

## Concerning the Use of Routine Meteorological Data in Estimating Atmospheric Diffusion Parameters

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(Manuscript received 14 February 1977, in revised form 26 October 1977)

### ABSTRACT

By using an integrated form of the Businger-Dyer flux-gradient equations, routine measurements of wind speed and vertical temperature difference in the surface layer are related to low-level atmospheric turbulence and stability. It is suggested that surface wind speed and vertical temperature difference can be used directly to determine the vertical plume diffusion parameter for unstable conditions without using the conventional Pasquill stability index. Their application in the conventional stability scheme, for both stable and unstable conditions, is also discussed in detail.

### 1. Introduction

The importance of specifying the stability of the lower atmosphere in air pollution dispersion analysis is widely recognized. Its determination in terms of routine meteorological data is quite often a matter of practical concern in dispersion model calculations. The most frequently used meteorological data include the surface wind speed, insolation level and vertical temperature difference. Although the methodologies with regard to the use of these data are well known, the underlying rationales are not always clear, and in some cases different methods yield significantly different stability estimates. This article describes an investigation of some of the problems involved in using such data. Much consideration will be directed toward the use of the surface wind speed and the vertical temperature difference as basic determinants of low-level atmospheric turbulence and stability.

It would be incomplete not to mention the sensible heat flux as a basic stability determinant. However, considering the present technical difficulties in making accurate heat flux measurements, especially at relatively low flux levels, it could hardly be categorized as a routine observation. In any event, it is well worthwhile examining the ways in which the traditional meteorological data, such as the insolation level and the vertical temperature difference, are used in the conventional stability classification schemes. And indeed, it is important to relate the sensible heat flux to other routine meteorological observations, e.g., solar altitude and cloud cover, as in the revised Pasquill stability classification scheme by Smith (Smith, 1973; Pasquill, 1974).

Since most of the routine meteorological observations are carried out in the surface layer, it serves as a logical

basis for the present investigation. More specifically, the flux-profile relationships based on similarity principles for the atmospheric surface layer will be called upon to relate routine meteorological parameters to atmospheric stability and turbulence near the ground in discussions to follow.

### 2. Formulation and numerical results

To estimate low-level wind turbulence and atmospheric stability in terms of routinely measured wind speed and temperature data, it is convenient to use the integrated form (Benoit, 1977) of the Businger-Dyer flux-gradient equations (Businger *et al.*, 1971) for the surface layer. For unstable atmospheric conditions, one can express the wind speed at height  $z$  as

$$u(z) = \frac{u_*}{k} \left\{ \ln(z/z_0) + \ln \left[ \frac{(\zeta_0^2 + 1)(\zeta_0 + 1)^2}{(\zeta^2 + 1)(\zeta + 1)^2} \right] + 2[\tan^{-1}(\zeta) - \tan^{-1}(\zeta_0)] \right\}. \quad (1)$$

The vertical potential temperature difference, which is often used in determining stability, can be written in the form

$$\theta(z_2) - \theta(z_1) = - \frac{\alpha H}{k \rho C_p u_*} \left[ \ln(z_2/z_1) + 2 \ln \left( \frac{\zeta_1^2 + 1}{\zeta_2^2 + 1} \right) \right]. \quad (2)$$

For stable conditions, the corresponding equations are

$$u(z) = \frac{u_*}{k} \left[ \ln(z/z_0) + \frac{\beta(z-z_0)}{L} \right], \quad (3)$$

$$\theta(z_2) - \theta(z_1) = - \frac{H}{k \rho C_p u_*} \left[ \alpha \ln(z_2/z_1) + \frac{\beta(z_2 - z_1)}{L} \right]. \quad (4)$$

In the above,  $k$  is the von Kármán constant,  $u_*$  the friction velocity,  $H$  the heat flux,  $\zeta = (1 - \gamma z/L)^{1/2}$ ,  $\zeta_0 = (1 - \gamma z_0/L)^{1/2}$ , and  $\zeta_i = (1 - \gamma_i z_i/L)^{1/2}$  for  $i = 1, 2$ , where  $z_0$  is the surface roughness parameter and  $L$  the Monin-Obukov parameter. The above equations are equivalent to those used earlier by Nickerson and Smiley (1975), with the only exception that in the neutral limit, when  $z/-L$  is very small, Eqs. (1) and (2) remain well-behaved while the Nickerson-Smiley forms become divergent.

