NOTES

A Comparison between a Steady-State Downdraft Model and Observations Behind Squall Lines

B. F. RYAN AND J. C. CARSTENS[1]

Division of Cloud Physics, CSIRO, Sydney, Australia

8 April 1977 and 26 November 1977

ABSTRACT

A simple parcel model has been used to examine the possibility that observed profiles of θ_e following the passage of a squall line are produced by air originating in middle-level clouds and descending in steady rain. The model predicts realistic profiles of θ and q.

The calculations suggest that in meteorological situations such as squall lines the thermodynamic properties of air associated with evaporating downdrafts can be parameterized by this simple model.

1. Introduction

In disturbed conditions there is a well-mixed layer that is transformed by the passage of a precipitating storm or a squall line disturbance into a cooler and more stable structure, and several observational programs have been designed to look at the changes in the subcloud layer following such occurrences. For example, Zipser (1969) showed that air of middle tropospheric origin descended in unsaturated downdrafts and noted that the recording of the lowest equivalent potential temperatures coincided with observations of heavy rain from middle-level clouds. Behind the squall front there were regions where the equivalent potential temperature was approximately constant with height.

Betts (1976) has examined data from a series of squalls that occurred during VIMHEX 1972 (Betts and Stevens, 1974; Betts and Miller, 1975) and found profiles of equivalent potential temperature similar to those found by Zipser (1969). He suggested that in the absence of mixing and radiative heat sources the trajectories of the air parcels should follow lines of near-constant equivalent potential temperature.

2. A simple model of a steady-state downdraft

The near-vertical profiles of equivalent potential temperature suggest that over a limited period of time the change in the thermodynamic properties of the air can be studied with a one-dimensional steady-state parcel model similar to that used by Kamburova and Ludlam (1966). Such a model has the following conservation equations.

Mass continuity

$$\rho_a w = \text{constant}, \tag{1}$$

where ρ_a is the density of air and w the vertical velocity of the air parcel.

Constant flux of drops

$$N_i(v_i+w) = \text{constant}, \tag{2}$$

where N_i is the number of drops per cubic meter in the size interval i and v_i is the fall speed of those drops.

Conservation of water mass

$$\rho_a wq + \sum N_i(v_i+w)\tfrac{4}{3}\pi r_i^3 \rho_w = \text{constant} \tag{3}$$

or

$$\rho_a wq + \rho_w RR = \text{constant}, \tag{4}$$

where r_i is the radius of the drop, q the mixing ratio, ρ_w the density of water and RR the rainfall rate.

Conservation of equivalent potential temperature

$$\theta_e = \text{constant}. \tag{5}$$

The change in the mass of a droplet of radius r_i with pressure is given by

$$\frac{dm_i}{dp} = \frac{4\pi r_i \Delta e DF}{R_w T(1-e/p)\rho_a g(v_i+w)} \tag{6}$$

(see Kamburova and Ludlam, 1966), where Δe is the vapor pressure excess, D the diffusion coefficient, R_w is the gas constant for water vapor, T the absolute temperature, p the pressure of the dry air, g the acceleration

[1] Present affiliation: Graduate Center for Cloud Physics Research, University of Missouri, Rolla.

