Note on the Use of Weibull Statistics to Characterize Wind-Speed Data[1]

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13 September 1977 and 26 December 1977

ABSTRACT

A hybrid density function is given for describing wind-speed distributions having nonzero probability of "calm." A Weibull probability graph paper designed specifically for plotting wind-speed distributions is used to determine distribution parameters to within a few percent of values obtained by the maximum likelihood technique. Data from the National Weather Service are used to demonstrate the use of the hybrid density function and the Weibull graph paper.

1. Introduction

The Weibull distribution has been used in describing wind-speed data (Justus *et al.*, 1976; Hennessey, 1977; Takle and Brown, 1976; Takle *et al.*, 1978). Hennessey discusses other previous applications of Weibull and related distributions to wind data. Some sites have wind characteristics not accounted for in the two-parameter Weibull distribution; Justus *et al.* (1976) acknowledge problems at low wind speeds, and Hennessey (1977) alludes to wind-speed distributions that are too deformed or heterogeneous for a two-parameter distribution. Herein we summarize procedures we have developed and used for coping with the problems caused by low or zero wind speeds, for computing the Weibull parameters, and for easily ascertaining the qualitative resemblance of a given wind-speed distribution to the Weibull.

2. Low wind speeds

Cup anemometers that provide much of the standard climatological data on wind speeds typically have relatively high wind-speed thresholds. Histograms of wind speed determined from these data will therefore typically exaggerate the probability of calm or very low wind speeds. On the other hand, for the Weibull probability density function [notation of Justus *et al.*, (1976)], we have

$$P_X^W(x) = \frac{k}{c}\left(\frac{x}{c}\right)^{k-1} \exp\left[-\left(\frac{x}{c}\right)^k\right], \quad x \geqslant 0$$
$$= 0, \quad \text{otherwise},$$

and for the cumulative distribution function

$$F_X^W(x) = 1 - \exp\left[-\left(\frac{x}{c}\right)^k\right], \quad x \geqslant 0$$
$$= 0, \quad \text{otherwise},$$

where c is the scale parameter, having the same units as x, and k is the (dimensionless) shape parameter. If this distribution is fit to wind-speed data, the probability of observing periods of zero wind speed is not properly accounted for because

$$F_X^W(0) = 0.$$

[1] Journal Paper No. J-8941 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa. Project 2116.

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0021-8952/78/0556-0559$05.00

