A Redetermination of the Rayleigh Optical Depth and its Application to Selected Solar Radiation Problems

DOUGLAS V. HOYT

Geophysical Monitoring for Climatic Change, NOAA, Boulder, Colo. 80302

(Manuscript received 27 September 1976)

ABSTRACT

Rayleigh optical depths are calculated for six standard atmospheres using the latest value of the depolarization factor ($\rho_n=0.0139$). Present uncertainties in the Rayleigh optical depth of ±0.16% exist for the values calculated in this paper but the new values are 3.5–3.7% lower than previously reported values. As a consequence it is shown that previous aerosol optical depth or turbidity values from Volz sunphotometer measurements, for example, are reported with systematic errors 2–100% too low. The new Rayleigh optical depths are shown to largely explain the systematic differences between the various wavelength pair determinations of total ozone by the Dobson spectrophotometer. Other possible applications of the results of this paper are also indicated.

1. Introduction

The Rayleigh optical depth is an important parameter in many atmospheric radiative transfer problems. In this paper the Rayleigh optical depth as a function of wavelength is calculated for variety of cases. Calculated Rayleigh optical depths are functions of several parameters, and uncertainties in these parameters contribute to uncertainties in the derived values of Rayleigh optical depths. The major source of uncertainty has been and continues to be the value of the depolarization factor. New experimental determinations of the depolarization factor indicate its value is lower than heretofore assumed. Adoption of the most recent value gives new and lower values for the Rayleigh optical depth. The interpretation and consequence of these new calculations are presented.

2. Theory and error analysis

The Rayleigh optical depth τ_R of the atmosphere is given by

$$\tau_R=\int_0^\infty \beta(\lambda,z)dz, \tag{1}$$

where β is the volume scattering coefficient of clear air as a function of wavelength λ and altitude z. The above integral is generally calculated from sea level (1013.25 mb) to the top of the atmosphere to obtain the normal Rayleigh optical depth. The volume scattering coefficient β is given by

$$\beta(\lambda,z)=\frac{8\pi^3}{3}\frac{(n_s^2-1)^2N}{\lambda^4N_s^2}\frac{6+3\rho_n}{6-7\rho_n}, \tag{2}$$

where n_s is the index of refraction of air, N the number density of air at any atmospheric temperature and pressure, N_s the number density at standard temperature and pressure or Avogadro's number, λ the wavelength of light, and ρ_n the depolarization ratio for the transverse scattering of unpolarized light. This relationship was first derived by King (1923). The quantity

$$f_L=\frac{6+3\rho_n}{6-7\rho_n} \tag{3}$$

is the correction factor for the depolarization of light that is attributable primarily to the anisotropy of atmospheric molecules. It accounts for at least part of the lack of complete polarization of sky light at 90° to the direction of the incident beam (Rayleigh, 1918).

Each of the terms in Eq. (2) can contribute to an error in the determination of the volume scattering coefficient β and hence in the Rayleigh optical depth τ_R. The index of refraction of air n_s was determined using Edlen's (1953) formula which Penndorf (1957) pointed out may be in error by as much as 0.1–0.2%. The value of Avogadro's number N_s is known to within 5.1×10^{-4}% (*Handbook of Chemistry and Physics*, 1975) and is not a significant source of error. At any one level the number density N of air may vary as much as 10% because of temperature and pressure variations. However, when integrated over the entire atmosphere in (1), these variations contribute very little to variations in τ_R as will be shown in the next section.

The depolarization correction factor f_L or depolarization factor ρ_n may contribute a significant systematic error to the determinations of β and τ_R if it is improperly

TABLE 1. Rayleigh optical depths versus wavelength for various standard atmospheres and Elterman's model.

