Empirical Predictors for Natural and Seeded Rainfall in the Florida Area Cumulus Experiment (FACE), 1970–1975

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ABSTRACT

The data obtained from the Florida Area Cumulus Experiment in the years 1970–75 are analyzed statistically. Specifically a set of empirically derived predictors for both seeded and unseeded rainfall is identified. First the experiment is briefly described and the data given. The concept of echo motion categories is presented. The responses to be predicted and the variables used as predictors are listed and described and the methods for obtaining the prediction models are given. Next comes a listing of the model equations obtained by those methods, along with some commentary on their possible physical meaning. Examples illustrate the use of some of these prediction models for estimating seeding effects and possible bias in selection of experimental days. A discussion of the echo motion covariate and the basic predictor variables, their histories, rationales and some theoretical indications of their importance completes the main body of the paper.

1. Introduction

In September 1975 the fourth summer experimentation period of FACE was completed, comprising randomized (by days) dynamic cumulus seeding in a 1.3×10^4 km^2 target area in south Florida. The design, physical background, execution and previous results of this experiment have been recorded in the literature (Simpson and Woodley, 1975). The rationale and procedures have been completely described by Woodley and Sax (1976). The main rainfall results for the entire period 1970–75 have been analyzed by Woodley *et al.* (1977). Briefly, positive significance in seeded-control rainfall differences have apparently been attained in a data analysis sense. Some of the stratifications and predictors were identified part way through the four seasons of experimentation. This paper sets forth statistical models using the relevant stratifications and predictors, so that these variables and methods are clearly recorded prior to the commencement of the 1976 experimental season of FACE on 1 June 1976.

In brief recapitulation of concepts and design, days suitable for launching seeding aircraft (two aircraft in 1975) are objectively chosen. Neighboring cumuli are seeded massively with airborne silver iodide pyrotechnic flares (100–1000 g per cloud) with the aim of inducing growth and merger of two or more clouds. The motivation of the experimental series is to determine whether and under what conditions rainfall can be increased by dynamic seeding and also to improve understanding of and ability to simulate cumulus processes and interactions.

Rainfall is evaluated by calibrated radar, checked by a gage network and five gage clusters. The range of validity of this type of measurement in estimating seeding effects has been reported by Woodley *et al.* (1975) and Olsen and Woodley (1975). Rainfall is compared for both floating and total targets. The floating target comprises all radar echoes undergoing a seeding pass by the aircraft and all those other echoes merging with them, so long as the echoes or complex remain within the total target. Identical flights are performed on randomized seed and control days, with the flare release button pressed and "flare" number recorded. The seed decision is known only to the randomizer who secretly arms or disarms the flare rack. The rainfall analyses are completed before the key scientists know whether or not a given day was seeded.

In 1973 a significant covariate was discovered, namely, echo motion. Experimental days were readily stratified into two categories (termed motion categories)

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as follows:

Category 1: Marching days
Category 2: Stationary days

Days could usually be classified using the radar scope tracings and notes thereon routinely provided by the National Weather Service radar observers. Category classification was conducted independently by two individuals not involved in the rainfall analyses. Simpson and Woodley (1975) showed by a one-way analysis of variance that category is a highly significant covariate for both floating and total target rainfall.

Subsequently Biondini (1976) reanalyzed the Florida single-cloud experiment (Simpson *et al.*, 1971), using echo motion as a covariate. He found strong evidence that seeding effects on single clouds depend on the motion category.

The motion category stratification and single-cloud reexamination provided the clues necessary for the FACE 1970–75 data analyses. These analyses comprise the subject of this paper.

2. Description of the method used for generating predictor models

The primary goal is to estimate, on the basis of observed quantities which are physically independent of seeding treatment, how much rain would have fallen in the total target, floating target and nonfloating target, had the seeding treatment been the opposite of what actually occurred. If this can be done, the problem of determining the effect of seeding would be essentially solved. A secondary goal is to elucidate the relationships among the various observed variables. For these purposes the following quantities were observed and recorded for each experimental unit (day).

a. *The responses*

Quantities of primary interest whose values we wish to predict:

1) Total target rainfall TT
2) Floating target rainfall FT
3) Nonfloating target rainfall NFT≡TT−FT

TABLE 1. FACE 1970–75 data.

