On the Calibration of Hailpads

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ABSTRACT

A model of the vertical collision of a sphere with a hailpad predicts that the dent volume is proportional to the impact kinetic energy, and gives a relationship between dent diameter and sphere diameter for ice spheres. Laboratory calibration experiments confirm the essence of the theory but cast some doubt on the validity of the assumption of a constant resistance pressure. Further experiments simulating windblown ice spheres show that for the conditions we considered, the horizontal partition of energy has a small effect on the minor axis diamter of the dent. Consequently, if the wind speed is unknown, no more than a 10% error may occur if the sphere diameter is determined using the no-wind relation. Finally, field calibration of the hailpads with hailstones falling in a natural hail shaft tend to support both the laboratory calibrations and the model predictions.

1. The humble hailpad

Hailpads consisting of a square piece of styrofoam covered with aluminum foil have been used to detect hail at the ground in many parts of the world. First employed by Schleusener and Jennings (1960) in northeastern Colorado, they have also been used in Oregon (Decker and Calvin, 1961), Illinois (Changnon, 1969), Italy (Vento, 1976), Alberta (Strong and Lozowski, 1977) and again in Colorado (Morgan and Towery, 1975). The principle of operation of the hailpad is exceedingly simple. Hail impinging on the pad creates an impression in the foil and the styrofoam, which remains as a permanent record of the impact after the hail has bounced off or melted. These dents are subsequently analyzed to provide a quasi-objective measure of the intensity of the hailfall, which is independent of the occurrence or extent of crop damage. Crop damage provides an inconsistent measurement of hailfall inasmuch as it is a function of non-hail factors such as the type and growth stage of the crop (Summers and Wojtiw, 1971).

The chief advantage of the hailpad over other surface hail sensors is its low cost (Towery and Changnon, 1974). Because of this, hailpads can be deployed over large regions with sufficient areal density to be able to resolve some of the finestructure which is now known to occur within a hailswath (Strong and Lozowski, 1977; Morgan and Towery, 1975). Its main drawbacks are its lack of time resolution and consequent inability to distinguish multiple hailfalls, its lack of robustness under the beating of very large hail, and the current lack of an accurate automatic technique for measuring the dents. Nevertheless, as a device for measuring hail at the ground for the purpose of evaluating hail suppression experiments, it has no serious competitors. These considerations have led to the establishment of a network of over 500 farmer operated hailpad stations within the 20 000 mi^2 target of the Alberta Hail Project (Deibert, 1975).

The purpose of this paper is to outline a theory and to describe some experiments which will provide some insight into the principles and problems of hailpad calibration.

2. A collision model

We start with the premise that the impression left by a hailstone must in some way be related to the physical parameters of the stone. The problem to be resolved is: Which are the important parameters and how are they related to the parameters of the dent? Ideally, the hailpad might be expected to act like a mould, recording an accurate impression of a portion of the hailstone's surface. With sufficient patience and ingenuity, one could then individually match the hailstones which fell on a pad with the dents which they made. Unfortunately, because of the elasticity and resilience of the pad materials, the exact shape of the hailstone is never perfectly captured and the hailstones themselves are not preserved. The task of hailpad interpretation, then, would consist ideally of making use of these approximate replicas of a portion of a hailstone's surface, in order to try to deduce such parameters as the numbers, size, shape, density, velocity and hardness of the impacting stones. Although it is

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unlikely that all of these parameters can be deduced from the hailpad, some plausible deductions concerning the relationship between dent size and hailstone size may be made with the aid of certain simplifying assumptions about the nature of the hailpad and the hailstones. One such assumption is that hailstones consist of hard, homogeneous ice spheres. This assumption is made implicitly or explicitly when a hailpad is calibrated using ball bearings rather than real hailstones.

The following model represents the collision of a single hard ice sphere with a homogeneous block of styrofoam whose surface is initially flat. Since there are several types of styrofoam material available, it is important to mention that we envisage a particular variety called STYROFOAM IB which is manufactured by the Dow Chemical Company. The dynamic effects of the stretching of the aluminum foil, and the complications arising out of multiple, superimposed impacts are ignored. The ice sphere is imagined to fall vertically onto the horizontal styrofoam surface, impacting without rotation at its terminal speed v_t. We thereby also ignore the complication of windblown hailstones.