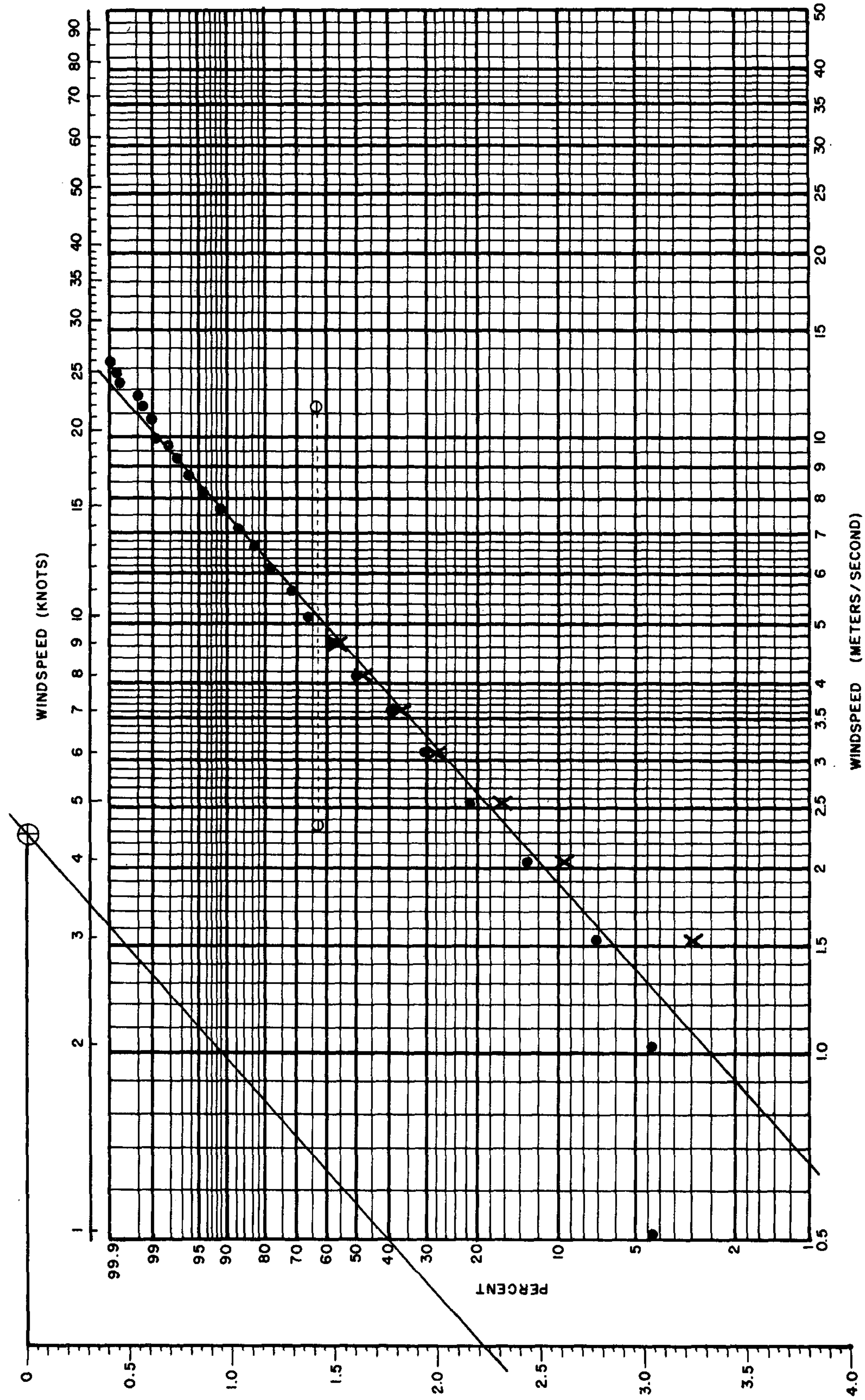

FIG. 1. Cumulative probability distribution of 3 h wind speeds at the Des Moines office of the National Weather Service for the period 1 January 1965–31 December 1974.