Date	TRT	CAT	FT	TT	NFT	LFT	LTT	LNFT	c	p	s	n	v
6–29–70	1	1	00.20	03.97	3.77	−1.609	01.379	01.327	03.00	0.208	3.00	0	09
6–30–70	2	2	03.97	08.55	4.58	01.379	02.146	01.522	13.00	0.468	4.10	3	00
7–2–70	1	2	01.37	02.39	1.02	00.315	00.871	00.020	02.00	0.072	5.00	0	00
7–7–70	2	2	00.96	09.26	8.30	−0.041	02.226	02.116	02.00	0.080	3.20	0	00
7–8–70	1	2	12.13	14.64	2.51	02.496	02.684	00.920	04.00	0.210	3.90	0	00
7–18–70	1	2	05.61	10.36	4.75	01.725	02.338	01.558	00.00	0.066	2.70	0	00
6–16–71	1	1	00.28	00.31	0.03	−1.273	−1.171	−3.507	00.00	0.000	3.40	1	13
7–1–71	2	1	00.32	01.94	1.62	−1.139	00.663	00.482	02.00	0.041	2.70	2	12
7–12–71	2	2	00.43	09.27	8.84	−0.844	02.227	02.179	01.00	0.003	2.50	0	00
7–13–71	1	1	01.94	03.68	1.74	00.663	01.303	00.554	02.00	0.001	3.40	0	12
7–14–71	1	1	02.05	06.03	3.98	00.718	01.797	01.381	01.00	0.007	2.60	0	10
7–15–71	2	1	01.18	02.31	1.13	00.166	00.837	00.122	03.00	0.100	3.40	1	08
7–7–73	1	2	01.80	03.17	1.37	00.588	01.154	00.315	03.00	0.001	3.00	1	00
7–16–73	2	2	02.20	04.79	2.59	00.788	01.567	00.952	05.00	0.374	3.30	0	00
7–17–73	2	1	01.93	04.42	2.49	00.658	01.486	00.912	03.00	0.607	3.60	1	12
7–20–73	1	1	07.93	10.04	2.11	02.071	02.307	00.747	04.00	0.262	4.90	3	10
7–25–73	1	1	00.95	01.63	0.68	−0.051	00.489	−0.386	02.00	0.080	3.40	0	10
8–6–73	2	2	05.80	12.13	6.33	01.758	02.496	01.845	01.00	0.083	1.95	0	00
8–9–73	2	1	02.67	03.54	0.87	00.982	01.264	−0.139	23.00	0.058	3.60	2	08
8–22–73	1	1	03.19	04.13	0.94	01.160	01.418	−0.030	00.00	0.051	2.85	0	15
8–25–73	1	1	03.47	05.45	1.98	01.244	01.696	00.683	03.00	0.678	2.80	1	11
8–27–73	1	1	00.37	00.83	0.46	−0.994	−0.186	−0.777	02.00	0.149	4.40	2	12
8–28–73	1	1	00.32	01.12	0.80	−1.139	00.113	−0.223	03.00	0.113	6.05	1	14
9–9–73	2	1	00.16	00.48	0.32	−1.833	−0.734	−1.139	01.00	0.026	3.55	0	10
7–16–71	3	2	01.23	08.61	7.38	00.207	02.153	01.999	00.00	0.034	3.40	0	00
7–21–72	3	1	00.12	00.27	0.15	−2.120	−1.309	−1.897	00.00	0.192	5.00	3	13
8–4–72	3	1	00.28	00.32	0.04	−1.273	−1.139	−3.219	00.00	0.000	1.35	0	10
8–9–72	3	1	02.64	03.74	1.10	00.971	01.319	00.095	00.00	0.011	1.70	0	12
8–18–72	3	1	01.41	03.33	1.92	00.344	01.203	00.652	00.00	0.096	2.30	1	08
6–26–73	3	1	03.37	05.90	2.53	01.215	01.775	00.928	01.00	0.005	3.25	1	12
7–9–73	3	2	03.67	06.59	2.92	01.300	01.886	01.072	01.00	0.101	2.60	0	00
7–26–73	3	2	01.91	03.99	2.08	00.647	01.384	00.732	02.00	0.247	3.75	0	00
8–11–73	3	1	01.87	02.27	0.40	00.626	00.820	−0.916	01.00	0.082	3.95	0	15
8–14–73	3	2	07.41	10.40	2.99	02.003	02.342	01.095	03.00	0.062	2.85	0	00
8–26–73	3	1	02.14	05.86	3.72	00.761	01.768	01.314	01.00	0.341	2.80	2	12
6–21–75	2	2	12.85	15.53	2.68	02.553	02.743	00.986	13.40	0.274	2.75	1	00
6–24–75	1	2	06.29	07.45	1.16	01.837	02.008	00.148	03.90	0.198	4.10	0	00

TABLE 1—(*Continued*)

Date	TRT	CAT	FT	TT	NFT	LFT	LTT	LNFT	c	p	s	n	v
6–25–75	2	1	06.11	10.39	4.28	01.810	02.341	01.454	05.30	0.526	4.35	2	06
6–27–75	1	1	02.45	04.70	2.25	00.89	01.548	00.811	07.10	0.250	4.25	0	08
6–30–75	2	2	03.61	04.50	0.89	01.284	01.504	−0.117	06.90	0.018	1.60	0	00
7–9–75	2	1	00.47	03.44	2.97	−0.755	01.235	01.089	04.60	0.307	2.30	1	08
7–16–75	2	1	04.56	05.70	1.14	01.517	01.740	00.131	04.90	0.194	3.35	0	12
7–18–75	2	1	06.35	08.24	1.89	01.848	02.109	00.637	12.10	0.751	4.85	2	08
7–19–75	1	1	05.06	07.30	2.24	01.621	01.988	00.806	05.20	0.084	4.20	2	10
7–20–75	1	1	02.76	04.05	1.29	01.015	01.399	00.255	04.10	0.236	4.40	0	10
7–23–75	1	1	04.05	04.46	0.41	01.399	01.495	−0.892	02.80	0.214	3.10	0	09
7–24–75	2	1	05.74	06.73	0.99	01.747	01.907	−0.010	06.80	0.796	3.95	0	10
7–26–75	1	1	04.84	09.70	4.86	01.577	02.272	01.581	03.00	0.124	2.90	0	08
7–29–75	1	1	11.86	15.10	3.24	02.473	02.715	01.176	07.00	0.144	3.05	1	05
7–30–75	2	1	04.45	06.21	1.76	01.493	01.826	00.565	11.30	0.398	4.00	0	12
8–13–75	2	2	03.66	07.58	3.92	01.297	02.026	01.366	04.20	0.237	3.35	0	00
8–15–75	1	1	04.22	08.51	4.29	01.440	02.141	01.456	03.30	0.960	3.70	0	08
8–16–75	2	1	01.16	04.17	3.01	00.148	01.428	01.102	02.20	0.230	3.80	0	08
8–19–75	1	2	05.45	08.13	2.68	01.696	02.096	00.986	06.50	0.142	3.40	0	00
8–25–75	1	1	02.02	02.20	0.18	00.703	00.788	−1.715	03.10	0.073	3.15	0	14
8–28–75	2	1	00.82	01.09	0.27	−0.198	00.086	−1.309	02.60	0.136	3.15	0	12
9–11–75	1	1	01.09	02.16	1.07	00.086	00.770	00.068	08.30	0.123	4.10	0	10
9–12–75	2	1	00.28	03.50	3.22	−1.273	01.253	01.169	07.40	0.168	4.65	0	10