The styrofoam is imagined to consist of a large number of tubular cells of cross-sectional area A, oriented perpendicular to the surface In actuality, styrofoam cells are more bubble-like than tubular, so that the justification for such a conceptual model must rest on the accuracy of its predictions. We assume that the cells do not behave elastically, but that each exerts a constant vertical retarding force F on the sphere, regardless of the state of compression of the cell. The styrofoam therefore exerts a constant pressure $p=F/A$ on the sphere, and the total retarding force increases as the area of the sphere in contact with the styrofoam increases. A cursory investigation of the literature on foamed plastics (Bender, 1965) indicates that there may be some justification for such a constant stress model provided that the stress plateau, which occurs beyond the compressive yield point in the stress-strain diagram, is achieved shortly after impact, and that it persists up to high strain values. The relative brittleness and the large open pore structure of STYROFOAM IB suggests that this may be the case, although we have no quantitative evidence for this contention apart from the present results. The sphere is eventually brought to a halt, and it is assumed subsequently to remain at rest, rather than bouncing out. Finally, we assume that the center of the sphere does not penetrate below the level of the undisturbed surface of the styrofoam.

Three stages in the collision sequence are illustrated in Fig. 1. At any time during the deceleration of the sphere, its equation of motion is

$$m\frac{dv}{dt}=-p\pi a^2, \tag{1}$$

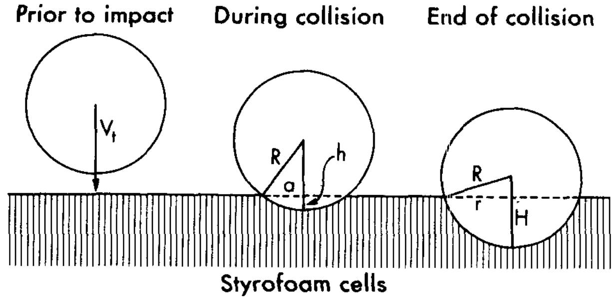

FIG. 1. Three stages during the collision of a sphere with a hailpad.

where m is the mass of the sphere, $v(t)$ the instantaneous velocity of the sphere and $a(t)$ the instantaneous radius of the dent. Since $v=dh/dt$, where h is the instantaneous depth of the dent, and by geometry, $a^2=2Rh-h^2$, the equation of motion becomes

$$\frac{d^2h}{dt^2}=Ah+Bh^2, \tag{2}$$

where $A=-2\pi Rp/m$ and $B=\pi p/m$.

The boundary conditions are

$$h=0 \quad \text{and} \quad dh/dt=v_t \quad \text{at} \quad t=0, \tag{3a}$$

$$h=H \quad \text{and} \quad dh/dt=0 \quad \text{at} \quad t=T, \tag{3b}$$

where T is the total duration of the collision, and H the depth of the final dent. The solution to Eq. (2) is in terms of elliptic functions (Davis, 1962). We need not derive this in detail, however, since our main interest is to relate the final depth H and radius r of the dent, to the mass m, radius R and fallspeed v_t of the sphere.

Multiplying Eq. (2) by dh/dt, integrating with respect to time, and applying the boundary conditions leads to

$$\tfrac{1}{2}mv_t^2=\pi pH^2(R-\tfrac{1}{3}H). \tag{4}$$

Assuming p to be a constant, characteristic of the styrofoam, Eq. (4) implies that for a sphere of given radius R the final dent depth H (and hence its radius r) are determined by the kinetic energy of the impact. This means that two spheres of equal diameter but differing density will produce dents of equal size provided that they hit the hailpad with equal kinetic energies. This result is particularly useful because it means that non-ice spheres may be used to calibrate hailpads provided that the terminal kinetic energy of the equivalent diameter ice sphere is duplicated. It also contradicts a suggestion (Changnon, 1969) that perhaps the impact momentum is the quantity to duplicate.

