De-Aliasing First-Moment Doppler Estimates

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ABSTRACT

A technique to remove the ambiguity in Doppler mean velocity estimates is described. The technique assumes that along a radial, or portion of a radial, the velocity estimates are quasi-uniformly distributed about the mean. If the data do not meet this criterion, the velocities are adjusted such that they are distributed about the mean.

1. Introduction

In many geophysical problems frequency or frequency shifts are related to velocities in a remotely sensed field. Discrete equi-spaced samples of a time-varying function result in a maximum unambiguous frequency equal to one-half the sampling frequency (f_s). Frequencies $>f_s/2$ are aliased into the fundamental Nyquist co-interval ($\pm f_s/2$) and are interpreted as velocities within $\pm f_s\lambda/4$, where λ is the wavelength of transmitted energy. This ambiguity, if uncorrected, results in an incorrect interpretation of the data field. Knowledge of field characteristics must be used to properly assign the value of n from admissable velocities $[=\pm 2n(\text{PRF})\lambda/4+V]$, all indicating the same frequency shift in the fundamental Nyquist co-interval. Here, V is the velocity computed within the Nyquist co-interval and $\pm 2n(\text{PRF})\lambda/4$ the term which must be added to V to obtain the correct velocity estimate. The problem, then, is to correctly assign to every velocity estimate the appropriate value of n. One technique is to select n such that the gradient between a velocity estimate and its already corrected neighbors is a minimum. An alternative technique described here recognizes that in many problems the motions under study represent deviations about a mean state and that the distribution of the radial estimates allows aliasing to be detected and corrected.

2. Technique

Although the technique may be applied to any sample data system, its application to pulse Doppler will be emphasized here for convenience of discussion and illustration. For simplicity, n is further restricted to values of ± 1 and 0. The range of unambiguous velocities derived from the complex time sample for a given pulse repetition frequency (PRF) is given by

$$\pm V_{\max}=(\text{PRF})\lambda/4. \tag{1}$$

As the unambiguous range is also sensitive to the PRF $[R_{\max}=\frac{1}{2}(c/\text{PRF})]$, the range-velocity ambiguities are coupled by

$$R_{\max}=\frac{c\lambda}{8V_{\max}}, \tag{2}$$

where c is the speed of light. In a remote probing system the PRF should be optimized such that both range and velocity folding is minimized. A point of this paper is that it is not necessary to remove the prospect of velocity folding (at the expense of increased range ambiguity); that a real-time technique exists which will recognize and correct velocity folding provided it is not extreme.

Within a height increment H_i, the mean radial velocity V_k is computed from

$$\bar{\bar{V}}_k(H_i)=\frac{1}{k}\sum_{j=i}^{k}\left(M^{-1}\sum_{l=i}^{M} V_l^e\right)_j, \tag{3}$$

where k is the number of radials and M the number of radial velocity estimates $(=V_l^e)$ within the height increment H_i in the jth radial.

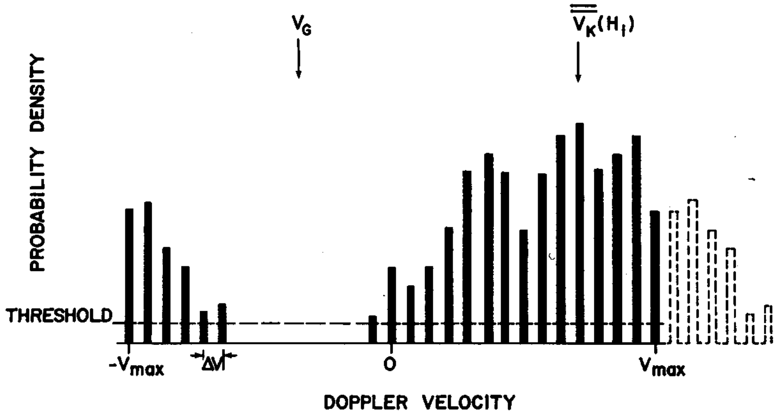

FIG. 1. Sample histogram of mean radial velocity estimates in a radial. The dashed lines represent the histogram resulting from the redistribution of these velocities $< V_G$ after the technique was applied.

As illustrated in Fig. 1, a histogram of velocities in the H_i increment is constructed with any convenient velocity increment (ΔV). A search is made to locate the largest gap (V_G) where there are no occurrences below a prescribed probability threshold. The procedure to obtain the appropriate value of n and hence, the correct velocity estimate V_i^c, is then based upon the following criteria:

$$\left.\begin{array}{ll} V_i^c = 2V_{\max} + V_i^e, & V_i^e < V_G < \bar{\bar{V}}_k(H_i) \\ V_i^c = V_i^e, & V_i^e < V_G > \bar{\bar{V}}_k(H_i) \\ V_i^c = -2V_{\max} + V_i^e, & V_i^e > V_G > \bar{\bar{V}}_k(H_i) \\ V_i^c = V_i^e, & V_i^e > V_G < \bar{\bar{V}}_k(H_i) \end{array}\right\}. \quad (4)$$

The velocities are now distributed about $\bar{\bar{V}}_k(H_i)$.

For application in tornadic storms, an additional check has been found useful. That is, if $|V_i^c - V_{i-1}^c| < |V_{\max}|$, then $V_i^c = V_i^e$ even if the condition in (4) were met. This requires that the radial data spacing not exceed $V_{\max}$ divided by the gradient of the velocity component in the radial direction. This is also the basis for the most widely recognized alternative unfolding procedure. In practice, this additional check has never caused an error and in one instance preserved the azimuthal shear produced by a tornado.

This algorithm has been tested on 564 546 Doppler velocity estimates from the National Severe Storms Laboratory (NSSL) [$V_{\max} = 34$ m s^{-1}] and the University of Chicago and Illinois State Water Survey (CHILL) [$V_{\max} = 26$ m s^{-1}] in which 1154 folded velocities were found. Close examination of the data revealed no algorithm failures. For these data the threshold was set to zero, the velocity increment to 1 m s^{-1}, and the height increment included the entire unambiguous range.

3. Summary and conclusion

One method of correcting folded Doppler velocity estimates is to minimize the gradient between an estimate and the surrounding previously checked estimates. A possible advantage of this technique is its sensibility to the range of velocities in a field, i.e., multiple-folding could exist. The disadvantages are that it is time consuming and that (compared to the technique described here) it is more sensitive to errors in the velocity estimates. A single bad data point is sufficient to incorrectly adjust all velocity estimates downstream.

The disadvantage of the technique described in this note is that if folding is too severe (velocities throughout the Nyquist co-interval) within each height increment, then it will not recognize those folded velocities. Also, as described here, no provision is made for multiple folding within a height increment, although the algorithm may be extended to do so. The advantages are its speed and decreased sensitivity to errors in the data field. In its application, a single PRF or multiple PRF scheme is selected based upon the expected meteorology such that moderate folding is allowed, thus allowing an increase in unambiguous range coverage. It is particularly well suited to the widely used unbiased real-time estimators, such as derived from the autocovariance estimate. Its utility to real-time single and multiple Doppler displays is obvious.