Legend for Table 1

TRT	treatment code, 1 = seeded, 2 = random control, 3 = nonrandom control
CAT	echo motion category code, 1 = moving, 2 = stationary
FT	floating target rainfall volume ($m^3 \times 10^7$)
TT	total target rainfall volume ($m^3 \times 10^7$)
NFT	nonfloating target rainfall volume ($m^3 \times 10^7$)
	LFT = log FT, LTT = log TT, LNFT = log NFT
c	echo coverage (percent)
s	seedability (km)
p	prewetness ($m^3 \times 10^7$)
n	earliness (h)
v	average speed of echo motion (knots)

Note: Three experimental days are edited out of Table 1: 17 July 1970, 10 September 1973 and 22 June 1975. There are missing data for the first two days. The last one is a very disturbed "outlier" with respect to c and p. None of the models in Tables 2–5 involve these days. Fig. 1 also has these data points edited out.

4) The logarithms of these quantities, LTT, LFT, LNFT, respectively.

b. The predictors

Quantities assumed to be physically independent of seeding treatment, which may have predictive power. They include:

1) The rainfall in the target area for the 1 h period prior to the first treatment pass. This is called *prewetness* and is symbolized by p.

2) The percent of the area within 100 n mi of the Miami radar covered by echoes at 1400 local time LT. This is called coverage and is symbolized by c.

3) The maximum seedability s of the one-dimensional cumulus model for that day (Simpson *et al.* 1967);

4) The number of hours n between 0900 and 1200 LT during which there were echoes in the target area. This is called "earliness."

5) The speed v of echo motion for motion category 1 days.

c. The two-valued (1, 2) concomitant variable, motion category

In many ways this is the most important "predictor" of them all. For our purposes here, it might better be termed a "stratification variable." All the prediction models were obtained within motion categories. It has been reported elsewhere (Biondini, 1976; Woodley *et al.*, 1977) that motion category is the most important single factor yet found to account for the variability of rainfall in FACE.

d. Seeding treatment (seed versus no seed)

The data are presented in Table 1.

The method for obtaining predictors is as follows: First the data are stratified by seeding treatment and motion category. The five predictors listed in 2b along with all quadratic and cross-product terms (such as pv, etc.) and the term $p^{\frac{1}{2}}$ were taken as possible candidates[3] and thrown into a stepwise regression program

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[3] These were the only prewetness (p) variance stabilization transformations used in the stepwise regression program.

to see which, if any, of the responses listed in 2a could be predicted with just a few terms. The result is a hierarchy of empirical prediction models of generally increasing complexity which come closer and closer to "explaining" the variability of the response variable of interest. Since the object is to come up with a "simple function" to use as a predictor, it was felt that the process needed to be terminated after the first few steps no matter whether there remained any "significant" predictors in terms of α levels for insertion. This is so particularly for the following two reasons: 1) there are very limited sample sizes, particularly for the category 2 cases, and 2) the α levels for insertion and deletion were purposely set at very high levels (0.50 for insertion, 0.75 for deletion). This makes it easy to insert and difficult to delete a variable. The maximum number of variables allowed into the models was, therefore, set, somewhat arbitrarily, at five for the category 1 models (seeded and unseeded), three for the category 2 unseeded models, and two for the category 2 seeded

TABLE 2. Results for Category 1: random controls. $N=14$.