λ (μm)	30°N January	30°N July	45°N January	45°N July	60°N January	60°N July	Elterman (1968)
0.30	1.181	1.181	1.180	1.180	1.178	1.178	1.222
0.32	0.8968	0.8969	0.8955	0.8959	0.8944	0.8944	0.9280
0.34	0.6938	0.6939	0.6928	0.6931	0.6919	0.6920	0.7179
0.36	0.5456	0.5457	0.5448	0.5450	0.5441	0.5441	0.5645
0.38	0.4352	0.4353	0.4346	0.4348	0.4340	0.4340	0.4503
0.40	0.3515	0.3516	0.3510	0.3512	0.3506	0.3506	0.3638
0.42	0.2872	0.2872	0.2868	0.2869	0.2864	0.2864	0.2972
0.44	0.2370	0.2370	0.2366	0.2368	0.2363	0.2364	0.2452
0.46	0.1973	0.1974	0.1971	0.1971	0.1968	0.1968	0.2042
0.48	0.1657	0.1657	0.1654	0.1655	0.1652	0.1653	0.1714
0.50	0.1402	0.1402	0.1400	0.1400	0.1398	0.1398	0.1450
0.55	0.09495	0.09497	0.09481	0.09486	0.09470	0.09470	0.09825
0.60	0.06663	0.06664	0.06653	0.06656	0.06645	0.06645	0.06894
0.65	0.04814	0.04815	0.04807	0.04810	0.04802	0.04802	0.04982
0.70	0.03566	0.03566	0.03561	0.03562	0.03556	0.03557	0.03690
0.80	0.02079	0.02079	0.02076	0.02077	0.02073	0.02073	0.02151
0.90	0.01293	0.01293	0.01291	0.01292	0.01289	0.01290	0.01338
1.00	0.008460	0.008461	0.008448	0.008452	0.008437	0.008438	0.008754
1.50	0.001661	0.001661	0.001658	0.001659	0.001656	0.001656	0.001718

measured. Measurements of the depolarization factor have been made by Gans (1921), Ananthakrishnan (1935), Volkmann (1935), Parthasarathy (1951), Massoulier (1963), Bridge and Buckingham (1964), Weber *et al.* (1967) and Rudder and Bach (1968) among others. From the summary given by Penndorf (1957) the value of ρ_n for air equals 0.0375±0.0047 for six measurements. In an earlier summary by Volkmann (1935) of seven measurements, the value of $\rho_n=0.0429 \pm 0.0033$. In general, as experimentalists have treated the problems of stray light more carefully their measured values of ρ_n have tended to become smaller. Kasten (1968) was aware of this problem and took $\rho_n=0.0295$ for air based upon recent measurements by Bridge and Buckingham (1964) and Dintzis and Stein (1963) for his treatment of Rayleigh scattering. This value of ρ_n, however, is higher than the values of Weber *et al.* (1967) and Rudder and Bach (1968) from whose works an upper limit for the value of ρ_n in air of 0.0139±0.0006 may be set.

The difference between these modern values and the set of previous values may be attributed to the failure of the earlier experimenters to isolate Rayleigh scattering from Raman scattering. The strongly depolarized Raman lines will produce an apparently larger depolarization and hence give systematically high values of the depolarization factor. If the value of ρ_n is taken to be equal to 0.0350, as Penndorf (1957) and Elterman (1968) do, the value of the correction factor f_L is 1.0608. When the value 0.0139±0.0006 is used, $f_L=1.0235$ and in the past there has been a systematic difference in the Rayleigh optical depths of about 3.6±0.11% with the lower value of the depolarization factor giving the lower values of Rayleigh optical depth.

3. Calculations of the Rayleigh optical depth for different standard atmospheres

Using Edlen's formula for n_s and standard atmospheres from *U.S. Standard Atmosphere Supplement 1966* for values of N, Eqs. (1) and (2) may be used to calculate values of the normal Rayleigh optical depth. January and July atmospheres for 30°, 45° and 60°N are used. As a check on the computer program, *U.S. Standard Atmosphere, 1962* and a value of the depolarization factor of 0.0350 are used to calculate the Rayleigh optical depth. These results were identical to those of Elterman (1968).

In Table 1 the Rayleigh optical depths for the six standard atmospheres and Elterman's case are tabulated as a function of wavelength. The value of ρ_n is taken to equal 0.0139. From the table, variation in τ_R of as much as 0.27% at any one wavelength can occur. At any one latitude the variation during the course of a year is 0.05%. These differences arise from variations in the density structure of the atmosphere as a function of time. The cooler the atmosphere the lower the values of τ_R.

The uncertainty in the Rayleigh optical depth due to the uncertainty in the depolarization factor alone is 0.11%. The uncertainty due to density variations is 0.05% so the total uncertainty in the tabulated values is about 0.16% if one uses any one column. The values of τ_R, nonetheless, are 3.5–3.7% lower than Elterman's or Penndorf's values, which are widely used in atmospheric radiation problems. The consequences of the new values will be discussed in the next section.

4. Applications of the results

The low Rayleigh optical depths calculated above affect the interpretation of several remote sensing

TABLE 2. Decadal values of Rayleigh optical depths used for the Dobson spectrophotometer.

