the maximum ratio of b_{500}/b_{380} decreases rapidly to 1.

To see if a stratospheric monodisperse aerosol can account for the observed large ratios of B_{500}/B_{380} , it is necessary to subtract the aerosol contributions in the mixing layer from the observed turbidity values. Since there is a good correlation between the integrating nephelometer and turbidity measurements it is assumed that the average scattering coefficient as measured by the nephelometer is representative of the aerosol in the mixing layer. It is further assumed that the extinction coefficient due to aerosol decreases logarithmically as described by Elterman (1968):

$$b(h_2) = b(h_1) \exp\left[-(h_2 - h_1)/H_p\right], \quad (7)$$

where H_p , the scale height, is set equal to 1.2 km, and $b(h_1) = b_{\text{scat}} - b_{\text{Rayleigh}} = 0.0017 \text{ km}^{-1}$, the extinction coefficient measured by the nephelometer on those days turbidity was measured.

The mixing layer is assumed to extend to 4 km, giving a total lower tropospheric aerosol attenuation coefficient of 0.0023. The average turbidity coefficient for the months of November, December, January, February is 0.036 and 0.019 for 500 nm and 380 nm, respectively, thus implying a background aerosol attenuation of 0.034 and 0.017 at 500 and 380 nm. This would give a ratio of $B_{500}/B_{380} \approx 2.00$, a value slightly higher than the maximum 1.9 theoretically predicted for a monodisperse aerosol with a radius of 0.4 μm .

Other contributions to the preferential attenuation of 500 nm light are ozone (Elterman, 1968) and water vapor (Eldridge, 1967). In order for O_3 to be responsible for $B_{500}/B_{380} \approx 2.0$, the O_3 concentration would

TABLE 2. Water vapor contribution to turbidity coefficients.

Precipitable H ₂ O vapor (cm)	Contribution to turbidity coefficients due to H ₂ O vapor		B_{500}/B_{380} after subtraction of H ₂ O vapor and tropospheric aerosol contributions
	500 nm	380 nm	
1.0	0.010	0.003	1.7
2.0	0.013	0.004	1.6
5.0	0.021	0.006	1.18

have to be ~ 3.0 times greater than reported by Elterman (1968). Ozone measurements made in Northern Arizona by Epstein *et al.* (1956) do not yield anomalously high concentrations and consequently cannot account for $B_{380} < B_{500}$. To investigate the effects of water vapor on turbidity, total water vapor burdens were calculated using radiosonde data obtained from Winslow, Ariz., a city located ~ 105 mi south of Page. The total water vapor burden for the months of November, December 1973 and January, February 1974 was in all cases approximately equal to 1 cm of precipitable water. Using tables constructed by Eldridge (1967), contributions to turbidity coefficients due to water vapor can be calculated. The results of these calculations are presented in Table 2, along with the resulting B_{500}/B_{380} ratios obtained after subtraction of water vapor and lower troposphere (0-4 km) aerosol contributions. From Table 2, it is evident that water vapor can make a significant contribution to measured turbidity coefficients when very clean atmospheres are being measured; consequently, water vapor absorption should be included in the equation [Eq. (1)] used to determine "turbidity," i.e., the attenuation coefficient due to aerosols. Since water vapor preferentially absorbs 500 nm over 380 nm radiation it is partially responsible for $B_{500}/B_{380} > 1$. However, Table 2 shows that B_{500}/B_{380} is still substantially greater than 1 after subtraction of water vapor and lower troposphere contributions. At 1 and 2 cm of precipitable water $B_{500}/B_{380} \approx 1.7$ and 1.6, respectively. The resulting ratio of $B_{500}/B_{380} > 1$ could be due to the stratospheric monodisperse aerosol layer described by Junge and Manson (1961). The aerosol layer with a 0.4 μm mean radius and a $\sigma_g = 1.0$ could yield a B_{500}/B_{380} ratio as high as 1.9. Based on above calculations it appears that:

- 1) Water vapor can make a significant contribution to the turbidity coefficient when clean atmospheres are being considered, and consequently should be included in turbidity calculations.
- 2) The observed ratio of $B_{500}/B_{380} > 1$ is due not only to water vapor absorption, but also to a stratospheric semimonodisperse aerosol.

5. Conclusions

In general, it is not possible to definitely interpret aerosol and absorption characteristics from turbidity measurements because of the large number of parameters involved. However, some explanations for the observed anomalous extinction are possible. Aerosol number distribution measurements made at various locations and altitudes up to 4 km, all yield data that follow the Junge curve with values of β from 1.2 to 2.1. Large β values are associated with greater aerosol loading. Using Mie theory, it was shown that as β increases the ratio of B_{500}/B_{380} decreases from unity. Fig. 1, the log probability plot of the turbidity coefficients, shows the theoretically predicted effects of a simultaneous increase in β and aerosol number concentration, i.e., B_{500} and B_{380} approach the same value (overriding the background of $B_{500} > B_{380}$) as the aerosol number concentration increases.