Step No.	Model	r^2	P_α (reg)
	Response—floating target rainfall		
1	$FT=0.6306+1.6062\ ps$	0.6283	7.219×10^{-4}
2	$FT=-0.0255+1.5132\ ps+0.0130\ cv$	0.7017	1.289×10^{-3}
3	$FT=0.1581+2.2105\ ps+0.0130\ cv-0.3520\ pv$	0.7305	3.388×10^{-3}
4	$FT=-0.2198+5.0387\ ps+0.1068\ cv$ $-1.4669\ pv-0.2169\ cs$	0.8344	1.455×10^{-3}
5	$FT=0.0667+6.6333\ ps+0.1346\ cv$ $-1.9219\ pv-0.3580\ cs+0.0129\ c^2$	0.8749	1.885×10^{-3}
Type 1 termination			
	Response—total target rainfall		
1	$TT=2.1181+1.9075\ ps$	0.6389	6.031×10^{-4}
2	$TT=1.0451+5.1996\ ps-18.2743\ p^2$	0.7692	3.146×10^{-4}
3	$TT=3.2413+5.5852\ ps-19.6146\ p^2-0.0696\ sv$	0.8093	6.259×10^{-4}
4	$TT=3.2251+5.4156\ ps-18.8808\ p^2$ $-0.0810\ sv+0.0085\ cv$	0.8306	1.604×10^{-3}
5	$TT=2.7493+6.5703\ ps-19.7165\ p^2$ $-0.0849\ sv+0.0165\ cv-0.5312\ cp$	0.8586	3.010×10^{-3}
Type 1 termination			
	Response—nonfloating target rainfall		
1	$NFT=4.4898-0.2713\ v$	0.2193	9.128×10^{-2}
2	$NFT=16.2833-2.8490\ v+0.1348\ v^2$	0.3673	8.069×10^{-2}
3	$NFT=17.1924-2.8992\ v+0.1327\ v^2-0.0316\ cn$	0.4633	8.942×10^{-2}
4	$NFT=16.2758-2.9193\ v+0.1358\ v^2$ $-0.0334\ cn+0.0592\ s^2$	0.5207	1.221×10^{-1}
5	$NFT=25.4751-3.0548\ v+0.1459\ v^2$ $-0.0325\ cn+0.7558\ s^2-5.0611\ s$	0.6344	9.622×10^{-2}
Type 1 termination			
	Response—log floating target rainfall		
1	$LFT=-1.4692+3.6423\ p^{1/2}$	0.4939	5.061×10^{-3}
2	$LFT=-1.8544+3.8934\ p^{1/2}+0.0036\ c^2$	0.6468	3.265×10^{-3}
Type 2 termination			
	Response—log total target rainfall		
1	$LTT=-0.06633+2.5933\ p^{1/2}$	0.5872	1.393×10^{-3}
2	$LTT=-0.2990+2.4980\ p^{1/2}+0.0446\ c$	0.6861	1.707×10^{-3}
3	$LTT=-1.4247+7.6455\ p^{1/2}+0.0492\ c-4.8480\ p$	0.7708	1.539×10^{-3}
4	$LTT=-1.8639+9.3282\ p^{1/2}+0.0322\ c$ $-6.4291\ p+0.1382\ n^2$	0.8340	1.472×10^{-3}
5	$LTT=-2.2700+10.4926\ p^{1/2}+0.0214\ c$ $-6.7626\ p+0.2734\ n^2-0.7165\ pn$	0.8724	2.038×10^{-3}
Type 1 termination			
	Response—log nonfloating target rainfall		
1	$LNFT=-0.4325+1.5736\ p^{1/2}$	0.2144	9.544×10^{-2}
2	$LNFT=-1.4660=4.9463\ p^{1/2}-4.1657\ p^2$	0.4200	4.998×10^{-2}
3	$LNFT=-2.1636+6.1073\ p^{1/2}-5.5320\ p^2+0.0483\ nv$	0.6190	1.795×10^{-2}
4	$LNFT=-2.7128+6.0073\ p^{1/2}-5.9963\ p^2$ $+0.0524\ nv+0.04665\ s^2$	0.6779	2.47×10^{-2}
5	$LNFT=-4.6421+10.2828\ p^{1/2}-0.1153\ p^2$ $+0.0721\ nv+0.1468\ s^2-2.2151\ ps$	0.7506	3.44×10^{-2}
Type 1 termination			

models. In most cases, these maxima were attained and the stepwise process terminated. This is hereafter referred to as *type 1 termination.* In a few cases, the stepwise regression program terminated by failing to find a significant variable to add (even at the 0.5 level) before the maximum number was obtained. This is called *type 2 termination.*

3. Presentation of prediction models and commentary

Table 2 gives the results for the control category 1 (moving) days—namely, a tabulation of the models produced—in the order of their production (step number) by the stepwise regression procedure. Also given is the square of the simple correlation, r^2, between the predictor (right-hand side of model equation) and response. This is sometimes called the proportion of the response's variation "explained" by the model. Another measure of the "goodness" or "adequacy" of the model is the probability level P_α (reg) of the regression's significance. This number may be interpreted under the standard regularity conditions as the smallest α level which is allowed for rejecting the null hypothesis H_0: the predictor and response are uncorrelated. It is the area under the right-hand tail of the standard F distribution resulting from the analysis of variance table for the regression. The test statistic F is related to r^2 via the equation

$$F=\frac{r^2(N-1-k)}{(1-r^2)k},$$

where N is the sample size and k the number of variables in the model.

Examination of Table 2 gives the impression that the floating target behaves similarly to the total target on category 1 control days. At least these responses seem to depend in roughly the same way on the same predictors. Since there was no seeding on these days, the similarity in these responses is heartening. Another impression is that the four random variables (FT, TT, p and c) are closely related on control category 1 days. This impression is strengthened when one looks at the models obtained by ordinary (nonstepwise) multiple linear regression of FT and TT on p and c:

$$\mathrm{FT}=0.199+6.3252p+0.128c$$
$$r^2=0.657,\quad P_\alpha(\text{reg.})=2.780\times10^{-3},$$

$$\mathrm{TT}=1.367+7.817p+0.102c$$
$$r^2=0.632,\quad P_\alpha(\text{reg.})=4.076\times10^{-3}$$

TABLE 3. Results for Category 2: random controls. $N=8$.