0021-8952/78/0395-0400$0.500

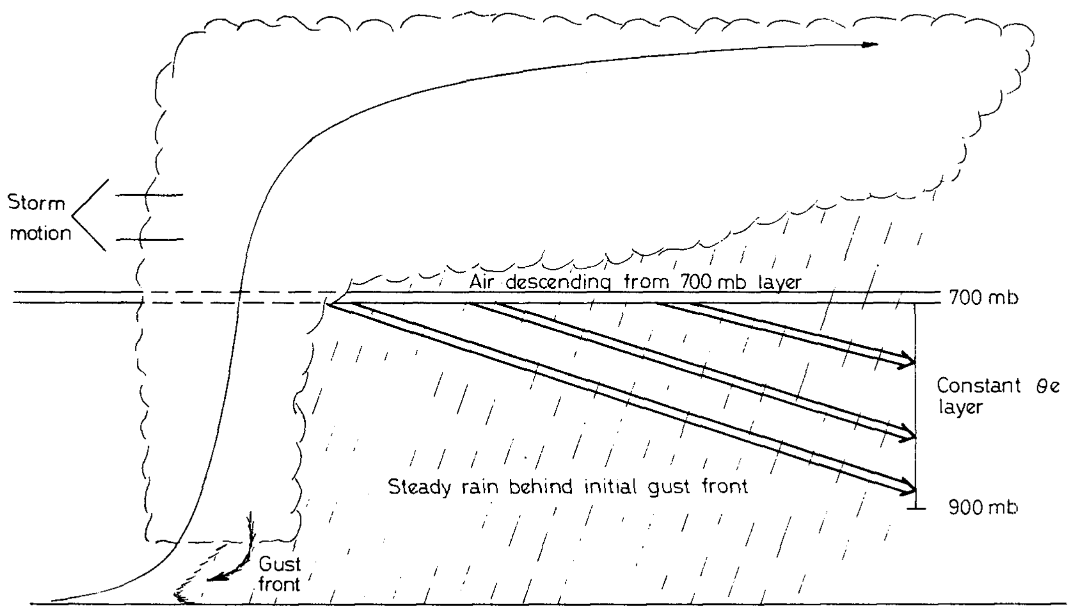

FIG. 1. Schematic diagram of the air flow relative to the travelling squall line. The descending air behind the squall is modeled as shown in the figure.

due to gravity and F the ventilation coefficient given by Beard and Pruppacher (1971);

$$F=\begin{cases} 0.78+0.308\ \mathrm{Sc}^{\frac{1}{3}}\ \mathrm{Re}^{\frac{1}{2}}, & \mathrm{Re}\gtrsim 2 \\ 1.00+0.108\ (\mathrm{Sc}^{\frac{1}{3}}\ \mathrm{Re}^{\frac{1}{2}})^2, & \mathrm{Re}<2 \end{cases}$$

where Re is the Reynolds number and Sc the Schmidt number.

With this simplified model the vertical profile of θ and q can be deduced once an initial raindrop distribution is chosen. VIMHEX 1972 data have been used to test the model predictions.

3. Application of the model

Fig. 1 shows a schematic sketch of the assumed air flow relative to a squall line. The mid-level tropospheric air behind the squall front is that being modeled in the present paper. It is assumed that low θ_e air in the vicinity of 700 mb is advected into the rain coming from the mid-level clouds. This air is subsaturated and descending as a result of buoyancy and pressure field forces. Five storms observed during VIMHEX have been reported in considerable detail by Betts *et al.* (1976) and Ruiz (1975). In all storms there was a region of constant θ_e between cloud base and the density current at the ground (see Fig. 2). Further, the rawinsonde data are considered to represent good line samples through the storm, and apart from storm 60 there was a period of steady rain preceding or during the rawinsonde ascent. However, there are no observational records on the downdraft w or the rainfall rate in the region of the atmosphere where the equivalent potential temperature was constant. It is therefore necessary to deduce these parameters in order to make use of the model.

If it is assumed that the depth of the constant equivalent potential temperature layer is proportional to the period of steady rain, a downdraft w can be deduced by making the assumption

$$\frac{\Delta p}{\Delta t}\approx -\rho g w, \tag{7}$$

where Δp is the depth of the layer and Δt the rain period. In the abovementioned storms the downdrafts estimated in this way varied from 0.5 to 1.5 m s^{-1} (see Table 1); it is of interest that Miller and Betts (1977) find that the mesoscale downdrafts predicted in numerical squall line simulations are of this order.

There is no way of deducing a droplet distribution at cloud base and therefore for the purposes of the present study a Marshall-Palmer distribution has been assumed, i.e., $N=N_0\exp(r/r_0)$ with r_0 chosen to be 228 or 119 μm. An estimate of the rainfall rate at cloud base was obtained from the observed surface conditions by using Eq. (4) (see Table 1). On the basis of these estimates various rainfall rates were assumed for the model calculations. When r_0 is also assumed N_0 can be deduced.