The prediction of the model was tested by dropping various size spheres of glass and steel onto hailpads, in such a way as to duplicate either the terminal kinetic energy or the terminal momentum of the equivalent diameter ice sphere. These spheres were hitting the hailpad at less than their own terminal velocity. Ice spheres were also dropped in a stairwell from a height of 34.5 m, thereby achieving 92–97% of terminal velocity on impact, depending on their size. The dents were measured, and a graph of dent diameter as a function of sphere diameter is shown in Fig. 2. For this work, spheres of 12.7 mm diameter and larger were used. In the work reported later in this paper, however, we used spheres down to 5 mm in diameter. Each point on the graphs represents an average of ten repetitions. Although the ice spheres had not quite attained their terminal kinetic energy at impact, the coincidence of the results for the three materials, when the kinetic energy is simulated (Fig. 2a), appears to confirm the prediction of Eq. (4). This is not the case if one attempts to duplicate the terminal momentum of the equivalent diameter ice sphere. When this is done, the relationship between the sphere and dent diameters is distinctly different for the three different materials (Fig. 2b).

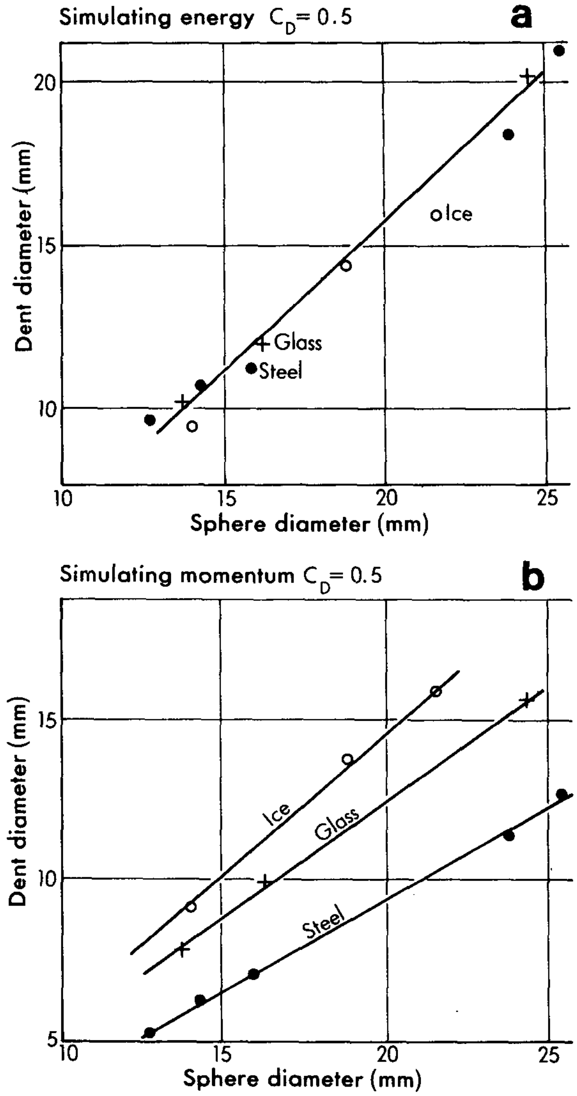

FIG. 2. Dent diameter as a function of sphere diameter when the kinetic energy (a) or the momentum (b) of an ice sphere is simulated with an equivalent diameter sphere of glass or steel. Here it was assumed that $C_D=0.5$ for smooth spheres in order to calculate the drop heights for the simulation.

This behavior may be peculiar to the type of styrofoam we used, and may not therefore be found when other types of styrofoam are calibrated. It is conceivable, for example, that the resistance pressure p, for some types of styrofoam, may be proportional to v_t; that is, the faster the material is initially compressed, the greater may be the resisting force per unit area. In this event, Eq. (4) would imply that the final dent depth H (and hence its radius r) are uniquely determined by the sphere radius R, and its momentum, rather than its kinetic energy.

The volume of a spherical cap of depth H and radius R is given by

$$V=\pi H^2(R-\tfrac{1}{3}H). \quad (5)$$

Using this result, Eq. (4) takes on a particularly simple form

$$\tfrac{1}{2}mv_t^2=pV. \quad (6)$$

Eq. (6) equates the initial kinetic energy of the sphere to the work done in compressing the styrofoam. V is the volume of the dent at the instant when the velocity of the sphere is zero. In view of this interpretation, it is easy to take into account the effect of a rebound of the sphere with velocity v_r, by adding a term $-\tfrac{1}{2}mv_r^2$ to the left-hand side of Eq. (6). The volume V would then be interpreted to be the final dent volume, provided that the styrofoam has no tendency to creep after the impact is over. Similarly, in order to take into account the dynamic effect of the aluminum foil, a term σA could be added to the right-hand side of Eq. (6), where σ is the "surface tension" of the aluminum foil and A the increase in the area of the foil due to stretching.

