On the Use of Power Laws for Estimates of Wind Power Potential

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ABSTRACT

The evaluation of wind power potential at a proposed aerogenerator site by extrapolation from measured winds at a reference level is investigated. It is shown that the total mean wind power density is not particularly sensitive to the selection of roughness length or power law exponent; over the entire likely range of these parameters the wind power in the mean flow at typical aerogenerator hub heights is within 1.4–4 times the power at a reference height of about 10 m. In lieu of in situ profile measurements, it is suggested that a power law exponent of 1/7 is adequate for realistic but conservative estimates of the available wind power except at extremely rough sites where the estimates may only be conservative.

1. Introduction

The search for viable alternative energy sources has led to many surveys of wind power resources using whatever wind data were available. The concomitant problem of estimating wind power densities from wind speed statistics has been explored most recently in Hennessey (1977); but other important questions, such as the variation of wind power density with height, remain unresolved. A solution to the problem of determining the best way to estimate the variation of wind power with height without measurements of either wind profiles or stability has recently been discussed in the literature (Justus and Mikhail, 1976).

The two simplest expedients for estimating the wind power at wind generator hub height from measurements at a single reference level are 1) to assume the logarithmic wind profile or 2) to assume a power law profile. There are advantages to using $z_0$ and the logarithmic wind profile relation directly rather than the power law and $\rho$ in extrapolating wind-power potential. For instance, $z_0$ but not $\rho$ is independent of height except in complex terrain. In the U. S. literature, power laws are used almost exclusively but without any generally accepted methodology. To cite a few examples, an exponent of 0.14 was used in a wind energy assessment for the mountainous island of Oahu (Hardy, 1977). On the other hand, a value of 0.23 was used in a study covering the continental United States (Justus et al., 1976). Another study adjusted all winds to a 10 m reference height using a power law exponent of 0.2 and then recommended that the readers extrapolate the data using whatever expedient they deem appropriate (Coty, 1976). Recently, the more complicated method of Justus and Mikhail (1976) which involves a power law exponent which varies with the reference wind speed was recommended for general use (Justus et al., 1976; Justus, 1976).

The variation of the power law exponent with surface roughness and stability is summarized in Munn (1966). Empirical methods of correcting for the effects of the variation of the power law exponent with atmospheric stability in order to develop site-specific power law relationships are available (e.g., Touma, 1977), but stability considerations are not too important in the determination of the effective annual-mean power law exponent. Variations in surface roughness between sites can be roughly accounted for when the terrain is flat and fairly homogeneous, but these conditions will seldom occur at wind power sites.

In this contribution we examine the various methods of extrapolating wind power with height for wind power surveys with a view toward developing a standard technique which can be used in all types of terrain and which will result in conservative estimates. Only the wind power density in the mean flow is discussed in this paper; the question of turbulent interactions with aeroengenerators being beyond its scope.

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2. Estimating wind power potential by extrapolation from a reference height

Two forms for the wind power profile at a projected wind power site are in common use. One is taken from a power law assumption,

\[ P/P_r = (z/z_r)^{\beta_0}, \]

and the other from the familiar logarithmic wind profile,

\[ P/P_r = \left[ \frac{\ln(z/z_0)}{\ln(z_r/z_0)} \right]^2, \]

where \( P/P_r \) is the ratio of the mean wind power density at height \( z \) to the mean wind power density at a reference height \( z_r \). In order to solve Eq. (1), \( \beta \) must be determined; and in order to solve Eq. (2) \( z_0 \) must be determined with \( z_0 < z_r \).

Observational studies (e.g., DeMarrais, 1959; Singer and Nagle, 1962; Miller et al., 1971) have indicated that although \( \beta \) may be a decreasing function of wind speed for low wind speeds, it is nearly constant for wind speeds relevant to power generation by the wind. However, it has recently been suggested that the following empirical relationship roughly approximates the behavior of \( \beta \) in terms of a reference level wind speed \( V_r \):

\[ \beta = \beta_0 + \beta_1 \ln V_r, \]