4. Results

Fig. 2 shows the observed profiles of θ_e before and after the passage of the storms. Storms 47 and 64 have a region where profiles of θ_e after the storm are very nearly constant whereas those after storms 35 and 108/109 show fluctuations of ± 0.5°C about the mean over the region of interest. Fig. 3 shows the smoothed profiles.

TABLE 1. Assumed and observed parameters used in the model.

	Observations					Values estimated from Eqs. (4) and (7)		Values assumed in model calculations		
	p Initial (mb)	p Final (mb)	q Initial (g kg^{-1})	q Final (g kg^{-1})	RR Ground (mm h^{-1})	RR* Cloud base (mm h^{-1})	W** (m s^{-1})	RR Cloud base (mm h^{-1})	W (m s^{-1})	r_0 (μm)
Storm 35	725	850	10.2	11.6	34	41	0.79	41	0.79	228
								41	1.5	228
Storm 47	725	825	9.5	11.8	8.6	19	0.56	19	0.56	228
								19	0.08	228
								24	0.56	228
Storm 64	700	825	8.0	9.7	5.1	20	1.2	20	0.9	228
								21	1.2	228
								26	1.2	228
								26	1.2	119
Storm 108/109	825	950	11.5	14.5	14	36	0.9	36	0.9	228

* Estimated from Eq. (4).
** Estimated from $\Delta p/\Delta t$.