Neglecting both of these terms for the present, Eq. (6) can now be used to determine a relationship between the radius of the ice sphere and the radius of the dent which it produces. From geometry

$$H=R[1-(1-\alpha^2)^{\frac{1}{2}}], \quad (7)$$

where $\alpha=r/R$, the ratio of the dent to sphere radii. We also have

$$m=\tfrac{4}{3}\pi R^3\rho_i, \quad (8)$$

where ρ_i is the ice density, and

$$mg=\tfrac{1}{2}\rho_a v_t^2 C_D \pi R^2, \quad (9)$$

where ρ_a is the air density, C_D the drag coefficient of the sphere and g the acceleration due to gravity. Substituting these relations into Eq. (4), we obtain the desired result, *viz.*,

$$R^{-1}[\tfrac{2}{3}-(\tfrac{2}{3}+\tfrac{1}{3}\alpha^2)(1-\alpha^2)^{\frac{1}{2}}]=\frac{16}{9}\frac{\rho_i^2 g}{\rho_a C_D p}. \quad (10)$$

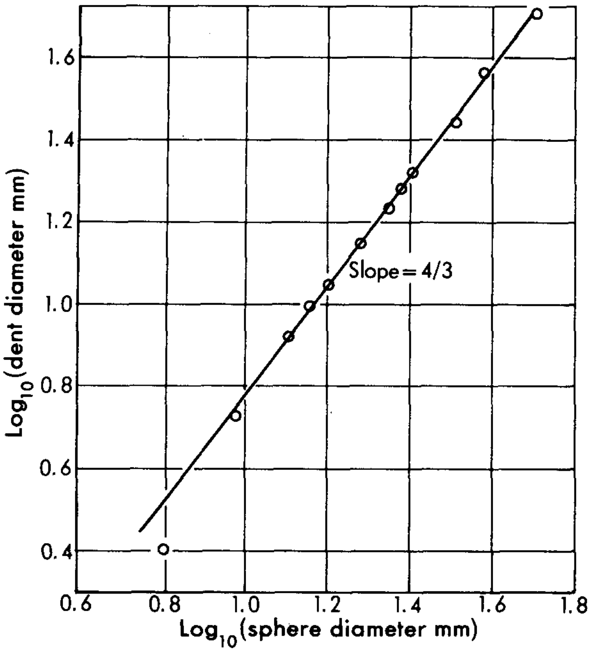

FIG. 3. Laboratory calibration relation for the hailpad.

The right-hand side of Eq. (10) is assumed to be a constant, independent of the radius R of the sphere. In general $\alpha \leqslant 1$, and the left-hand side of Eq. (10) can be expanded in a binomial series. Retaining only the dominant term yields

$$r=\left(\frac{64}{9}\frac{\rho_i^2 g}{\rho_a C_D p}\right)^{1/4} R^{5/4}. \tag{11}$$

This second prediction of the theory, i.e., that to a first approximation the dent radius varies as the 5/4 power of the sphere radius, has also been tested. Using glass and steel spheres with assumed drag coefficients of 0.45 to simulate the terminal kinetic energy of equivalent diameter rough ice spheres of drag coefficient 0.6, a relation between sphere diameter and dent diameter was established (Fig. 3). The drag coefficient of 0.45 for smooth glass and steel spheres is an average value over the Reynolds number range 10^3 to 10^5 (Prandtl and Tietjens, 1957). The value of 0.6 for rough ice spheres is based on measurements by List (1959) on roughly spherical hailstones. Except for the smallest size category, the data points of Fig. 3 are well-fitted by an equation of the form $r=CR^{4/3}$ with C a constant. The theoretical prediction given by Eq. (11) is a fair approximation to this result; however, the lack of better agreement appears to result largely from the failure of the assumption that p is a property of the styrofoam independent of the size of the impacting sphere.