For unstable conditions, there are currently two sets of parameters that have been widely used. The Businger parameterization (Businger *et al.*, 1971) assumes  $k = 0.35$ ,  $\gamma = 15$ ,  $\gamma_1 = 9$  and  $\alpha = 0.74$ , while the Dyer-Hicks parameterization (Dyer, 1974; Dyer and Hicks, 1970) corresponds to  $k = 0.41$ ,  $\gamma = 16$ ,  $\gamma_1 = 16$  and  $\alpha = 1.0$ . For the stable case, the Businger parameterization appears to be the only one supported by a substantial data base, and can be expressed in terms of  $\alpha = 0.74$  and  $\beta = 4.7$ .

For a given roughness parameter  $z_0$ , the coupled equations (1) and (2), or (3) and (4), can be solved simultaneously such that the friction velocity  $u_*$  and the heat flux  $H$  can be expressed in terms of the one-level wind speed  $u(z)$  and the vertical temperature difference,  $\theta(z_2) - \theta(z_1)$ , that can be measured on a routine basis.

One immediate application of the solutions to Eqs. (1) and (2) can be realized by using the functional form of  $\sigma_w/u_*$  proposed recently by Panofsky *et al.* (1977): The relationship

$$\sigma_w/u_* = 1.3(1 + 3z/-L)^{1/2} \tag{5}$$

has been found to fit the data well at least for  $0 \leq z/-L \leq 7.5$ . In Fig. 1, computed  $\sigma_w$  values at the 30.5 m level, using Eq. (5) and the solutions based on the

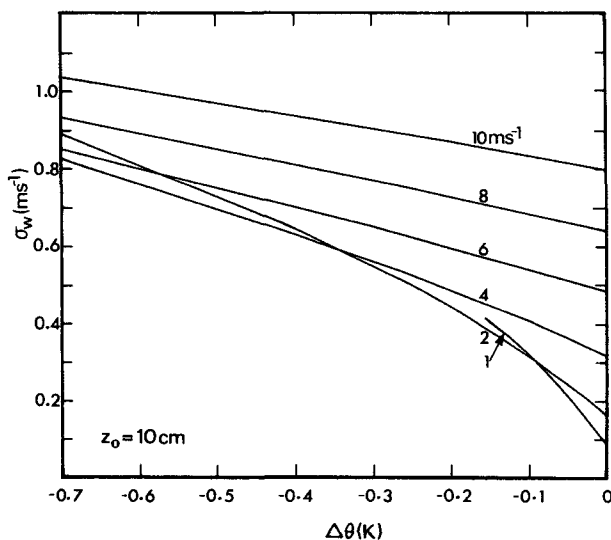


FIG. 1. Variation of  $\sigma_w$  with potential temperature difference between the 6.1 and 30.5 m levels for different values of surface wind speed  $u$ , for the unstable case.

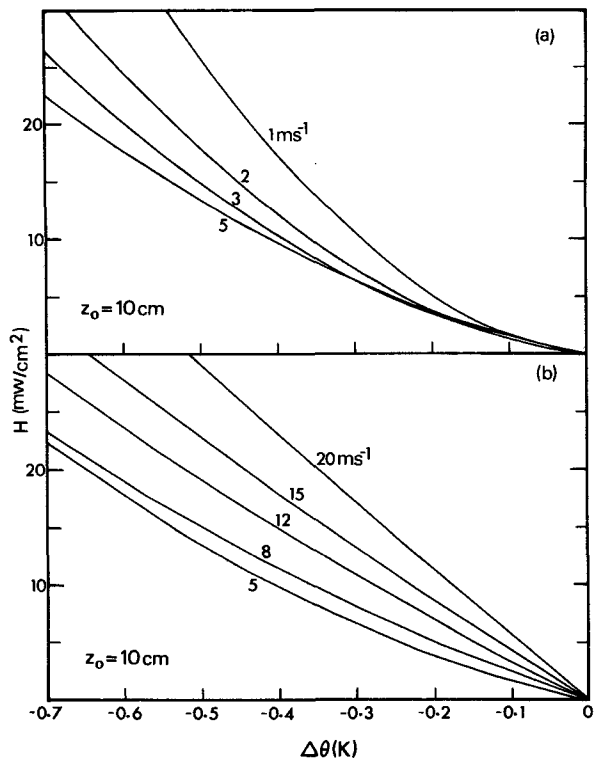


FIG. 2. Computed upward heat flux as a function of  $\Delta\theta$  for various surface wind speeds with  $z_0 = 10$  cm. The  $5 \text{ m s}^{-1}$  curve is plotted in both (a) and (b) for comparison.