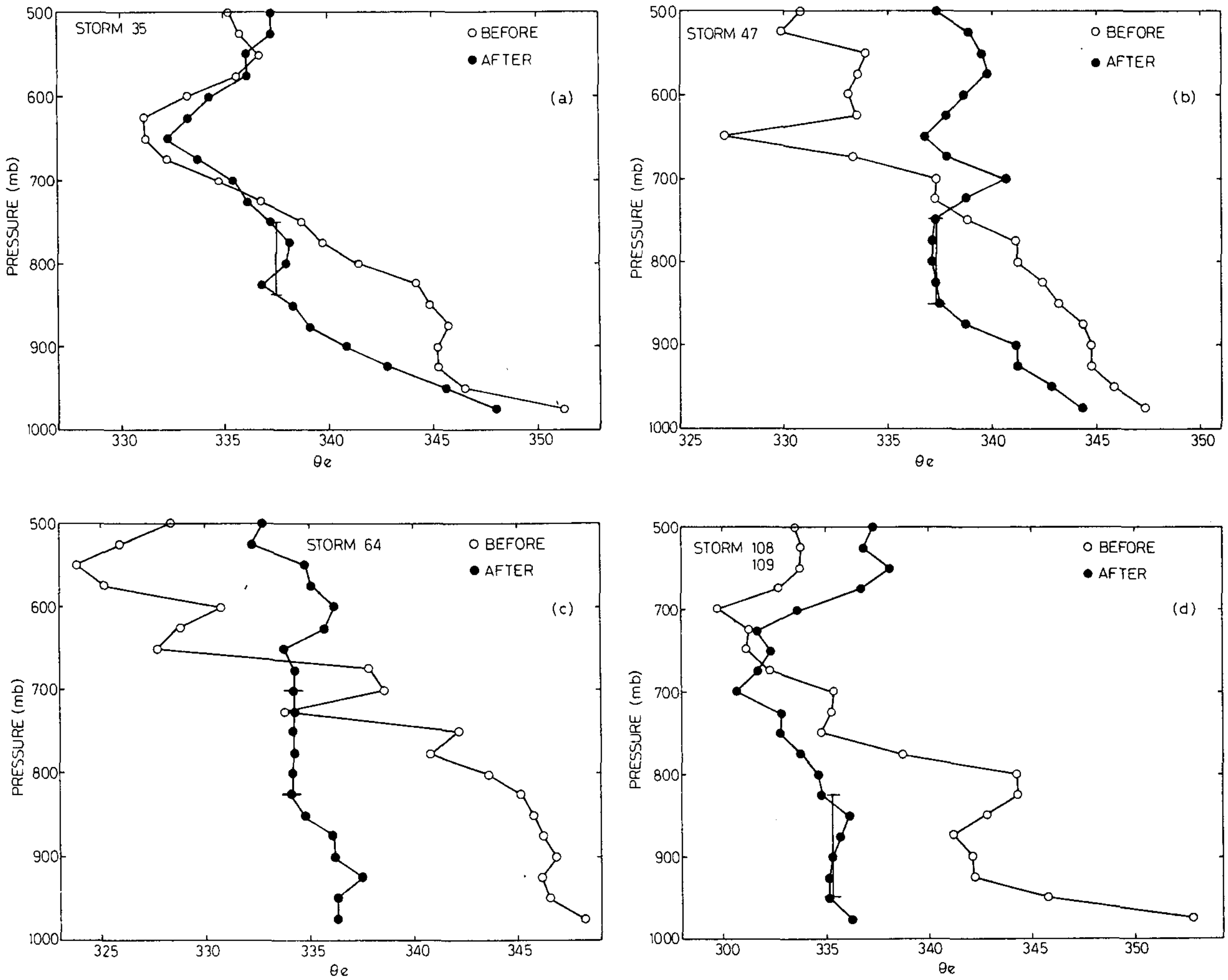

FIG. 2. Equivalent potential temperature θ_e as a function of pressure before and after the passage of storms 35, 47, 64 and 108/109.

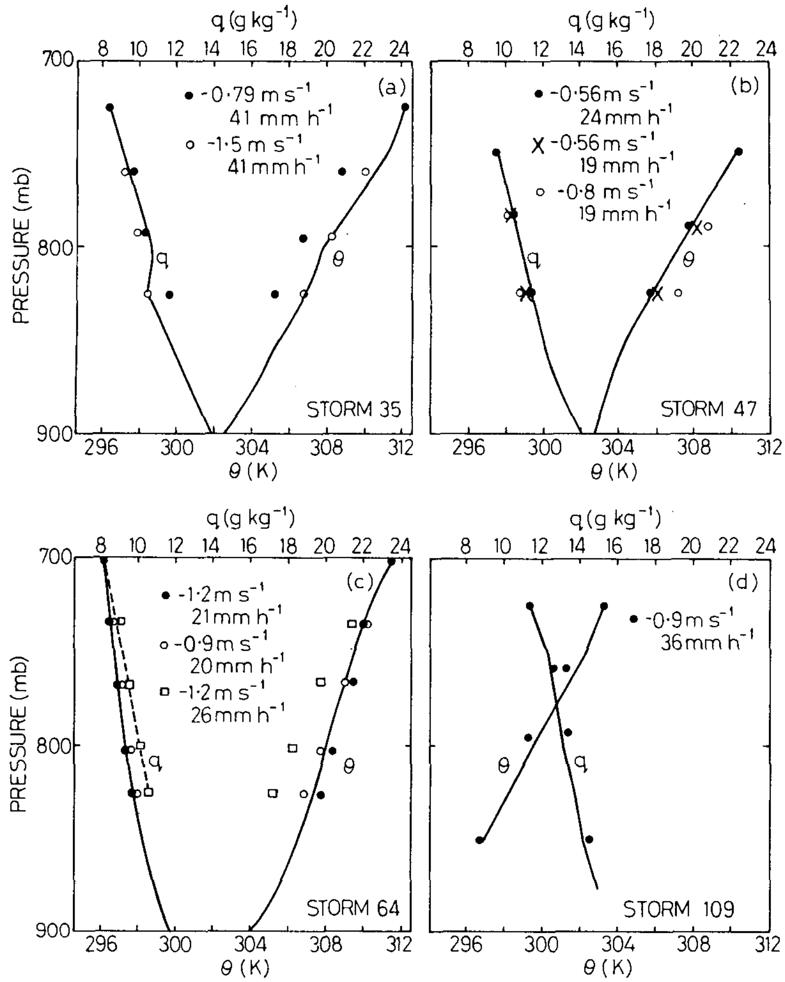

FIG. 3. Observed profiles (solid lines) of θ and q as a function of pressure following the passage of storms 35, 47, 64 and 108/109. The calculated values of θ and q are given at 25 mb intervals for a number of initial downdrafts and rainfall rates.