The observed calibration relation of Fig. 3 can be used in Eq. (10) to solve for p as a function of the radius of the impacting sphere. What is obtained is an effective p averaged over the impact; it is the value of p which the model styrofoam must exert if it is to yield the same dent radius that is measured experimentally. It should not be confused with the instantaneous pressure which the actual styrofoam/aluminum foil combination is exerting at any time during the collison. Fig. 4 illustrates that although p is approximately constant over several narrow radius intervals, it is not constant over the entire range of radii considered. For sphere radii <0.6 cm, p increases rapidly as R decreases. This means that the model styrofoam must become more resistive for such small spheres. Physically this probably happens because the aluminum foil on the real hailpad absorbs a greater proportion of the kinetic energy as the sphere size diminishes. Thus the dent size in the styrofoam is smaller than it would be without the foil, giving rise to the result that the model styrofoam appears more resistive and that p is larger. This hypothesis is further supported by a visual inspection of the hailpads, which shows that at a sphere radius of about 0.6 cm, the aluminum foil begins to tear. A second sudden decrease in p occurs at a sphere radius of about 1.2 cm. Again a visual inspection demonstrates that at this radius the styrofoam no longer simply crushes but also begins to crack. For the largest spheres, the finite thickness of the hailpad ($\sim$2.5 cm) becomes important and one can probably no longer ignore the mechanical effects of the underlying surface during the collision.

One possible consequence of the $p^{-1/4}$ dependence of Eq. (11) is that the calibration should not be sensitive to small variations in the mechanical properties of the styrofoam from batch to batch. We have found this to be the case in practise with the particular styrofoam we use. Of course, one cannot therefore infer that differing styrofoam/aluminum foil combinations will have similar calibration characteristics.

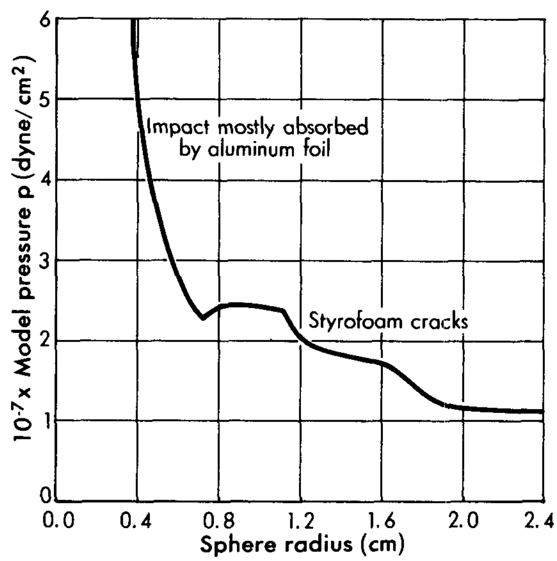

FIG. 4. Styrofoam pressure as a function of sphere radius.

Since the model in its present form is strictly applicable only to foil-free styrofoam hailpads, we decided to perform all further tests on pads without their foil wrapping. For the vertical impact of a sphere dropped from a height L in vacuo, Eq. (4) can be used to show that

$$\rho_b L = (3p/4g)[1-(1-\alpha^2)^{\frac{1}{2}}]^2 \times\{1-\tfrac{1}{3}[1-(1-\alpha^2)^{\frac{1}{2}}]\}, \quad (12)$$

where ρ_b is the density of the sphere. If $\rho_b L$ is plotted as a function of α, a single relation should exist for all spheres, irrespective of their size or composition. This result is essentially confirmed in Fig. 5, which shows experimental results of drop tests with steel and glass spheres of varying sizes. The solid curve in Fig. 5 is given by Eq. (12) in which $p=8.6\times10^6$ dyn cm^{-2}. This value of p was chosen to give a reasonable fit to the data points for the 4.76 mm radius steel sphere. The scatter of the experimental points suggests that the resulting pressure p is still not quite a constant property of the styrofoam even without the foil wrapping.