where \( \beta_0 \) and \( \beta_1 \) are constants (Justus and Mikhail, 1976). It was also suggested that these constants are universal and have the values of approximately 0.37 and -0.0881, respectively. However, there is no theoretical basis for this assumption of universality; and, in fact, there is strong theoretical and empirical evidence (e.g., Plate, 1971) that \( \beta \) is a function of the roughness of the surface, which implies that \( \beta_0 \) and \( \beta_1 \) are also functions of surface characteristics. Using linear regression analysis, we estimated \( \beta_0 \) and \( \beta_1 \) from a collection of 80 wind profiles (most of which are 1 h averages) measured over a grassy field (Blackadar and Panofsky, 1970). The wind speeds varied from about 1 to 11 m s\(^{-1}\) and were measured at the two heights of 13 and 26 m. The least-squares estimates of \( \beta_0 \) and \( \beta_1 \) were 0.45 and -0.17, respectively. From 120 wind profiles (10 min averages) taken from the 7 and 23 m heights (wind speed range 0-15 m s\(^{-1}\)) at the 123 m Risg Tower (see Panofsky and Petersen, 1972; Hennessy, 1974; Peterson, 1975), \( \beta_0 \) and \( \beta_1 \) were estimated to be 0.29 and -0.09, respectively. These measurements indicate that the parameters \( \beta_0 \) and \( \beta_1 \) depend on the particular properties of the site and are in no sense universal. In any case, Eq. (3) is not a satisfactory model for \( \beta \) as it can be inferred from noting that the coefficient of determination \( r^2 \) for the linear regression of \( \beta \) on \( \ln V_r \) is only 0.40 for the Blackadar and Panofsky (BP) data and only 0.14 for the Risg 123 m tower data.

According to Justus and Mikhail (1976), the exact value of these constants can be found by dividing each by the factor \([1 - 0.0881 \ln(z_r/10)] \), where \( z_r \) is the reference height in meters.

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FIG. 1. Ratio of total wind power density in the mean flow at 50 m to that at reference heights of 10 or 5 m as functions of surface roughness \( z_0 \) and power law exponent \( \beta \) for neutral stratification.

A far superior empirical relationship can be found simply by assuming that the wind speed at some height is related to the reference level wind speed by

\[ V = \gamma_0 + \gamma_1 V_r. \]

For the BP data the coefficient of determination for this model is 0.99; for some data (28 one-hour average wind profiles) from a 12 m tower at Risg (see Peterson et al., 1976) it was found to be 0.98; and for the Risg 123 m tower it was found to be 0.88. However, in Eq. (4), just as in Eq. (3), the parameters are site specific rendering these models of little practical use because the parameters are not related in any obvious way to easily quantifiable characteristics of a proposed wind power site. If models (1) or (2) are to be used to estimate wind power potential, it is necessary to be able to make estimates of \( \beta \) or \( z_0 \) based on cheaply and easily determinable features of the site not requiring prior wind profile measurements. If wind profile measurements were made, they could be used to determine part of the wind-power profile directly. This would permit extrapolation to higher levels with far greater accuracy than the use of Eqs. (1) or (2).

The data from the three sites analyzed here yielded the following average values of \( \beta \): Blackadar and Panofsky data, grassy field, \( \bar{\beta} = 0.14 \) between 13 and 26 m; Risg, 123 m tower, heterogeneous surface, \( \bar{\beta} = 0.13 \) between 7 and 23 m; Risg, small tower, water surface, \( \bar{\beta} = 0.12 \) between 5 and 12 m. The values of \( \beta \) agree tolerably well with those predicted by the nomogram of Davenport (Plate, 1971). The close agreement in \( \beta \) between the land sites and the water site is due partially to the fact that the observations over water were taken at lower heights.
3. Sensitivity of wind power estimation to assumed values of $p$ or $z_0$

The power law exponent $p$ is a function only of surface roughness $z_0$ and stability (Richardson number). Fig. 1 shows the ratio of total wind power density in the mean flow at 50 m to that at reference levels of 5 or 10 m as a function of either $z_0$ or $p$ for neutral stratification. It can be seen that, for a wide range of values of $z_0$ and $p$ likely to be encountered in the atmosphere, this ratio varies from about 2-4 for a reference level of 5 m (providing $z_{50}/z_0 > 30$) and from about 1.5-4 for a reference level of 10 m (providing $z_{50}/z_0 > 10$). This indicates that estimates of wind power density which are extrapolated from measurements at some reference level well above the surface roughness elements are not very sensitive to variations in $z_0$ or $p$. This lack of sensitivity means that even extreme errors in the estimates of $z_0$ or $p$ will result in errors in the estimation of wind power potential of not much more than a factor of 2. Since an experienced observer can usually estimate $z_0$ to within a factor of about 3, actual errors in the estimate of wind power potential should be much less. Given $z_0$, improved estimates of $p$ are possible (see Fig. 1).