of potential temperature and mixing ratio which were observed over the region where θ_e was conserved. The values of θ and q calculated from Eqs. (1)–(5) are shown at 25 mb intervals on the same figure. The calculated

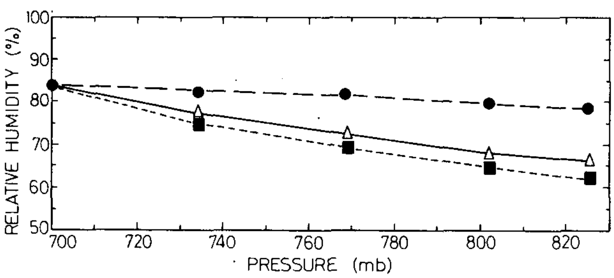

FIG. 4. Relative humidity as a function of height for storm 64. The circles represent a Marshall-Palmer distribution with $r_0=119$ μm and rainfall rate of 26 mm h^{-1}, the triangles a Marshall-Palmer distribution with $r_0=228$ μm and a rainfall rate of 26 mm h^{-1}, and the squares an initial Marshall-Palmer distribution with $r_0=228$ μm and a rainfall rate of 21 mm h^{-1}.

profiles are found to be sensitive to the choice of r_0 but relatively insensitive to a variation in the rainfall rate of up to about 25%. This is shown for storm 64 in Fig. 4, where the relative humidity is shown as a function of pressure for different values of r_0 and rainfall rate. It is worthwhile noting that the initial rainfall rate defined by the conservation Eq. (4) implies that if the computation was extended to the surface the model would have the observed rainfall rate. The model is also dependent upon the value assumed for the downdraft.

Clearly, the model cannot give a unique solution to the problem with the limited data available. For storms 47 and 64 the observations showed both q and θ, to be very nearly linear functions of pressure over the range for which θ_e was observed to be constant. It is perhaps significant that the model calculations produce a profile for q that is approximately linearly related to pressure; consequently, since θ_e is conserved, θ must be proportional to $p^{0.72}$. For these two storms the assumed choice of rainfall rate and downdraft leads to predicted values close to the observed profiles of θ and q. For storms 35 and 108/109, on the other hand, the θ_e profiles are not nearly so well approximated by the constant θ_e assumption and it is therefore difficult to match a calculated profile to observations.

5. Discussion

The accuracy of the model predictions is likely to depend on the validity of the assumptions made regarding the droplet spectrum. The first assumption is that tropical rain has a Marshall-Palmer distribution; however, there is some evidence (e.g., Blanchard, 1953) to suggest that the droplet size distribution in nonfreezing rain is not well represented by an exponential law. The second assumption is that for the time scales involved with the model coalescence and breakup can be ignored. Fig. 5 shows the calculated changes in the two droplet distributions used for storm 64. By the time the parcel has traversed a 100 mb interval the Marshall-Palmer distribution is starting to become distorted, although not so distorted that its general characteristics are destroyed. For droplet radii $>300\ \mu m$ the slope is well preserved. However, if in reality the Marshall-Palmer distribution were strictly preserved the model calculations would lead to errors in the thermodynamic properties in a way that would tend to underestimate q and overestimate θ.