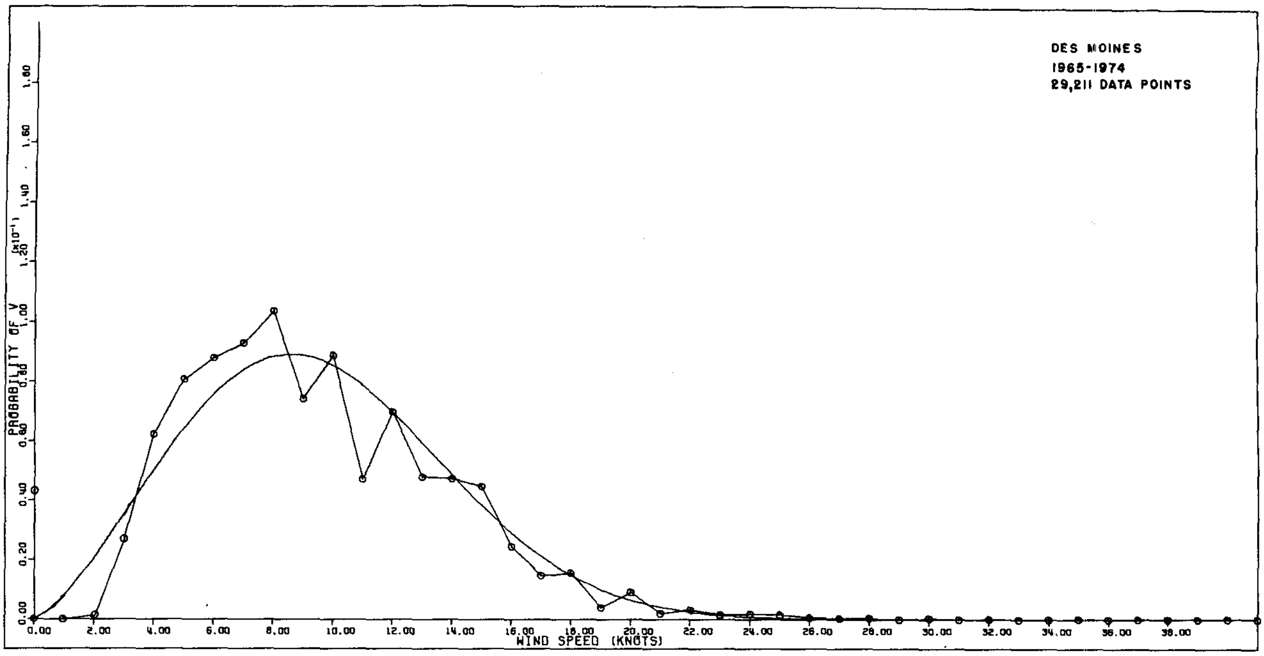

FIG. 2. Probability of occurrence of 3 h wind speeds used in Fig. 1.

The problem of properly including calm periods into the distribution is reduced (but not eliminated) by defining a hybrid density function,

$$P_X^H(x)=F_0\delta(x)+(1-F_0)P_X^W(x),\ \text{for all real } x, \quad (1)$$

where F_0 is the probability of observing zero wind speed and $\delta(x)$ the Dirac delta function. The corresponding distribution function is

$$F_X^H(x)=\begin{cases}F_0+(1-F_0)F_X^W(x), & x\geqslant 0\\ 0, & \text{otherwise},\end{cases} \quad (2)$$

where $F_X^W(x)$ is the Weibull cumulative distribution function. The mean and variance of this distribution are then, respectively,

$$\bar{x}_H=c(1-F_0)\Gamma(1+1/k), \quad (3)$$

$$\overline{(x-\bar{x})^2_H}=c^2(1-F_0)\Gamma(1+2/k)-\bar{x}_H^2, \quad (4)$$

where $\Gamma(Y)$ is the gamma function. This method simply removes those measurements determined to be "calm," and fits the Weibull distribution to the nonzero wind speeds. The zero wind speeds are then reintroduced to give the proper mean and variance and to renormalize the distribution. By setting $F_0=0$, Eqs. (1)–(4) reduce to the equations used by Justus *et al.* (1976). The two cases, $F_0=0$ and F_0 equal to the probability of zero wind speeds, give the two extremes for the shape of the distribution on the low-speed end; limits of the low wind-speed ambiguity can be estimated by using the hybrid distribution.

It may be argued that F_0 contains some true nonzero sub-threshold wind speeds, and that by eliminating these, the remaining data set would not necessarily be expected to describe a Weibull distribution. For data sets having high probability of calm and a low value of scale parameter (c), the Weibull function is not likely to provide a good fit to the data. This situation and other "deformed" distributions can be identified by the use of Weibull probability paper.

3. Computation of Weibull parameters

From the form of $F_X^W(x)$, it can be shown that

$$\ln\{-\ln[1-F_X^W(x)]\}\ \text{vs}\ \ln x,$$

where x is wind speed, will give a linear relationship (Justus *et al.*, 1976). We have used this relationship and the shape-parameter scaling technique of Nelson and Thompson (1971) to construct a Weibull probability paper (Fig. 1).[4] If the distribution of nonzero wind speeds represents a Weibull distribution, the plot of the cumulative probability on Fig. 1 will be a straight line. The value of c is then given by the crossing of the 63.2% level (denoted by the fiducial marks and dashed line), and the value of k is determined by transferring the slope of the line from the fiducial point above the graph to the scale at the left. A plot such as in Fig. 1 can be of assistance in evaluating the credibility of the Weibull distribution in various wind-speed ranges.

A more accurate determination of c and k can be made by a linear least-squares fit to the data as suggested by Justus *et al.* (1976). A third method, and most accurate in our experience, of determining the Weibull parameters is the maximum-likelihood technique given

[4] Copies of this paper are available from the authors.

Table 1. Weibull parameters and mean wind speeds determined by various methods.