The most recent study of the variation of the value of $p$ with a stability parameter can be found in Touma (1977). In this study data from several nuclear power plant sites were analyzed. The power law exponent was computed between the 10 and 60 m levels, and the Pasquill stability classes were determined from the 10-60 m temperature difference using the method mandated by the Nuclear Regulatory Commission (NRC) Regulatory Guide 1.23. Results derived from this study are presented in Table 1.

The stability averaged annual values of $p$ are appreciably higher than those for the annual values under neutral conditions because of the frequent occurrence of stable stratification (30-55% of all cases). However, for wind power surveys, researchers are looking for sites with strong winds on which to site aerogenerators with cut-in speeds from about 3.6 to 6.6 m s$^{-1}$ (see Section 5); so a better estimate of an average $p$ value for any wind power site will be closer to its average neutral value than to its stability-averaged value.

On the other hand, stability variations notwithstanding, any attempt at precisely estimating $p$ using available techniques [either the nomogram of Daverport (Plate, 1971) or Fig. 1] still requires some estimate of the surface roughness $z_0$. Although wind-power estimates are not too sensitive to $z_0$ or $p$ (provided the reference level is more than 10 times the roughness length), assuming a value of 1/7 for $p$ should usually yield conservative but reasonable estimates of total wind power, except where the roughness of the site is large compared to the height of the reference level.

4. Estimation of wind power potential for flow over heterogeneous terrain

The assumption that the wind speed increases logarithmically with height is useful near the ground over relatively flat, homogeneous terrain, but it is not valid for complex terrain (e.g., Jackson and Hunt, 1975; Peterson et al., 1976; Jensen and Peterson, 1978). However, some of the best wind power sites will be located in rough terrain, on ridgelines, hills, mountains, capes or escarpments, where little is known about the shape of the wind profile. A major factor in determining the wind profile over a potential wind-power site such as a hilltop is the pressure field, so the problem cannot be simplified to just studying the interaction of the wind with the roughness of the surface as it can be for homogeneous terrain. Given only a reference level wind, present modeling techniques cannot adequately predict the wind and power profiles over complex terrain. Three-dimensional potential-flow theory and wind tunnel models may help in determining whether or not wind speeds do significantly increase with height above a hill, but determination of the actual power characteristics of the wind at a particular site still require tedious field measurements of the wind profile. Until such measurements are actually made, it seems prudent to follow some conservative practice such as using the 1/7 power law.

5. Estimating various wind-power densities at aerogenerator hub height

In this section the effects of using various methods of extrapolating wind speed on wind power estimates are illustrated. The data are from the meteorological tower at Risø which has anemometers at several levels including 7, 23 and 56 m. A complete description of this tower and its instrumentation can be found in several papers (e.g., Panofsky and Petersen, 1972). The 56 m level is near the hub height of most of the currently planned aerogenerators. Using 120 observed wind speed profiles selected from the record so as to result in a sample spread uniformly throughout one year (1967), the mean and standard deviation of both the total wind power density and the usable wind power density were computed for the 56 m height. Next, the 56 m wind power density was estimated by various methods of

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**Table 1.** Annual average power law exponents $p$ at several locations for 1) the weighted average over all stability classes and 2) the neutral case (Pasquill stability class D). Derived from data given in Touma (1977).

<table>
<thead>
<tr>
<th>Site</th>
<th>Missouri</th>
<th>Kansas</th>
<th>Iowa</th>
<th>Texas</th>
<th>Michigan</th>
<th>Lake Erie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrain</td>
<td>Rolling</td>
<td>Rolling</td>
<td>Rolling</td>
<td>Rolling</td>
<td>Rolling</td>
<td>Windward shore</td>
</tr>
<tr>
<td>type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)</td>
<td>0.23</td>
<td>0.26</td>
<td>0.25</td>
<td>0.31</td>
<td>0.18</td>
<td>0.26</td>
</tr>
<tr>
<td>2)</td>
<td>0.13</td>
<td>0.19</td>
<td>0.19</td>
<td>0.17</td>
<td>0.12</td>
<td>0.21</td>
</tr>
</tbody>
</table>

* Computed from Tables II and III, Touma (1977).
* Computed from Table V, Touma (1977).
extrapolating the 7 m wind power density, and estimates of the total and usable power density were computed. These results are presented in Table 2.