Betts (1976) in his paper on the parameterization of the tropical subcloud layer suggested that following the passage of a storm the profiles of θ_e and θ_{es} should be approximately constant. In an evaporating downdraft the model calculations give profiles of θ and q that are

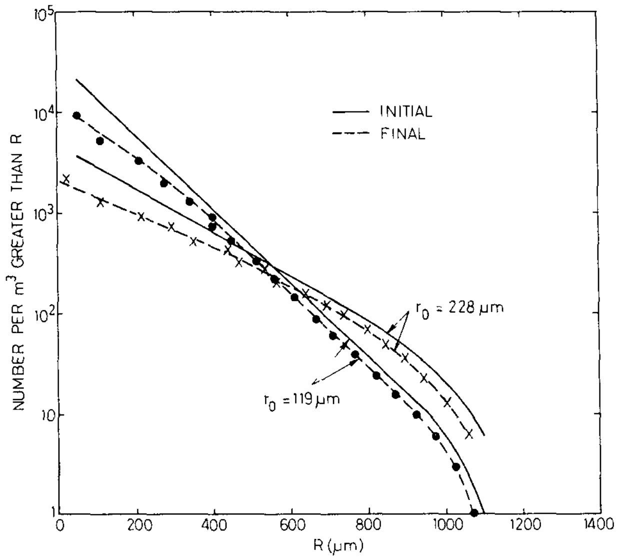

FIG. 5. The initial cumulative Marshall-Palmer distribution for $r_0=199\ \mu m$ and $r_0=228\ \mu m$ and the cumulative distribution given by the model after the parcel has descended 100 mb.

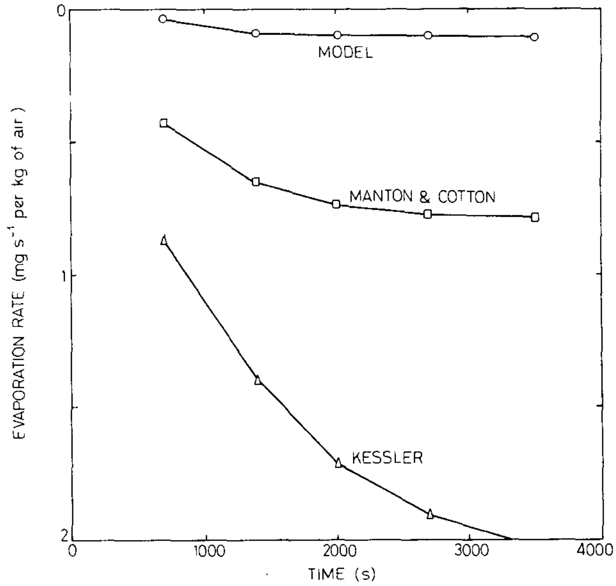

FIG. 6. The evaporation rate as a function of time for an initial rainfall rate of 17 mm h^{-1} and a downdraft of 1 m s^{-1} according to the model and to the Kessler (1969) and Manton and Cotton (1977) parameterizations.

consistent with observations, but the calculated profiles of θ_{es} are not as constant as those that are observed.

An interesting sidelight from the model is the suggestion that the instantaneous evaporation rate (dm/dt) should be a slowly varying function of time. This result has been compared with the evaporation rates derived from the Kessler (1969) $[G_K]$ and Manton and Cotton (1977) $[G_M]$ parameterizations. In terms of the present formulation Kessler's parameterization is given by

$$G_K=-9.7\times10^{-2}r_0^{2.6}N_0\left[\frac{\Delta e}{R_wT(1-e/p)}\right],$$

and Manton and Cotton's formulation is given by

$$G_M=-8.75\left(\frac{\rho_w\rho_a g}{\mu^2}\right)^{\frac{1}{4}}Dr_0^{2.75}N_0\left[\frac{\Delta e}{R_wT(1-e/p)}\right],$$

where μ is the viscosity and the other symbols have been defined earlier. N_0 at each time step in the model has been calculated by assuming that the Marshall-Palmer distribution slope is valid for droplets having a radius $>100\ \mu m$, and as discussed earlier this assumption will lead to an overestimate of N_0.