The background ratio of $B_{500}/B_{380} \approx 2.0$ remains to be explained. A monodisperse aerosol with a geometric standard deviation of 1.0 at 0.4 μm would yield a maximum ratio B_{500}/B_{380} of about 1.9 instead of the measured 2.0. However, it is extremely unlikely that an aerosol layer would exist with a geometric standard deviation of 1 and since the ratio of B_{500}/B_{380} decreases to 1 with increased geometric standard deviation, it appears the 20 km stratospheric aerosol layer cannot by itself account for the observed B_{500}/B_{380} .

Another contributor to preferential attenuation of 500 nm over 380 nm radiation is atmospheric water vapor. Total water vapor burdens, obtained from radiosondes made at Winslow, Ariz., were approximately 1 cm of precipitable water for those months when a high B_{500}/B_{380} ratio was observed. Correcting B_{500} and B_{380} for water vapor absorption lowered B_{500}/B_{380} from 2.0 to about 1.6–1.7.

As a result of the above calculations, it is felt that an additional correction due to water vapor concentrations should be added to the standard turbidity equation. For clean atmospheres such as observed in Page, Ariz., water vapor may be a significant contribution to the observed turbidity coefficients. However, water vapor alone cannot be responsible for the high B_{500}/B_{380} ratios. It is believed that the observed ratio of $B_{500}/B_{380} > 1$ is due, not only to water vapor

absorption, but also to a stratospheric semi-monodisperse aerosol.

Acknowledgments. We thank Dr. Edwin C. Flowers of EPA for checking the calibration of our Eppley sun photometer. This work was part of the Air Quality Subproject of the Lake Powell Research Project, supported by National Science Foundation under Grant GI 34838.

REFERENCES

- Eldridge, R. G., 1967: Water vapor absorption of visible and near infrared radiation. *Appl. Opt.*, **6**, 709–713.
- Elterman, L., 1968: UV, visible, and IR attenuation for altitudes to 50 kilometers. Rep. AFCRL-68-0153, Air Force Cambridge Research Laboratories, Bedford, Mass., 1–49. [NTIS No. AD 671933]
- Environmental Data Service, NOAA, 1974: Atmospheric turbidity and precipitation chemistry data for the world 1972. [National Climatic Center, Federal Building, Asheville, N. C. 28801]
- , 1975: Atmospheric turbidity and precipitation chemistry data for the world 1973. [National Climatic Center, Federal Building, Asheville, N. C. 28801]
- Epstein, E. S., C. Osterberg and A. Adel, 1956: A new method for the determination of the vertical distribution of atmospheric ozone from a ground station. *J. Meteor.*, **13**, 319–334.
- Herman, B. M., and D. N. Yarger, 1965: The effect of absorption in a Rayleigh atmosphere. *J. Atmos. Sci.*, **42**, 644–651.
- Junge, C. E., 1963: *Air Chemistry and Radioactivity*. Academic Press, 382 pp.
- , and J. E. Manson, 1961: Stratospheric aerosol studies. *J. Geophys. Res.*, **66**, 2163–2183.
- McCormick, R. A., and D. M. Baulch, 1962: The variation with height of the dust loading over a city as determined from the atmospheric turbidity. *J. Air Poll. Control Assoc.*, **12**, 492–496.
- , and K. R. Kurfis, 1966: Vertical diffusion of aerosols over a city. *Quart. J. Roy. Meteor. Soc.*, **92**, 392–396.
- Mie, G., 1908: Beitrage zur Optik truber Medien. *Ann. Phys.*, **25**, 377–445.
- Minnaert, M., 1954: *The Nature of Light and Colour in the Open Air*. Dover, 225–226.
- Porch, W. M., E. J. Mannery, R. J. Charlson and P. W. Hodge, 1971: Use of astronomical telescopes to measure aerosol pollution. *Nature*, **233**, 326–327.
- Robinson, E., 1969: Effect on the physical properties of the atmosphere. *Air Pollution*, Vol. 1, 2nd ed., Arthur C. Stern, Ed., Academic Press, 349–400.
- Shaw, G. E., J. A. Reagan and B. M. Herman, 1973: Invert solutions of atmospheric extinction using direct solar radiation measurements made with a multiple wavelength radiometer. *J. Appl. Meteor.*, **12**, 374–380.
- Van de Hulst, H. C., 1957: *Light Scattering by Small Particles*. Wiley, 453 pp.