Businger parameterization, are displayed as functions of the temperature difference  $\Delta\theta$  measured across the 6.1 and 30.5 m elevations. These temperature sensor levels have been used in the Regional Air Monitoring System network in St. Louis. Following the usual convention in air pollution meteorology,  $u(z)$  is taken to be the surface wind speed at the 10 m level. The surface roughness parameter  $z_0$  is assumed to be 10 cm. The end point of the curve for  $1 \text{ m s}^{-1}$  surface wind corresponds to the upper limit of the  $z/-L$  range at 7.5. Using the method of Draxler (1976), the  $\sigma_w$  values displayed in Fig. 1 can be directly converted to the vertical plume diffusion parameters  $\sigma_z$  for low-level elevated sources.

The above simple method based on measured  $\Delta\theta$  and surface wind speed then serves as a very useful alternative to the conventional methods of determining the vertical diffusion parameter. It has the obvious advantage of not requiring a determination of the Pasquill stability as in the conventional approach. The explicit dependence on  $z$  may also prove to be useful for computing the dispersion of a low-level Gaussian puff.

To illustrate the potential use of measured  $\Delta\theta$  in atmospheric stability determination, heat flux values have been calculated from Eqs. (1) and (2) for different surface wind speeds and  $\Delta\theta$  values. The same roughness parameter and temperature sensor geometries used to calculate the  $\sigma_w$  values (Fig. 1) have been assumed.

TABLE 1. Comparison of Pasquill stability parameter with  $-\Delta\theta/u^2(z)$  for five values of  $\Delta\theta$ . (a)–(e) denote  $\Delta\theta$  values of  $-0.2, -0.3, \dots, -0.6$  K, respectively. Estimates based on Businger parameters are tabulated along with those (in parentheses) based on Dyer-Hicks parameters.

$P$	(a)	(b)	$-\Delta\theta/u^2(z)$ (c)	(d)	(e)
0.5	————	————	————	0.35 (0.22)	0.20 (0.12)
1.0	————	————	0.28 (0.14)	0.14 (0.08)	0.09 (0.06)
1.5	————	0.30 (0.14)	0.11 (0.06)	0.06 (0.04)	0.04 (0.03)
2.0	————	0.09 (0.05)	0.04 (0.03)	0.03 (0.03)	0.03 (0.02)
2.5	0.08 (0.05)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)
3.0	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)

The results displayed in Fig. 2 are based on the Businger parameters. The Dyer-Hicks parameters would lead to somewhat higher heat flux values resulting in slightly lower predicted stability parameters as will be discussed later.

It is important to realize that the physical properties depicted by the curves in Fig. 2 are dependent on the assumed temperature sensor levels in the surface layer. From the figure, it is seen that, for the specific  $\Delta\theta$  measurement levels and roughness parameter considered, the higher wind speed corresponds to a greater negative temperature lapse rate for relatively low wind speeds, if the heat flux level is held constant (or roughly speaking, a constant insolation level). The trend is reversed, however, for wind speeds greater than  $5 \text{ m s}^{-1}$  as shown in Fig. 2b. One can show, however, that if the  $\Delta\theta$  measurement is made at much lower levels, the temperature lapse rate will decrease in magnitude with increasing wind speed for practically all wind speeds with the heat flux held constant. Typically, if the  $\Delta\theta$

measurement is made across the levels of 1 and 4 m, greater wind speed will lead to smaller lapse rate for all surface wind speed greater than  $2 \text{ m s}^{-1}$ , assuming a constant heat flux. Furthermore, the wind speed at which this cross over or reversal of trend occurs depends on the surface roughness  $z_0$ . The lower the roughness parameter, the higher the threshold value of wind speed. For  $z_0 = 3 \text{ cm}$ , for example, this threshold occurs at about  $8 \text{ m s}^{-1}$  assuming the  $\Delta\theta$  measurement levels of 6.1 and 30.5 m. This rather complex relationship between  $\Delta\theta$  measurements, surface roughness and wind speed then points to the potential pitfalls and necessary care in using surface layer  $\Delta\theta$  data to determine atmospheric stability. As will be shown shortly (Fig. 3), for a sufficiently large negative  $\Delta\theta$ , the Pasquill stability can vary widely from A to D depending on the surface wind speed.