Wavelength pair	Wavelength	τ_R (previous studies)	τ_R (present study)
A	3055	0.491	0.4756
	3254	0.375	0.3634
C	3114.5	0.453	0.4373
	3324	0.343	0.3318
D	3176	0.416	0.4026
	3398	0.312	0.3018

problems including turbidity observations, total ozone observations, lidar observations, radiation and heat budget studies and so forth. The application of the Rayleigh optical depths to these measurement problems will be discussed below.

a. Turbidity observations

One of the most common and popular methods of deriving information on the aerosol optical depth is through the use of narrow-band (quasi-monochromatic) measurements of the solar irradiance. Examples of such narrowband instruments are the Volz sunphotometer (Volz, 1959) and spectrobolometer (e.g., Abbot and Fowle, 1908). The aerosol optical depth τ_a may be found from

$$\tau_a(\lambda)=\ln\frac{I_0(\lambda)}{I(\lambda)s}-\tau_R(\lambda)-\tau_g(\lambda), \qquad (4)$$

where I_0 is the extraterrestrial spectral irradiance at wavelength λ (the extraterrestrial response of the instrument), s is the factor to correct I_0 to one astronomical unit, I the measured spectral irradiance or response of the instrument, τ_R the Rayleigh optical depth and τ_g a possible optical depth due to gaseous absorption. All quantities on the right of Eq. (4) can be either measured or calculated.

If the values of τ_R are overestimated, as in the past, the values of the aerosol optical depth will be underestimated. For example, for the Volz sunphotometer the aerosol optical depth is 0.0150–0.0163 greater than it should be at 3800 Å and 0.0048 to 0.0052 greater at 5000 Å. In *Atmospheric Turbidity Data for the World* (1971), occasional negative optical depths occur at 5000 Å of no more than about -0.005. The negative values may be explained as due to an overestimation of the Rayleigh optical depth which thereby gives rise to negative optical depths of aerosols and hence apparent anomalous transmission of the atmosphere. The corrections proposed to the Rayleigh optical depth in this paper do not affect the values of α, the wavelength dependence of the aerosol optical depth using the Volz sunphotometer.

Roosen *et al.* (1973) illustrate many instances from the Smithsonian Institution solar constant program where the transmission exceeds the Rayleigh limit. Although this paper will modify the Rayleigh limit to allow higher transmissions to occur for nearly dust-free atmospheres, it will not explain all the anomalous transmission observations of the Smithsonian Institution. As Roosen *et al.* point out, most of these high transmission values can be explained by scattered light in the instruments.

Many of the very low turbidity values reported for remote sites such as Mauna Loa Observatory, Hawaii, or the South Pole may be too low by as much as 100% of their values (*Atmospheric Turbidity and Precipitation Chemistry Data for the World*, 1972; 1973). Furthermore, all the turbidity values reported for the Volz sunphotometer network in the United States are systematically in error by being 2–100% or more smaller than the true values. Recent broadband filter measurements of turbidity, for example, at Mauna Loa Observatory (Robinson, 1976), also have systematic errors of about 25–30%. It is therefore safer to report atmospheric transmission values (e.g., Abbot and Fowle, 1908) rather than aerosol optical depths if the Rayleigh optical depths are not well known. This statement is particularly true for remote, nearly aerosol-free atmospheres.

b. Total ozone measurements

The deduction of total ozone using the Dobson spectrophotometer requires a knowledge of the Rayleigh extinction. Previous measurements of total O_3 rely upon using Rayleigh optical depths with $\rho_n \approx 0.033$. If indeed the Rayleigh optical depth is lower than previously assumed, measurements using the A, C and D wavelength pairs will be affected. These systematic differences in the measurements of total O_3 compared with the measurements by the AD wavelength pairs have been noted by Dobson (1963) and DeLuisi (1967). Table 2 lists the wavelengths, previous Rayleigh optical depths divided by ln 10, and the present Rayleigh optical depths divided by ln 10 for the July 45°N atmosphere. If one now takes the present values for the Rayleigh optical depth and derives the multiplicative correction factors that will correct the A, C and D wavelength values of total ozone to the AD value, one gets the values in Table 3 for 0.34 cm of total ozone.

TABLE 3. Correction factors, calculated and measured.

Wavelength pair	Calculated correction factor	Measured correction factor
A	1.006	1.002
C	1.015	1.022
D	1.025	1.015

The measured correction factors in Table 3, calculated from the data of Basher (1976) based upon the measurements of Dobson and Normand (1962), have been empirically obtained and previously unexplained. The calculated correction factors agree to within ±1% of the empirically determined values and fall on both sides. There will be errors in the correction factors because the ozone absorption coefficients are slightly temperature-sensitive (Vigroux, 1953), the correction factors are weak functions of the solar zenith angle increasing by about 0.6% at 60°, and the correction factors were taken for 0.34 cm of O_3. Furthermore since the Rayleigh optical depth varies with season, better agreement between the calculated and measured correction factors than given should not be expected. That the agreement between the correction factors as measured and calculated is as good as it is suggests that the new values of the Rayleigh optical depth may be more nearly correct than previous values.

