Variations in Measured Values of Lateral Diffusion Parameters

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ABSTRACT

The behavior of the quantity $S=\sigma_y/x\sigma_\theta$ for surface releases of tracers is examined. Variations in this quantity for different field programs are shown to be attributable largely to variations in sampling time τ and averaging time t; larger values of S are found to be associated with longer τ and t. Other previously disregarded quantities, *viz.*, mean wind speed and atmospheric stability, are shown to have potentially important effects. Two spectral models are presented which reproduce many of the observed features.

1. Introduction

The specification of σ_y^2, the variance of a dispersed substance in the cross-wind direction, has long been a fundamental problem in the study of turbulent diffusion. In recent years, the role of σ_θ^2 in determining this quantity has received increasing attention; σ_θ^2 is the variance in the horizontal wind direction. The dimensionless ratio

$$S=\frac{\sigma_y}{x\sigma_\theta}, \tag{1}$$

has been examined in a number of studies (e.g., Pasquill, 1971; Draxler, 1976), and various empirical relationships for the dependence of S on downwind distance x or travel time T have been proposed. In general, there is considerable scatter of the experimental data about the "universal" curves which were suggested, but agreement within about a factor of 2 or better was usually obtained.

Attempts to derive a theoretical expression for the variation of S with x normally begin with a consideration of statistical analyses of the diffusion process. If $F_L(n)$ is the Lagrangian frequency spectrum, then one form of Taylor's theorem is (Ogura, 1959; Pasquill, 1974)

$$[\sigma_y^2]_{\tau,T}=[\sigma_v^2]_{\infty,0}T^2\int_0^\infty F_L(n)\times\left[1-\frac{\sin^2\pi n\tau}{(\pi n\tau)^2}\right]\frac{\sin^2\pi nT}{(\pi nT)^2}dn, \tag{2}$$

where T is the travel time required for the particles to reach the measuring location, τ the time over which the dispersion is sampled and σ_v the variance of the lateral wind velocity. The subscripts on the bracketed σ_v^2 refer to an infinitely long sampling time and a vanishingly small averaging time, while the τ and T subscripts on the bracketed σ_y^2 indicate a finite sampling time τ and an effective averaging time T.

Hay and Pasquill (1959) transformed (2) into an Eulerian expression by introducing β, the ratio of the Lagrangian and Eulerian time scales, and writing

$$[\sigma_y^2]_{\tau,T}=[\sigma_v^2]_{\tau,0}T^2\int_0^\infty F_E(n)\frac{\sin^2(\pi nT/\beta)}{(\pi nT/\beta)^2}dn, \tag{3}$$

where $F_E(n)$ is the Eulerian frequency spectrum of the lateral wind component. They argued that (3) is valid when τ is long enough to include the effects of the whole spectrum of turbulence, and that even for smaller τ Eq. (3) is still applicable provided $\tau \geqslant T$. If σ_θ is sampled over a time T but with an averaging time of zero, then S is given more specifically by the ratio $[\sigma_y]_{\tau,T}/x[\sigma_\theta]_{\tau,0}$. In the limit of large τ, Pasquill (1975) showed that (3) leads to an expression for S given by

$$S=\frac{[\sigma_v]_{\infty,T/\beta}}{[\sigma_v]_{\infty,0}}, \tag{4}$$

where the relation $\sigma_v T\approx\sigma_\theta x$ has been used. For a given turbulent spectrum, (4) is a function of T/t_L only, where t_L is the Lagrangian integral time scale. Alternatively, if the turbulent spectrum is a function of the reduced frequency $f=nz/\bar{u}$, where z is the height and $\bar{u}$ the mean wind speed, then one may also show that this ratio is a function of x alone. Such an approach has led Draxler (1976) to propose a set of curves for S as a function of T, using representative values of t_L to account for different stabilities. Pasquill (1976) has summarized the variation of S with x, and suggested values to be used independent of terrain

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roughness, release height and sampling duration up to 1 h.

In the following sections we wish to show there are, in fact, systematic differences between different sets of experimentally determined values of S, as defined in (1). Moreover, these differences can be largely explained by a reexamination of the statistical "filtering" which affects σ_v and σ_θ values as determined in various field programs. For example, the restriction $\tau \geqslant T$ is not sufficient to ensure that S be independent of sampling time. Rather, S will be shown to exhibit a complicated dependence upon τ, $\bar{u}$, t, x and stability. In addition, a spectral model is presented which qualitatively accounts for many of the observed characteristics of S.

2. Sampling time and velocity effects

In (2), $[\sigma_y^2]_{\tau,T}$ is the quantity which is generally determined in a dispersion experiment. The variance of wind direction may be measured along with σ_y during the duration of the tracer release. This release time is just τ, as in (2), and the wind data are also usually averaged over some period t.

If $x\sigma_\theta \approx \sigma_v T$, then

$$x^2[\sigma_\theta^2]_{\tau,t}=[\sigma_v^2]_{\infty,0}T^2\int_0^\infty F_E(n)\times\left[1-\frac{\sin^2\pi n\tau}{(\pi n\tau)^2}\right]\frac{\sin^2\pi nt}{(\pi nt)^2}dn, \quad (5)$$

so that one may define an experimentally determined quantity

$$S=\frac{[\sigma_y]_{\tau,T}}{x[\sigma_\theta]_{\tau,t}}=\left\{\frac{\int_0^\infty F_L(n)\left[1-\frac{\sin^2\pi n\tau}{(\pi n\tau)^2}\right]\frac{\sin^2\pi nT}{(\pi nT)^2}dn}{\int_0^\infty F_E(n)\left[1-\frac{\sin^2\pi n\tau}{(\pi n\tau)^2}\right]\frac{\sin^2\pi nt}{(\pi nt)^2}dn}\right\}^{\frac{1}{2}}. \quad (6)$$

This is actually the quantity which Draxler (1976) and Pasquill (1975) considered in their analyses of various field programs. It should be noted that while the high-pass filter functions in the numerator and denominator of (6) are the same, the low-pass filters are not. In Pasquill's study, some general bounds on S were given, and two field programs were specifically treated, each of which had essentially identical sampling times. Draxler incorporated a larger number of tests in his analysis, but did not distinguish between different values of τ, in keeping with the assumptions made in deducing (4).