For the reason stated above, the  $\Delta\theta$  measurement in the surface stress layer by itself is in general not a useful stability parameter. In this regard, it is interesting to compare the combined parameter  $-\Delta\theta/u^2(z)$ , which is proportional to the so-called modified bulk Richardson number, with the conventional Pasquill stability parameter. Given  $\Delta\theta$  and  $u(z)$ , one can estimate the upward heat flux from Fig. 2 and thereby determine the corresponding Pasquill stability parameter  $P$  (Pasquill, 1974; Smith, 1973). A brief comparison of  $-\Delta\theta/u^2(z)$  with  $P$  is given in Table 1. It is seen that the quantity  $\Delta\theta/u^2(z)$  behaves quite differently from the Pasquill stability parameter. In fact, for a constant temperature difference, it increases (toward zero) much more rapidly as wind speed increases than the Pasquill stability parameter  $P$ ; and it is practically

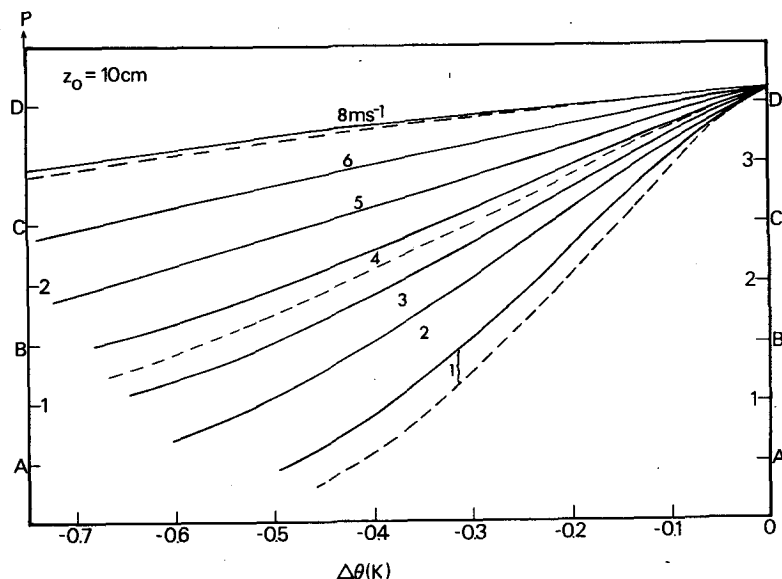


FIG. 3. Stability parameter plotted against  $\Delta\theta$  for various surface wind speeds for unstable conditions. The solid lines are derived using the Businger parameters, and the dotted lines for surface wind speeds of 1, 4 and  $8 \text{ m s}^{-1}$  are based on the Dyer-Hicks parameters.

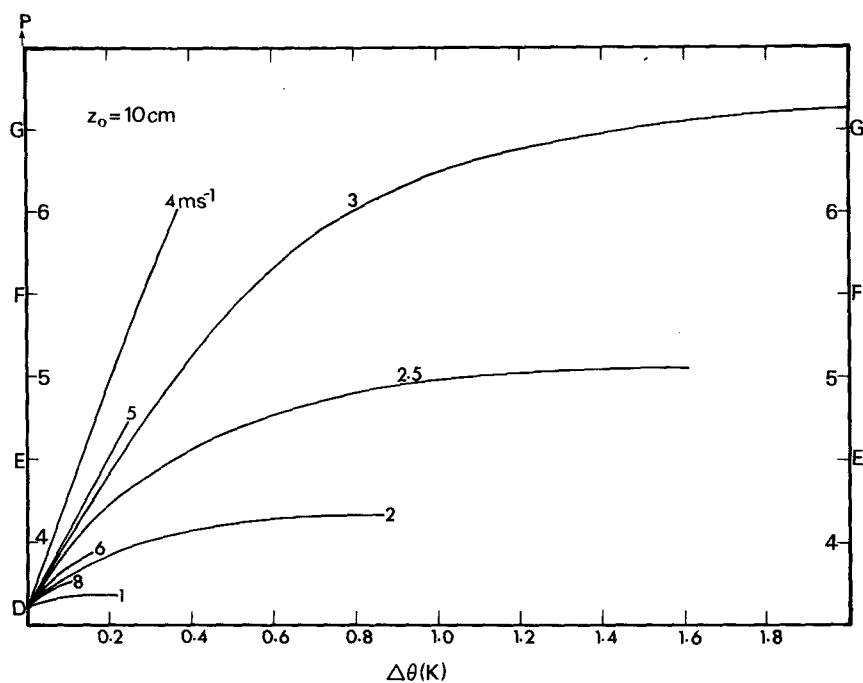


FIG. 4. Stability parameter plotted against  $\Delta\theta$  for various surface wind speeds for stable conditions using the Businger parameters.

independent of  $\Delta\theta$  in near-neutral stability. The unfilled items in Table 1 correspond to the uncertain region where extrapolations of the similarity relationships below  $1 \text{ m s}^{-1}$  wind speed would be required.