Step No.	Model	r^2	p_α (reg)
	Response—floating target rainfall		
1	$\mathrm{FT}=2.1717+0.0361\ c^2$	0.4808	5.651×10^{-2}
2	$\mathrm{FT}=1.8199+0.0638\ c^2-0.9526\ n^2$	0.7891	2.042×10^{-2}
3	$\mathrm{FT}=3.3384+0.1104\ c^2-0.9919\ n^2-0.7002\ c$	0.8265	5.305×10^{-2}
Type 1 termination			
	Response—total target rainfall		
1	$\mathrm{TT}=7.8485+0.0198\ c^2$	0.1661	3.163×10^{-1}
2	$\mathrm{TT}=13.3697+0.1837\ c^2-2.5218\ c$	0.7325	3.701×10^{-2}
3	$\mathrm{TT}=13.4882+0.2149\ c^2-2.6928\ c-0.6929\ n^2$	0.9177	1.234×10^{-2}
Type 1 termination			
	Response—nonfloating target		
1	$\mathrm{NFT}=6.7263-0.3372\ c$	0.3519	1.211×10^{-1}
2	$\mathrm{NFT}=10.2009-2.0664\ c+0.1180\ c^2$	0.8117	1.538×10^{-2}
3	$\mathrm{NFT}=8.7511-2.1366\ c+0.1170\ c^2+0.2204\ s^2$	0.9176	1.236×10^{-2}
Type 1 termination			
	Response—log floating target rainfall		
1	$\mathrm{LFT}=0.2323+0.1358\ c$	0.4115	8.645×10^{-2}
2	$\mathrm{LFT}=0.7570+0.01669\ c-0.0814\ s^2$	0.5155	1.634×10^{-1}
3	$\mathrm{LFT}=0.6672+0.0824\ c-0.2338\ s^2+4.9620\ p^{\frac{1}{2}}$	0.8361	4.752×10^{-2}
Type 1 termination			
	Response—log total target rainfall		
1	$\mathrm{LTT}=2.0124+0.0019\ c^2$	0.1108	4.204×10^{-1}
2	$\mathrm{LTT}=2.7218+0.0229\ c^2-0.3240\ c$	0.8063	1.652×10^{-2}
3	$\mathrm{LTT}=2.7309+0.0253\ c^2-0.3372\ c-0.0535\ n^2$	0.8884	2.245×10^{-2}
Type 1 termination			
	Response—log non-floating target rainfall		
1	$\mathrm{LNFT}=1.7596-0.0694\ c$	0.2107	2.52×10^{-1}
2	$\mathrm{LNFT}=2.7512-0.5694\ c+0.0337\ c^2$	0.7397	3.456×10^{-2}
3	$\mathrm{LNFT}=1.4593-0.5756\ c+0.0325\ c^2+0.5033\ s$	0.9946	5.550×10^{-5}
Type 1 termination			

A comparison of these models with the corresponding two variable models obtained from the stepwise procedure shows that we have not lost too much in terms of r^2 or P_α(reg.) by going to these somewhat simpler models. It is difficult to choose among the various models on the basis of what we presently know. Perhaps some future theoretical work will help decide, but it is more likely that any choice will be the result of applying the models to independent data.

Table 3 gives the results for the category 2 unseeded days. The internal consistency of the floating target and total target is, if anything, stronger than for the category 1 days. It is, in fact, remarkable that at the end of the third step, we end up with the same variables in the model. [Since there are 15 candidate variables for inclusion, the probability of this occurring "by chance," assuming all candidates were equally likely to be chosen, is $(3/15\times2/14\times1/13)^2=4.83\times10^{-6}$]. It is even more remarkable that the signs and magnitudes of the coefficients are so much in accord. With respect to the predictors studied here, the floating target on the category 2 unseeded days acts much like a slightly scaled-down version of the total target.

Evidence has been amassed and presented elsewhere (Biondini, 1976; Woodley *et al.*, 1977) that motion category portends naturally distinct precipitation modes in FACE. A comparison of Tables 2 and 3 gives strong qualitative support to this evidence and, at the same time, offers some tantalizing clues to some of the operating characteristics of these two modes. For example, the fact that prewetness enters strongly and positively in the category 1 models and hardly at all in the category 2 models is surely indicative of some significant differences in the physics behind the observations. Similarly, the fact that earliness enters strongly and negatively in the category 2 models and hardly at all in the category 1 models seems to indicate a difference in the physics.

Similar calculations (not tabulated here) were undertaken for categories 1 and 2 of the seeded cases. The most important result was that the responses depended on different variables than did the corresponding responses in the control cases.

4. Examples of the use of empirical predictors

Empirical predictors, such as those presented in the previous section, have a number of possible uses. This section will present a few examples illustrating two of those uses. It will be shown how the predictors can be used to estimate the effects of seeding on the various responses and to examine the question of a possible "bias" in the distribution of experimental units across treatments.

The basic idea is to use the empirical predictors to estimate what would have happened had the treatment been other than that which actually occurred. The procedure will be illustrated here by using five of the models given in the previous section for unseeded rainfall to predict what would have happened on the seed days had there been no seeding. The effects of seeding are estimated by looking at the difference between the actual measured response and the predicted response from the chosen model, i.e., the residuals (observed minus predicted). The location and spread of these residuals when compared on seed versus no-seed days can be taken as an indication of the difference between the seed and no-seed samples. This inference procedure is straightforward in principle, provided there is no systematic difference (bias) in the seeded and unseeded samples other than the seeding treatment itself. These bias effects may show up in the predictors and, if so, can be detected and estimated by comparing the predictors themselves on seed versus no-seed days.

Table 4 gives a summary of some of the results from five of the predictor models. The first column lists the source of the model from the tables in the previous section, the second column gives the predictor equation. The next three columns give the basic statistics (sample size, sample mean, sample variance) for the control residuals (actual rainfall minus predicted rainfall). Next come the basic statistics for the seeded residuals. These can be interpreted as the "seeding effects" appropriate for the model in question. Each residual is the actual (seeded) rainfall minus what the predictor says would have occurred had there been no seeding. For the category 1 models there are two sets of data. The first set was calculated with the very disturbed day of 22 June 1975 deleted; the second set (in parentheses below the first) includes that day. The next column tests the seeded versus control residuals for equality of variances (F test) and equality of means (Welch, 1938). The variance tests are two-tailed, the means tests one-tailed. Next come the basic statistics for the predictors themselves—first the control and then the seeded cases. Then come the results of testing the predictors for equality of variances and means.