The determination of the total ozone amount using the AD wavelength pairs will be little affected by the new Rayleigh optical depths. In fact, the AD wavelength pair determination of total O_3 as it is now conducted will underestimate the total O_3 by only about 0.1% compared with the true value.

c. *Other radiation calculations and measurements*

The Rayleigh optical depth is used in a wide variety of other measurement problems and theoretical calculations. For example, the flux values for a Rayleigh atmosphere are frequently calculated (e.g., Coulson *et al.*, 1960; Braslau and Dave, 1972). The results of these calculations will need to be modified to account for the lower Rayleigh optical depths. In radiation budget studies of the earth (e.g., Katayama, 1966; Sasamori *et al.*, 1972) a knowledge of the Rayleigh optical depths is needed. In these cases, however, the Rayleigh optical depth is only one component of the total optical depth and in general the aerosol optical depth will be overestimated so that the total transmission of the atmosphere will be nearly correct. Therefore, errors in the Rayleigh optical depth will not seriously affect the computations of this kind where total transmission is sought.

Another problem requiring a knowledge of the Rayleigh optical depth is in lidar measurements. In these measurements the Rayleigh backscattering is subtracted from the total scattering to get the residual backscattering of the laser beam due to aerosols. Past measurements then have underestimated the contributions due to aerosols but will probably deduce the correct aerosol height profile.

5. Conclusions

Recent measurements of the depolarization factor of air show that its value is lower than heretofore assumed. Consequently, the reported values of the Rayleigh optical depth have been too high by 3.5 to 3.7%. It is shown further that the Rayleigh optical depths remain uncertain to ±0.16% due to uncertainties in the depolarization factor for air and seasonal variations in the density of the atmosphere. Other sources of error are negligibly small.

The Rayleigh optical depths are calculated for six standard model atmospheres and the results applied to a variety of remote sensing problems. It is shown that narrow-band measurements of the atmospheric aerosol optical depth are in error. In particular the results of the Volz sunphotometer measurements in the United States underestimate the aerosol optical depth by about 0.016 at 3800 Å and 0.005 at 5000 Å. Depending upon the value of the actual aerosol optical depth these errors range from 2–100% or more of the reported values. Incorrectly reported values of the aerosol optical depth also occur for broadband filter measurements.

The results of the Rayleigh optical depth calculations were also applied to determinations of the total O_3 amount using the Dobson spectrophotometer. The reported and previously unexplained systematic differences in the measurements of total O_3 by different wavelength pairs are explained within the limits of uncertainty of the correction factors by use of the new Rayleigh optical depths. It is also noted that the Dobson spectrophotometer AD wavelength pair determination of total ozone underestimates the true amount of ozone by 0.1%.

Acknowledgment: The author wishes to thank Dr. John DeLuisi for his aid in explaining various aspects of the theory of measuring total ozone with the Dobson spectrophotometer.

REFERENCES

Abbot, C. G., and F. E. Fowle, Jr., 1908: *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. 2. 245 pp.

Ananthakrishnan, R., 1935: Redetermination of the depolarization of light scattering in gases and vapors. *Proc. Indian Acad. Sci.*, 2, 153–160.

Atmospheric Turbidity Data for the World, 1971: Published by Environmental Data Service, Asheville, N. C., 126 pp.

Atmospheric Turbidity and Precipitation Chemistry Data for the World 1972, 1974: Published by Environmental Data Service, Asheville, N.C.

Atmospheric Turbidity and Precipitation Chemistry Data for the World 1973, 1975: Published by Environmental Data Service, Asheville, N.C.

Basher, R. E., 1976: The approximation of particulate scattering coefficients in the determination of total ozone. *Quart. J. Roy. Meteor. Soc.*, **102**, 611–617.

Braslau, N., and J. V. Dave, 1972: Effect of aerosols on the transfer of solar energy through realistic model atmospheres. Part I: Nonabsorbing aerosols. IBM RC 4114, 100 pp.

Bridge, N. J., and A. D. Buckingham, 1964: Polarization of laser light scattered by gases. *J. Chem. Phys.*, **40**, 2733–2734.