In deriving Eq. (12), the assumption has been made that the kinetic energy of impact of the sphere equals its potential energy at height L. Because of frictional drag, this is not true for spheres dropped in air, and we have compensated for this as follows. If a sphere is dropped from rest in air and its acceleration is sufficiently rapid that the constant drag coefficient regime is achieved virtually instantaneously, then it will achieve a speed w after falling a distance L', where

$$L' = -\frac{w_t^2}{2g}\ln\left(1-\frac{w^2}{w_t^2}\right) \quad (13)$$

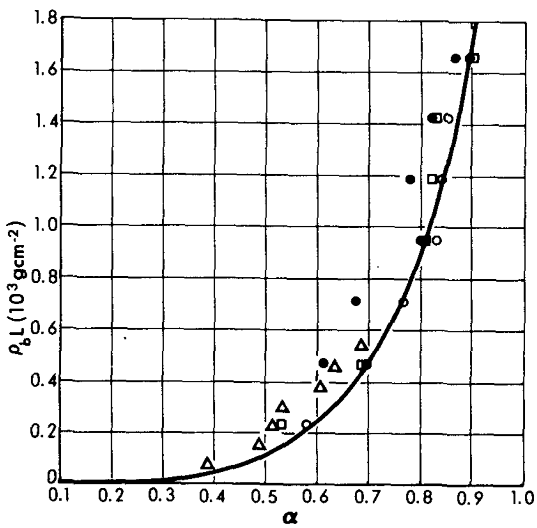

FIG. 5. Nondimensional dent diameter versus the product of sphere density and drop height. The meaning of the symbols is as follows: (●) steel, $\rho_b=7.83$ g cm^{-3}, $R=2.37$ mm; (○) steel, $\rho_b=7.84$ g cm^{-3}, $R=4.76$ mm; (□) steel, $\rho_b=7.78$ g cm^{-3}, $R=9.12$ mm; (△) glass, $\rho_b=2.52$ g cm^{-3}, $R=12.26$ mm.

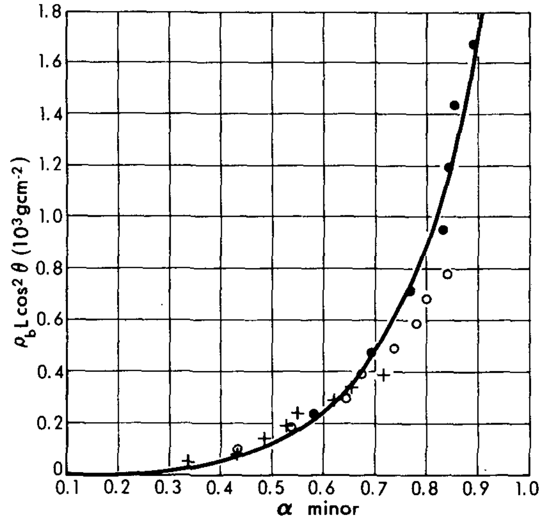

FIG. 6. Nondimensional minor axis diameter of dent versus the product of sphere density, drop height and the square of the cosine of the impact angle θ for $\theta=0°$ (●), $\theta=45°$ (○), $\theta=60°$ (+).

and w_t is the terminal speed of the sphere. In view of this, a sphere dropped from a height L' in air will achieve the same impact energy as one dropped from a height L *in vacuo* where

$$L=\frac{w_t^2}{2g}\left[1-\exp\left(-\frac{2gL'}{w_t^2}\right)\right]. \quad (14)$$

In plotting the experimental results in Fig. 7, we used the value of L given by Eq. (14).

3. Wind-driven hailstones

Hailstones with a terminal speed v_t, driven by a horizontal wind $v_t \tan\theta$, will be incident on a horizontally oriented hailpad at an angle θ to the vertical or the normal to the pad surface. The total kinetic energy of impact, $\frac{1}{2}mv_t^2\sec^2\theta$, can be envisaged as the sum of a partition due to the normal velocity component, $\frac{1}{2}mv_t^2$, and a partition due to the tangential velocity component, $\frac{1}{2}mv_t^2\tan^2\theta$. The details of a theory to describe such a collision are not easy to work out because the dent is not necessarily of a simple geometrical shape. However, it is possible that Eq. (6) might still hold.