5. Bases and history of covariate and predictors

a. Motion category

The primary differences in the natural target rainfall on category 1 (marching) versus category 2 (stationary) days is illustrated in Fig. 1. The category 2 days average twice as wet in the total target as do the category 1 days. Although category was only explicitly identified as a significant covariate in FACE just after the 1973 experiment (Simpson and Woodley, 1975), the inverse relation between air motion over a heat source and cumulus development has a venerable observational history over a generation (Malkus and Bunker, 1952; Malkus, 1952, 1963; Garstang and LaSeur, 1968 Bhumralkar, 1973b) and, moreover, can be derived from first principles—*viz.*, the hydrodynamic and thermodynamic equations, as will be documented.

TABLE 4. Summary of partial results from five of the predictor models.*

		Basic statistics on the residuals							
		Control residuals			Seeded residuals			P_α levels for seeding effects	
Source	Predictor for control rainfall	n	$\bar{x}$	s^2	n	$\bar{x}$	s^2	Means	Variances
Table 2, step 2, total target	$TT=1.0451+5.1996\,ps-18.2743\,p^2$	14	−0.0004	1.7768	19 (20)	1.8986 (2.3603)	14.9098 (5.8640)	0.0328 (6.48×10^{-4})	1.76×10^{-4} (0.0166)
Table 2, step 4, floating target	$FT=-0.2198+5.0387\,ps+0.01068\,cv-1.4669\,pv-0.2169\,cs$	14	−0.0005	0.9216	19 (20)	1.8175 (1.563)	10.4464 (11.1689)	0.0173 (0.0331)	3.28×10^{-5} (2.24×10^{-5})
Table 3, step 3, total target	$TT=13.4882+0.2149\,c^2-2.6928\,c-0.6929\,n^2$	8	0.0325	1.1129	6	−0.0717	29.5633	0.4839	5.04×10^{-4}
Table 3, step 3, floating target	$FT=3.3384+0.1104\,c^2-0.9919\,n^2-0.7002\,c$	8	0.0175	2.6611	6	2.9417	14.2380	0.0774	0.0323
Table 3, step 3, non-floating target	$NFT=8.7511-2.1366\,c+0.1170\,c^2+0.2204\,s^2$	8	0.118	0.6671	6	−4.4000	10.3204	0.0149	0.0023

	Basic statistics on the predictors							
	Control predictors			Seed predictors			P_α levels for bias effects	
Source	n	$\bar{\bar{R}}_c$	s^2	n	$\bar{\bar{R}}_s$	s^2	Means	Variances
Table 2, step 2, total target	14	4.4404	5.9358	19 (20)	3.1208 (2.7690)	2.3121 (4.6656)	0.0999 (0.0567)	0.0444 (0.3340)
Table 2, step 4, floating target	14	2.2586	4.6439	19 (20)	1.2904 (1.7507)	2.3231 (6.4389)	0.0766 (0.3244)	0.2776 (0.1038)
Table 3, step 3, total target	8	8.9188	12.1362	6	7.7617	9.5270	0.5554	0.4042
Table 3, step 3, floating target	8	4.1675	12.5090	6	2.5000	0.6583	0.2631	2.69×10^{-3}
Table 3, step 3, non-floating target	8	4.7475	7.4891	6	6.6483	10.0696	0.3045	0.3603

* Values in parentheses include very disturbed day of 22 June 1975; all others exclude this day.

The first theoretical development was in a series of "heated island" papers by one of these authors and her colleagues (Malkus and Stern, 1953; Stern and Malkus, 1953). From the linearized, two-dimensional hydrodynamic equations, an expression for the amplitude and shape of an "equivalent mountain" related to the heating produced by a flat island was derived, *viz.*,

$$A=\frac{\tau}{\Gamma-\alpha}, \tag{1}$$

$$x_0=\frac{U^3}{gsk}. \tag{2}$$

In Eq. (1), for the maximum theoretical mountain amplitude A, τ is the temperature excess (°C), Γ the dry adiabatic lapse rate and α the undisturbed lapse rate far upwind of the island. In (2), x_0 is the conventional decay distance of an exponential function, U the undisturbed wind speed, g the acceleration of gravity, s the undisturbed static stability and k the eddy conductivity of heat. These relationships are clarified in Fig. 2.

Theoretical calculations for the small (5 km wide) island of Nantucket under normal summertime conditions predicted a 700 m high "equivalent mountain" with a 1.5 m s^{-1} wind speed, while with a 9 m s^{-1} wind, the "mountain" was only 25 m high! An extensive aircraft and surface measurement program verified the predictions with regard to the structure of the air flow and the cumulus convection, which was developed to the towering congestus stage in the light wind cases and was suppressed or absent in those of strong wind. The inverse relation between convective development and wind speed was further substantiated by observational programs on the larger (15 km by 20 km) island of Barbados (Chaffee, 1964; Garstang, 1972; Garstang *et al.*, 1975).