Coulson, K. L., J. V. Dave, and Z. Sekera, 1960: *Tables Related to Radiation Emerging from a Planetary Atmosphere with Rayleigh Scattering*. University of California Press, Berkeley, 548 pp.

DeLuisi, J. J., 1967: The effect of haze upon Umkehr measurements. Sci. Rep. No. 3, Dept. of Meteorology, Florida State University, 564 pp.

Dintzis, F. R., and R. S. Stein, 1963: Observations on depolarization ratios of scattered light by some low molecular weight gases and *n*-paraffin vapors. *J. Chem. Phys.*, **40**, 1459–1460.

Dobson, G. M. B., 1963: Note on the measurement of ozone in the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **89**, 409–411.

——, and C. W. B. Normand, 1962: Determination of the constants used in the calculation of the amount of ozone from spectrophotometric measurements and an analysis of the results. *Ann. I.G.Y.*, **16**, 161–191.

Edlen, B., 1953: Dispersion of standard air. *J. Opt. Soc. Amer.*, **43**, 339–344.

Elterman, L., 1968: UV, visible, and IR attenuation for altitudes to 50 km, 1968. AFCRL-68-0153, 49 pp.

Gans, R., 1921: Asymmetrie von Gasmolekeln. Ein Beitragzur Bestimmung der Molekularen Form. *Ann. Phys.*, **65**, 97–123.

Handbook of Chemistry and Physics, 1975: CRC Press, Cleveland, Ohio (56th ed.)

Kasten, F., 1968: Rayleigh-Cabannes-Streuung in trockener Luft unter Berucksichtigung neuerer Depolarisations-Messungen. *Optik*, **27**, 155–166.

Katayama, A., 1966: On the radiation budget of the troposphere over the Northern Hemisphere (I). *J. Meteor. Soc. Japan*, **44**, 381–401.

King, L. V., 1923: On the complex anisotropic molecule in relation to the dispersion and scattering of light. *Proc. Roy. Soc. London*, **A104**, 333–357.

Massoulier, A., 1963: Nouvelles mesures de la depolarisation de la diffusion Rayleigh dans les gaz et les vapors. *J. Phys. Radium*, **24**, 342–344.

Parthasarathy, S., 1951: Light scattering in gases. *Indian J. Phys.*, **25**, 21–24.

Penndorf, R., 1957: Tables of the refractive index for standard air and the Rayleigh scattering coefficient for the spectral region between 0.2 and 20.0 μ and their application to atmospheric optics. *J. Opt. Soc. Amer.*, **47**, 176–182.

Rayleigh, Lord, 1918: On the scattering of light by a cloud of similar small particles of any shape and oriented at random. *Phil. Mag.*, **35**, 373–381.

Robinson, G. D., 1976: Examination of some solar radiation records from the Mauna Loa Observatory. CEM Rep. No. 4189-543, 33 pp.

Roosen, R. G., R. J. Angione and C. K. Klemcke, 1973: Worldwide variations in atmospheric transmission: 1. Baseline results from Smithsonian observations. *Bull. Amer. Meteor. Soc.*, **54**, 307–316.

Rudder, R. R., and D. R. Bach, 1968: Rayleigh scattering by ruby-laser light in neutral gases. *J. Opt. Soc. Amer.*, **58**, 1260–1266.

Sasamori, T., J. London and D. V. Hoyt, 1972: Radiation budget of the Southern Hemisphere. *Meteorology of the Southern Hemisphere*, *Meteor. Monogr.*, No. 35, Amer. Meteor. Soc., 9–23.

U.S. Standard Atmosphere 1962: Govt. Printing Office, Washington, D.C., 278 pp.

U.S. Standard Atmosphere Supplement 1966: Govt. Printing Office, Washington, D.C., 289 pp.

Vigroux, E., 1953: Contributions a l'étude experimentale de l'absorption de l'ozone. *Ann. Phys.*, **8**, 709–762.

Volkmann, V. H., 1935: Messungen des Depolarisationsgrades bei der molekularen Lichtzerstreuung. *Ann. Phys.*, **24**, 457–484.

Volz, F. E., 1959: Photometer mit selen-photoelement zur spektralen messung der sonnenstrahlung und zur bestimmung der Wellenlangenabhangigkeit der dunsttrubung. *Arch. Meteor. Geophys. Bioklim.*, **B10**, 100–139.

Weber, A., S. P. S. Porto, L. E. Cheesman and J. J. Barrett, 1967: High-resolution Raman spectroscopy of gases with cw-laser excitation. *J. Opt. Soc. Amer.*, **57**, 19–28.