By dropping spheres vertically onto a foil-free hailpad whose surface is inclined at an angle θ to the horizontal, it is possible to simulate a wind-driven collision without the need for a wind. The principal error in such a simulation is possibly that the direction of gravity and the normal to the hailpad surface do not coincide. Although this may affect the rebound energy of the spheres, such a possibility is ignored for now. According to our hypothesis, in order for this to be a realistic simulation in other respects, the total kinetic energy

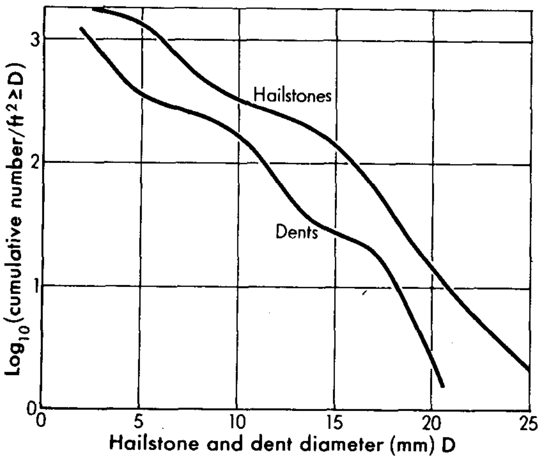

FIG. 7. Cumulative size distributions of hailstones and dents during field calibration of the hailpad.

of the steel sphere must equal that of a wind-driven ice sphere of equal size and impacting at the same angle This energy is $\frac{1}{2}mv_t^2\sec^2\theta$, and in order to achieve it, the steel sphere must be dropped *in vacuo* from a height L given by

$$L=\frac{\rho_i v_t^2}{2\rho_b g}\sec^2\theta. \quad (15)$$

The appropriate drop height in air can be derived using Eq (14). Accordingly, the height from which the steel sphere is dropped must increase with θ. This will have the effect of elongating the dent, but it is not obvious what the effect will be on the minor axis of the dent. It could be argued that the horizontal partition of the energy associated with the horizontal velocity component will have no effect on the depth of the dent, and hence on the size of the minor axis. Therefore if the partition of the energy associated with the vertical velocity component is alone responsible for determining the minor axis diameter of the dent, it might be expected by analogy with Eq. (13) that a relation of the form $\rho_b L\cos^2\theta=f(\alpha)$, where f is some function of α, should also prevail for wind-driven impacts. Fig. 6 shows the results of experiments in which a 9.5 mm diameter steel sphere was dropped onto a hailpad at angles of 0°, 45° and 60°. The abscissa is α_{minor}, the ratio of the minor axis diameter of the dent to the diameter of the sphere. The solid curve is the same solution to Eq. (12) that is depicted in Fig. 5. The experimental results suggest that for small values of α_{minor}, a single relation prevails for all the angles considered. However, for larger values of α_{minor}, a difference among the relations for the different angles begins to appear. We believe that this likely occurs because the horizontal partition of energy begins to play a significant role in enlarging the minor axis of the dent. By ignoring this effect, an error of up to about 10% can be expected in the sphere diameter inferred from the dent diameter. Such an error would lead to about a 30% error in the inferred mass and a 40% error in the vertical partition of the inferred kinetic energy. Clearly, nonspherical hailstones will complicate the interpretation of wind-driven dents even further.

4. Calibration under natural conditions

The theoretical model and the laboratory calibrations have treated the case of single impacts of hard ice spheres on an undented hailpad. In order to assess the possible complications introduced by multiple superimposed impacts of real windblown hailstones, with their attendant variations in shape and hardness, an attempt was made to calibrate the hailpad under natural conditions. This was done by exposing a hailpad in a storm, and simultaneously collecting and photographing a sample of the hail which fell adjacent to the pad. The size distributions of the dents and the hailstones were then compared.

This procedure was followed on 18 August 1974 in a severe hailstorm near Spruceview, Alberta. The shape of the hailstones was conical to spherical, and they ranged in size up to 2.3 cm. This value was the maximum stone dimension observed in a photographic image described below. The hail fell without rain, and it was accompanied by light winds (<10 mph). A foil-covered hailpad was exposed for the entire 10 min duration of the hailfall, and hail was collected at the same time in a net which retained all hailstones greater than a few millimeters in diameter. These stones were stored in containers filled with liquid hexane at about −65°C. After the hailfall had ended, a photograph was also taken of the hail lying on the ground. The hailstone size distributions derived from the ground photograph and the collected sample were in reasonable agreement. Differences between the two were probably due to melting of the hailstones on the ground. The problem