Figs. 6 and 7 show two examples of the comparative calculations. The comparison shows that the evaporation rate deduced from the Manton and Cotton (1977) parameterization approximates that from the numerical solution much more closely than the evaporation rate from the Kessler formulation. The basic difference between the two parameterizations is that Manton and Cotton allow for the variation of the atmospheric

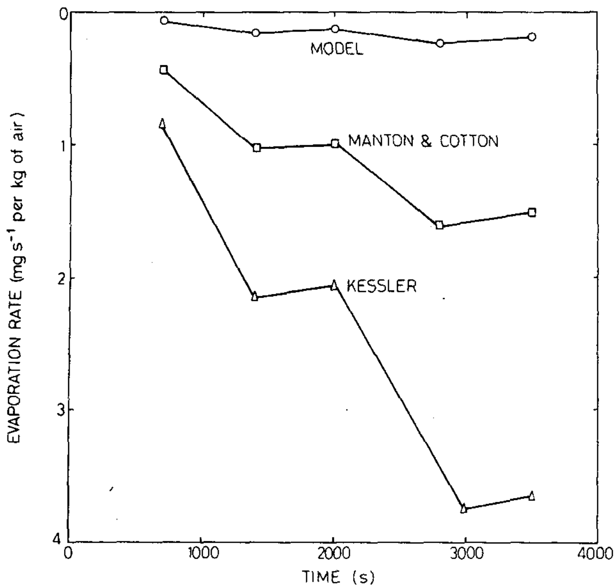

FIG. 7. As for Fig. 6 but for an initial rainfall rate of 114 mm h^{-1}.

parameters μ, ρ_a and D with pressure. We must conclude that the neglect of the pressure and temperature dependence of these parameters can produce significant errors in the evaporation rate.

6. Conclusion

While the limited data available do not permit a unique solution the agreement between predicted and observed profiles of θ and q suggest that the model has some validity in an evaporating downdraft. The results suggest that in those meteorological situations such as the squall line, a one-dimensional model of sufficient simplicity to be included in larger scale models is capable of realistically predicting the thermodynamic properties of the air associated with evaporating downdrafts.

REFERENCES

Beard, K. V., and H. R. Pruppacher, 1971: A wind tunnel investigation of the rate of evaporation of small water drops falling at terminal velocity in air. *J. Atmos. Sci.*, **28**, 1455–1469.

Betts, A. K., 1976: The thermodynamic transformation of the tropical subcloud layer by precipitation and downdrafts. *J. Atmos. Sci.*, **33**, 1008–1020.

——, and R. D. Miller, 1975: VIMHEX-1972 rawinsonde data. Tropospheric 25 mb wind and thermodynamic data. Low-level 10 mb thermodynamic data. Dept. Atmos. Sci., Colorado State University.

——, and M. A. Stevens, 1974: Rainfall and radar echo statistics VIMHEX 1972. Dept. Atmos. Sci., Colorado State University, 151 pp.

——, R. W. Grover and M. W. Moncrieff, 1976: Structure and motion of tropical squall-lines over Venezuela. *Quart. J. Roy. Meteor. Soc.*, **102**, 395–404.

Blanchard, D., 1953: Raindrop size distributions in Hawaiian rains. *J. Meteor.*, **10**, 457–473.

Kamburova, P. L., and F. H. Ludlam, 1966: Rainfall evaporation in thunderstorm downdraughts. *Quart. J. Roy. Meteor. Soc.*, **92**, 510–518.

Kessler, E., 1969: *On the Distribution and Continuity of Water Substance in Atmospheric Circulations. Meteor. Monogr.*, No. 32, Amer. Meteor. Soc., 188 pp.

Manton, M. J., and W. R. Cotton, 1977: On the parameterization of rain in cloud models (to be published).

Miller, M. J., and A. K. Betts, 1977: Traveling convective storms over Venezuela. *Mon. Wea. Rev.*, **105**, 833–848.

Ruiz, O., 1975: Mesoscale study of the tropical subcloud layer. Atmos. Sci. Pap. No. 237, Colorado State University, 133 pp.

Zipser, E. J., 1969: The role of organized unsaturated convective downdrafts in the structure and rapid decay of an equatorial disturbance. *J. Appl. Meteor.*, **8**, 799–814.