This earliest model was linearized, two-dimensional, steady state and dry. Hence, the skeptic would be justified in questioning its applicability to precipitating convection over Florida. The steady-state assumption was removed by Smith (1955; 1957) and the linearity by Estoque and Bhumralkar (1969) with no change in the key relationship. A pioneering attempt to parameterize the nonadiabatic effects of condensational

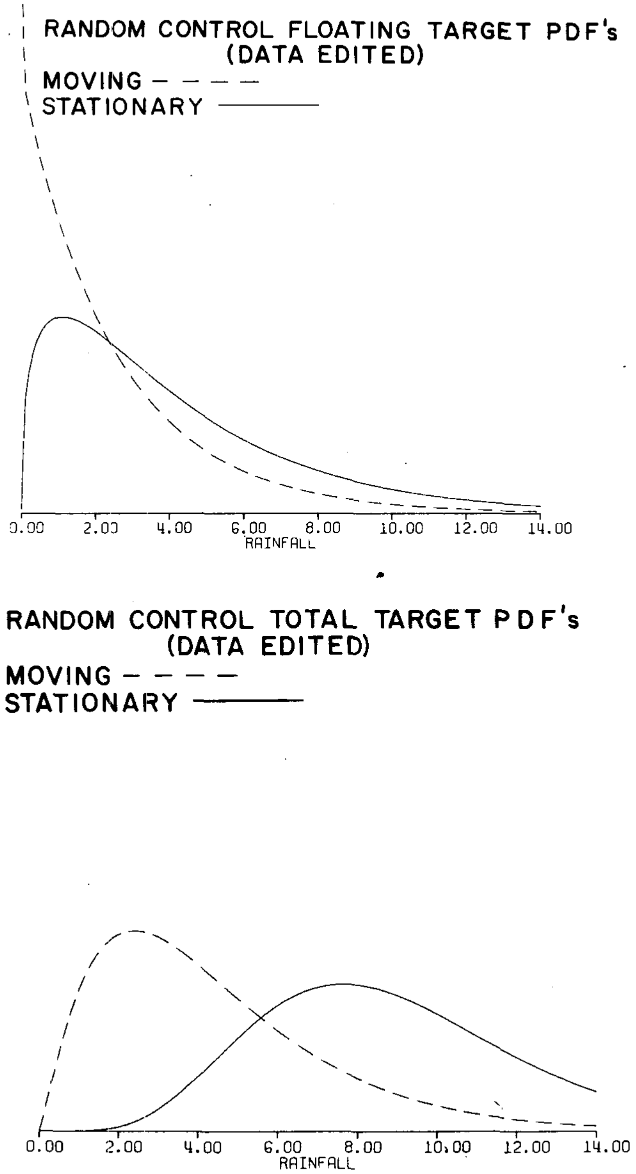

FIG. 1. Illustration of the rainfall population differences between moving (category 1) and stationary (category 2) situations in south Florida using probability density functions for rain volume. Curves are the best-fit gamma distributions to the actual data, determined by maximum likelihood. Abscissa is rain volume ($m^3 \times 10^7$). Ordinate is in units of probability density such that the total area under each curve is 1. The time unit for the rain volume is the 6 h following the first "seeding run." Three experimental days are edited out of the figure, *viz.* 17 July 1970 (no floating target), 10 September 1973 (c, n, v missing) and 22 June 1975 (seeded, outlier with respect to covariates c and p). Part (a) floating target 1970–75; part (b) total target 1970–75.

heating and evaporational cooling in a numerical, nonlinear, time-dependent island model was made by Bhumralkar (1973a), with a detailed observational program on flat Grand Bahama Island, 130 km by 20 km (Bhumralkar, 1973b). The crucial results from both theory and observations are that on light wind days, a line of confluence appears at about 1300 LT over the island, which soon develops showering cumuli. On strong wind days, no theoretical or observed confluence occurs; the cumuli are suppressed with little or no rain.

All the above models suffer from the limitation of two-dimensionality. Pielke and collaborators (e.g., Pielke, 1974; Cotton and Pielke, 1976; Cotton *et al.*, 1976) are developing a fully three-dimensional, time-dependent treatment of the airflow over the south Florida peninsula, with more realistic boundary layer simulations than preceding models.

b. *Predictors*

The five predictors listed in Section 2b do not have a whole generation's history as did the echo motion covariate. Only one, seedability (s), is derivable from a dynamic model. Nevertheless, the first four were defined and used either prior to or beginning from the original design of FACE. The first three were all used as covariates and/or predictors in the Florida randomized single-cloud experiments of 1968–70 (Simpson *et al.*, 1971). For the single clouds, prewetness was defined (Cotton, 1970) as the rain from the cloud for the 10 min prior to the seeding run. This prewetness was found to be a significant predictor for the control cloud rainfall. A second predictor, coverage (c), was also investigated; this predictor was found to contribute only a negligible further reduction in variance for the control single clouds. In the single-cloud experiments, model-predicted seedability was found to correlate beautifully with seeding effect on cloud height (Simpson *et al.*, 1967) and well with the increment in rainfall from the seeded clouds relative to the controls (Holle, 1974; Simpson, 1976). That seedability works as a predictor for unseeded rainfall is at first sight perplexing; however, in virtually all cases of good seedability the upper troposphere is quite unstable, with a slight inversion present at about 500 mb (Simpson and Woodley, 1971). Under these conditions, wide towers, or narrower towers with dynamic or thermal forcing, could grow to great heights.

The variables c (echo coverage) and n (earliness) have a history as stratification variables with dynamic seeding in Florida going back to the original design of FACE in 1970 or earlier. They were empirically recognized as associated with large rainfall, since they were used in an effort to screen out naturally rainy days from the experimental sample. The single cloud results suggested a lessened effect of dynamic seeding in Florida on naturally disturbed or rainy days. In 1968, a disturbed day was defined as a day on which c exceeded 13% (Simpson and Woodley, 1971). The subtraction of earliness from seedability s in the $s-n$ aircraft launch criterion represents an effort to avoid experimentation on disturbed days. Except for 4 of the 48 random GO days in FACE 1970–75 this screening achieved its goal.