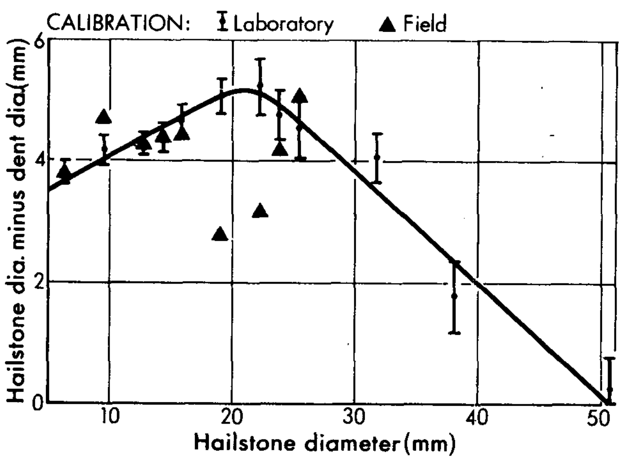

FIG. 8. Comparison of the laboratory calibration with the field calibration. The solid line is a hand fit to the laboratory points.

of smaller hailstones being obscured by larger ones did not seem to be a significant one.

The collected hailstones were subsequently spread out in a cold room, photographed, and sized in millimeter intervals, according to the maximum dimension of their photographic image. Since most of the hailstones were approximately spherical, the difference between the maximum and minimum dimensions was generally insignificant. The hailpad was analyzed by counting and measuring all of the dents either over the entire pad, or over a portion of the pad, provided that the number of dents in a given size interval on that portion exceeded 100. Unfortunately, at the time of this analysis the theory described in this paper had not been developed so that the dent sizes were classified in 1/20 inch intervals, according to their equivalent circular diameter. Because the wind was light and the dents were essentially circular, the difference between the equivalent circular diameter and the minor axis diameter was probably less than 10% for most dents. The classification was accomplished by superimposing standard circles of known size on the dents and visually attempting to match the areas. Because of this method of hand analysis, differences between analysts did occur. However, among the four analysts who examined this hailpad, these did not amount to overall shifts in the distribution by more than about 0.5 mm.

The cumulative size distributions of both the dents and the hailstones are illustrated in Fig. 7. Using this graph, the dent diameters can be related to the hailstone diameters, under the assumption that a hailstone of "diameter" D_H corresponds to a dent of "diameter" D_D, if there are equal numbers of hailstones larger than D_H and dents larger than D_D. The relation between hailstone size and dent size obtained in this way is compared with the laboratory calibration in Fig. 8. The agreement is relatively good, in spite of the high areal density of hailstones, giving rise to multiple, superimposed dents on the hailpad. Even the two outlying triangular points imply an error in inferred hailstone diameter of only about 10%. Further work is still required to examine the effects on the calibration of differing hailstone shapes and hardness, as well as high winds.

5. Conclusions

The model of the collision of a spherical hailstone with a hailpad predicts that the dent volume is proportional to the kinetic energy of the impact. This result has received some experimental confirmation for foil-free styrofoam pads provided that significant breaking of the pad does not occur. The addition of an aluminum foil wrapping to the styrofoam may modify this relation especially for hailstones with low energies. In order to take this effect into account in the theory, one might attempt to add a term to Eq. (6) which would account for the surface energy needed to stretch the aluminum foil.

A relationship has been shown empirically to exist between the partition of impact energy associated with the normal velocity component and the parameter α_{minor} (the ratio of the minor axis diameter to the sphere diameter) for oblique impacts. Under the conditions employed in our experiments, this relation is approximately independent of the sphere size and density and of the impact angle. Accordingly, for wind-driven hailstones, it is possible to relate the dent diameter to the hailstone diameter without a knowledge of the wind speed or the angle of impact of individual stones, provided one is prepared to accept a possible error of up to 10% in the inferred hailstone sizes.

Finally, it should be pointed out that the conclusions reached in this paper are applicable only to the type of styrofoam material which we have employed, and to a range of energies typical of those of freely falling hailstones. For other conditions, such as those employed by Chisman and McNaughtan (1972), for example, a quite different type of impact behavior may occur.

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