	F_0	Graphical c (m s^{-1})	Graphical k	Graphical $\bar{x}_H$ (m s^{-1})	Weighted LL SQ c (m s^{-1})	Weighted LL SQ k	Weighted LL SQ $\bar{x}_H$ (m s^{-1})	Maximum likelihood c (m s^{-1})	Maximum likelihood k	Maximum likelihood $\bar{x}_H$ (m s^{-1})	Calculated mean $\bar{x}$ (m s^{-1})
Ames* (1963–70)	0.0052	5.40	2.24	4.76	5.50	2.33	4.85	5.49	2.24	4.84	4.89
Des Moines (1965–74)	0.0429	5.20	2.22	4.41	5.21	2.42	4.42	5.54	2.38	4.71	4.69

* Januaries only.

by Johnson and Kotz (1970). The estimators $\hat{k}$ and $\hat{c}$ of k and c, respectively, satisfy the equations

$$\hat{c}=[N^{-1}\sum_{i=1}^{N} A_i x_i^{\hat{k}}]^{1/\hat{k}}, \tag{5}$$

$$\hat{k}=[\{\sum_{i=1}^{N} A_i x_i^{\hat{k}} \ln x_i\}\{\sum_{i=1}^{N} A_i x_i^{\hat{k}}\}^{-1}-N^{-1}\sum_{i=1}^{N} A_i \ln x_i]^{-1}, \tag{6}$$

where N is the total number of observations of nonzero wind speed and A_i the number of observations of wind speed x_i. Solution of these equations requires an iterative process.

4. Examples

Several examples of the use of the methods of Sections 2 and 3 are given elsewhere (Takle *et al.*, 1978). Figs. 1 and 2 represent 10 years of 3 h wind-speed data from the Des Moines office of the National Weather Service. The significant F_0 value of 0.043 is representative of other National Weather Service data for Iowa (Takle *et al.*, 1978). The linear fit of Fig. 1 was determined by eye. The smooth curve of Fig. 2 was drawn by using Weibull parameters determined by the maximum likelihood technique.

The values of $F_X^H(x)$ are shown by the dots in Fig. 1 and values of $F_X^W(x)$ are denoted by the ×'s for the low end of the distribution where $F_X^W(x)$ differs from $F_X^H(x)$. The range of probabilities between $F_X^H(x)$ and $F_X^W(x)$ for low wind speeds provides a measure of the uncertainty in the shape of the distribution at the low wind-speed end.

Table 1 gives the hybrid parameters and mean wind speeds determined by various methods for the Des Moines data and for a data set consisting of hourly wind speeds at 32 m for eight Januaries at Ames [see Takle *et al.* (1976) for description of instruments and exposures]. The Weibull parameters determined in Fig. 1 provided the graphical values for the Des Moines entry in Table 1. The linear least-squares method of Justus *et al.* (1976) was used for the weighted-LLSQ entry in Table 1, with each point weighted by the number of measurements of its respective wind speed. The maximum-likelihood entry results from the use of Eqs. (5) and (6). The hybrid mean wind speeds were calculated from Eq. (3) and

$$\bar{x}=(1/N)\sum x.$$

The parameter k is quite sensitive to the shape of the distribution at the low and high ends. The mean as calculated from Eq. (3), however, is much more sensitive to the value of c than to the value of k. Therefore, the subjectivity in the determination of k using Weibull probability paper is not too serious for calculating mean wind speed, although it may be serious for inferences about high and low wind speeds.

5. Conclusions

A slight variation of the Weibull density function to include zero wind speeds improves its applicability to wind-speed data. A graph suitable for plotting nonzero wind speeds is given that allows determination of Weibull parameters without the use of numerical curve fitting and that provides a convenient visual display of the data. Comparisons with standard estimation techniques show the graphical method used in conjunction with the hybrid Weibull function to give mean wind speeds within approximately 5% of actual means.

Acknowledgments. The authors wish to thank Dr. J. J. Goebel of the Iowa State University Statistical Laboratory for several helpful discussions and suggestions. W. M. Davis and Jay Morford provided computational assistance. Financial support was provided under a grant from the Iowa Energy Policy Council.

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