By combining the data used in Fig. 2 with Fig. 6.13 in Pasquill's treatise (Pasquill, 1974; Smith, 1973), one arrives at the set of curves shown in Fig. 3. The end points of the curves in the large negative  $\Delta\theta$  region correspond to the high end of the heat flux scale at  $29 \text{ mW cm}^{-2}$  in the Smith figure. Fig. 3 clearly demonstrates the usefulness of the combination of surface wind speed  $u$  (10 m) and low-level vertical temperature difference  $\Delta\theta$  as basic determinants for atmospheric stability. One distinct advantage of using the measurement of  $\Delta\theta$  is the high degree of precision readily achievable in the conventional system of matched thermistors, now widely in use. It should be pointed out that the constant wind speed curves (Fig. 3) are not unique because of the fact that the Businger parameterization used in Eqs. (1) and (2) is not the unique choice. The parameters derived by Dyer and Hicks (1970) with a von Kármán constant of 0.41 are also based on careful measurements and need be considered. It is therefore important to investigate how the two sets of parameters compare in their prediction of the stability level for a given set of  $u$  and  $\Delta\theta$ . Calculations parallel to the ones described above using the Dyer-Hicks parameters yielded somewhat higher heat flux values. The most significant discrepancies are found in the low surface wind speed region. Typically, for  $u$  (10 m) of  $1 \text{ m s}^{-1}$ , the maximum discrepancies are of

the order of 25% with the Businger parameters predicting the lower heat flux values and hence the higher stability parameters. For surface wind speed above  $4 \text{ m s}^{-1}$ , the predicted stabilities are in very good agreement for all practical purposes. For comparison, the  $\Delta\theta$ - $P$  curves based on the Dyer-Hicks parameters are plotted for  $u$  (10 m) = 1, 4 and  $8 \text{ m s}^{-1}$  in dotted lines in Fig. 3.

Similarly, for the stable atmospheric conditions, solutions to the coupled equations (3) and (4) can be combined with the revised Pasquill stability evaluation scheme to generate the set of curves in Fig. 4. They are based on the parameterization of Businger *et al.* (1971). The end points of the curves for surface wind speed  $4 \text{ m s}^{-1}$  and above correspond to the end of the negative heat flux scale of Smith at  $-3 \text{ mW cm}^{-2}$ , while those for surface wind speed below  $3 \text{ m s}^{-1}$  correspond to the analytical limits imposed by the specific log-linear flux-gradient relationships used, implying that no real, physical solutions to the coupled equations exist beyond these end points.

It should be noted that the stability scheme presented above is specific for  $z_0 = 10 \text{ cm}$  and  $\Delta\theta$  determined from the 6.1 and 30.5 m heights. A convenient generalization of the scheme to other roughness parameters has been outlined by Smith (1973) and Pasquill (1974) based on the approximate assumption that the ratio  $\sigma_z(z_0)/\sigma_z(z_0 = 10 \text{ cm})$  is independent of heat flux. To generalize the scheme to other  $\Delta\theta$  measurement configurations, conversions can be made on the basis of the unintegrated flux-gradient equations. The detailed

description of the generalization scheme will be published in a separate article.

### 3. Concluding remarks

In the foregoing discussion, the rationales for the usefulness of the vertical temperature difference as a basic determinant of low-level atmospheric diffusion parameters have been based on similarity principles for the surface layer. The validity of such an approach will therefore depend on how closely the actual situation approaches the conditions of stationarity and horizontal homogeneity. For this reason, it is not expected to have the same broad application as the insolation level used in the conventional Pasquill stability classification scheme, which is determined generally by methods quite independent of the condition of horizontal homogeneity. On the other hand, by avoiding the relatively uncertain link between solar radiation and sensible heat flux, the  $u\text{-}\Delta\theta$  scheme might potentially be more accurate for the more ideal situations. It is also expected to be more responsive to the local site meteorology, especially during evolutionary periods when the conventional stability classification based on insolation may not be responsive enough in dealing with the effect of time lag pointed out recently by one of the authors (Wang, 1977). As a topic for further research, the vulnerability of the  $u\text{-}\Delta\theta$  scheme to varying degrees of deviation from horizontal homogeneity should be

studied. It also remains an important and urgent task to assess in detail the accuracy and applicability of the various stability classification schemes. Undoubtedly, much more comprehensive analyses and a very substantial collection of experimental data will be required.

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