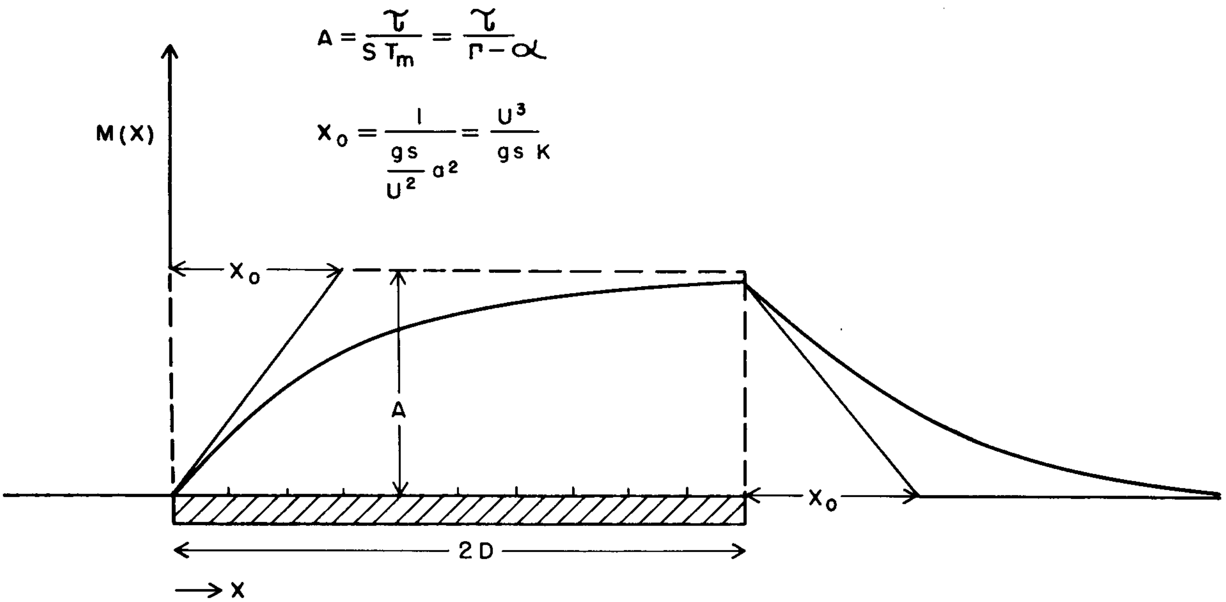

FIG. 2. Equivalent mountain corresponding to rectangular-shaped temperature profile. Elevation of mountain $M(x)$ as function of x is shown by heavy line. Vertical scale (cm) may be determined in a particular case by solving for $A=\tau/(\Gamma-\alpha)$, where τ is amount by which island surface temperature exceeds that of water. Distance of exponential decay, $x_0=U^3(gsk)^{-1}$ [cm], in this example is 0.3 times island width $2D$. This would represent a distance of 3 km if $2D$ were 10 km, $U=2$ m s^{-1}, $s=10^{-7}$ cm^{-1} and $K=2.6\times10^5$ cm^2 s^{-1}, which are typical values of parameters. For 2°C temperature elevation of island, the maximum height reached by "mountain" would be 667 m.

Of all the five predictors, then, the relationship of the first three to unseeded rainfall was long either known or suspected. The soundness of v (echo motion) has already been demonstrated in Section 5a.

Thus, neither the covariate nor predictors used here have been arbitrarily "pulled out of a hat" or "juggled" to reduce the variance. The covariate v has both a theoretical and observational basis, while four of the five predictor variables have a long empirical association with the response variable, rain in the target. Only seedability, involved in dynamic seeding experiments for a decade (Simpson, *et al.*, 1967), is correlated here for the first time with control rainfall.

6. Conclusions

In this paper we have purposely refrained from drawing conclusions of the population inference type even in those cases wherein very "significant" probability levels (P_α levels) are attained. The P_α levels quoted herein should be interpreted in a loose "data analysis" sense, that is, a rough measure of the strength of certain indications *in this data set*—rather than the population inference sense, i.e., the probability level of attaining at least as strong results in independent replications of the experiment. There are two reasons for this reticence: 1) the data need to be thoroughly examined to see if the model assumptions behind the tests are appropriate (this will be included in a follow-on paper); 2) more importantly, the empirical "predictors" listed herein have not been tested as predictors (they might better be termed "estimators"). The 1976 field phase of FACE hopefully will provide a chance to at least partially remedy this drawback. None of the authors of this paper feel that solid population inferences using these predictors are warranted unless and until the predictors are "verified" in some way.

Even if the predictors fail to predict quantitatively in the future, there are important qualitative, data-analysis-type conclusions to be drawn from their use. For example, the variables which were entered and the way in which they were entered give strong evidence (perhaps the strongest yet discovered) that motion categories indicate two distinct precipitation regimes in southern Florida and that seeding effects are different for these two regimes. To the extent that we may take the indications of Table 4 seriously, we may say that on category 1 days, seeding effectively increases the rainfall in both the floating target and total target, while producing no evident effect in the nonfloating target. On category 2 days, seeding evidently produced a positive effect in the floating target, but this was apparently compensated by a negative effect in the nonfloating target, leaving the total target virtually unchanged. There is also some indication of a negative bias in the choice of days; the seeded days were perhaps somewhat naturally drier than the control days. This needs further investigation.

The generation, use and interpretation of covariates and predictors are topics of great and growing concern not only for FACE, but throughout the field of weather modification. It is hoped that the procedures outlined here will find appropriate consideration throughout that community.

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