The distinction between usable and total wind power densities has been drawn in Hennessey (1977). Total wind power density in the mean flow is computed from

$$P = \frac{1}{2} \rho V^3,$$

where $V$ is the mean wind speed and $\rho$ the air density. On the other hand, usable wind power density in the mean flow is computed from

$$P = \begin{cases} 
0, & V < V_0 \\
\frac{1}{2} \rho V^3, & V_0 \leq V \leq V_1 \\
\frac{1}{2} \rho V_1^3, & V_1 < V < V_2 \\
0, & V \geq V_2
\end{cases},$$

where $V_0$ is the cut-in speed, the speed at which the aerogenerator begins generating electricity; $V_1$ is the rated speed at which the maximum possible power is generated; and $V_2$ is the cut-out speed at which the aerogenerator is shut down.

For this illustration usable wind power densities are computed for both the NASA 100 kW and the GE 1500 kW aerogenerators. These two types span the range of currently planned large-scale wind energy conversion systems (WECS).

The following four methods were used to estimate the 56 m wind speed to make wind power estimates from these tower data:

1) Values of $\rho$ were computed from the observed winds at the 7 and 23 m levels, and then the wind power density was extrapolated to the 56 m level for each of the 120 observed 7 m wind power densities using these "observed" values.

2) Extrapolation of the 7 m wind speed using the 1/7 power law.

3) Extrapolation using $\rho$ values determined by the method of Justus and Mikhail (1976) [see Eq. (3)] computed from each 7 m wind speed observation.

4) Extrapolation based on a conservative estimate of the surface roughness length $z_0$, using either the logarithmic wind profile or the equivalent value of $\rho$ estimated from Fig. 1 or Davenport's nomogram (Plate, 1971).

The prevailing winds at Risø are westerly, so for a conservative estimate of $z_0$ we took the low value of 2 cm reported by Panofsky and Petersen (1972) for the westerly quadrants. This 2 cm roughness length results in $\rho = 0.15$ using the nomogram of Davenport; so, at least at Risø, this method is nearly equivalent to the 1/7 power law.

In the vicinity of the Risø tower the terrain features are no more than a few meters high, and it is not unlike a relatively smooth site which might be chosen for an installation along a lee coastline. There is water on three sides which indicates that there will be internal boundary layers in the overlying air flow causing the wind speed profiles to depart from the classical logarithmic shape typical of flow over homogeneous terrain. The size of the roughness elements changes rather drastically around this tower. It is, therefore, reassuring that the various power estimates based on the observed $\rho$ values in the 7–23 m layer are as accurate as they are.

As expected, the traditional 1/7 power law is best at this relatively smooth site. Except for this feature, there is little difference between the two methods based on constant $\rho$ or $z_0$ values.

Clearly, the method for wind power surveys developed in Justus and Mikhail (1976) results in the worst estimates, overestimates in this case. The method is unnecessarily complicated and will poorly estimate the wind power potential in many cases, so it is untenable on practical as well as theoretical grounds.

6. Conclusions

The foregoing analysis leads to the following conclusions:

1) The extrapolation of wind power potential with height is a rather trivial matter whenever the terrain is not too complex.

2) If the wind is measured at a level at least an order of magnitude higher than the roughness elements, then estimates of total wind power at projected aerogenerator hub heights are not very sensitive to variations between

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4 Usable power is more general than the potential output defined by Justus et al. (1976) which depends on the complete operating characteristics of specific machine types.

4 For the NASA 100 kW type aerogenerator the hub height characteristic speeds $V_0$, $V_1$, and $V_2$ are, respectively, 3.6, 8.0 and 26.8 m s$^{-1}$ (Justus et al., 1976), and for the GE 1500 kW they are, respectively, 6.6, 13.1 and 29.9 m s$^{-1}$ (Justus, 1976).
the methods for extrapolating from the reference-level wind speed.

3) The method of Justus and Mikhail (1976) should not be used because it is not only theoretically unsound, but it also introduces the greatest uncertainty into the estimates of wind power potential.

4) Use of the 1/7 power law should yield usually conservative yet reasonable wind power estimates in situations where the roughness length for the site is at least an order of magnitude smaller than the height of the reference level.

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REFERENCES