For the present study, data from five sets of experimental diffusion measurements made over flat terrain were examined. The programs considered were Project Prairie Grass (Barad, 1958), the Green Glow and Hanford-30 series (Fuquay *et al.*, 1964), some measurements made at NRTS (Islitzer and Dumbauld, 1963), and a previously unpublished series made at Hanford, Wash., and designated the Hanford-67 series (Nickola, 1977). In each, tracer materials were released and sampled near ground level, and σ_θ was measured near the release height for the duration of the release. Only those data were considered for which $T=x/\bar{u}\leqslant\tau$, where x is the distance to the sampling arc from the release point. This restriction ensured that the measured σ_y was essentially characteristic of a dispersing plume, rather than a puff.

The results are shown in Fig. 1. The plotted curves of S as a function of x are seen to order monotonically with sampling time τ. The data points for each series of tests are averages measured at the distances indicated. While variations about these averages are generally large enough so that the differences between the means of two series may sometimes be less than one standard deviation, the trend is clear. The values suggested by Pasquill (1976) are marked by squares, and are seen to correspond approximately to those associated with a sampling time of 10 min and an averaging time of 1 s.

The observed behavior is consistent with that predicted by (6), as can be seen from the following qualitative considerations. Fig. 2 shows $nF(n)$ (solid line) as a function of n, where $F(n)$ is some hypothetical Eulerian or Lagrangian spectrum. The denominator of (6) is proportional to the area enclosed by the dashed lines; an averaging time of $t=5$ s has been assumed. The numerator is proportional to the area enclosed by the dashed line on the left and the dash-dotted line on the right. As τ increases, the low-frequency ends of the filtered spectra move toward the left, increasing the numerator proportionately more than the denominator. If the Lagrangian and Eulerian spectra were identical and t is shorter than the travel time T, this would result in an increase of S with τ, independent of the particular spectrum shape. The situation is complicated by the fact that these spectra are generally not equal, but a similar dependence of S on τ can nevertheless be shown to be true over a wide range of assumed spectral behavior. Some examples will be presented later.

Pasquill (1971) has noted that ideally σ_θ should be determined with $t=0$, whereas in the experiments represented in Fig. 1 t varied from 1.0 to 20 s. From (6) it is apparent that smaller averaging times will result in smaller values of S, other conditions being equal. This probably accounts for part of the large differences between the Prairie Grass results and those obtained from the other field programs. However, the experimental curves do not order monotonically with t as they do with τ, so it is evident that the latter quantity can substantially influence the behavior of S as well. In the absence of detailed knowledge of the relevant spectra in (6), it is impossible to specify which of these contributing factors, t or τ, will have

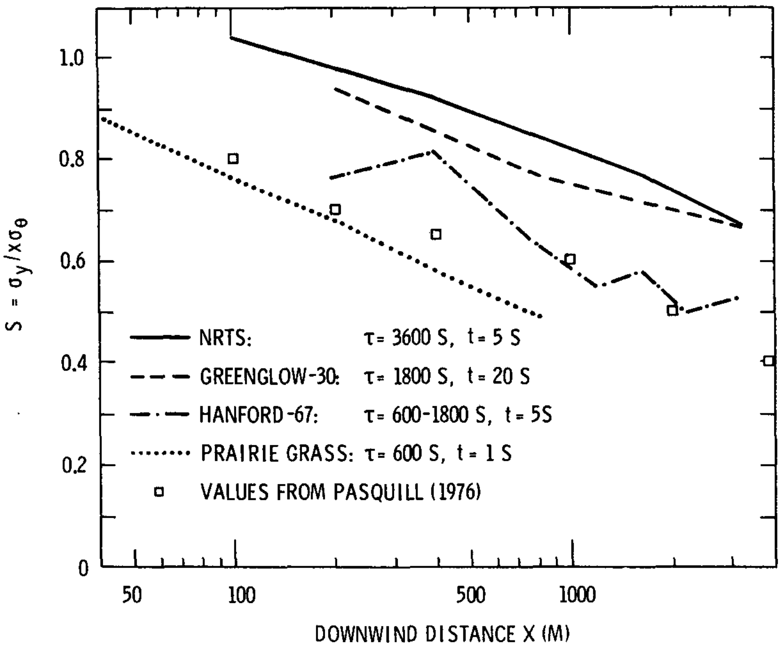

FIG. 1. Variation of $S=\sigma_y/x\sigma_\theta$ with distance as determined from several field programs.

the more important effects. It seems reasonable that both may be significant, and comparisons of results from various field programs should take variations in these quantities into account explicitly. This practice has not been followed in previous analyses.

There is some evidence (e.g., Kaimal *et al.*, 1972) that the Eulerian turbulent velocity spectra are functions of the dimensionless frequency $f=nz/\bar{u}$, where z is the height of measurement. If the same dependence is true for the Lagrangian spectra, then (6) may

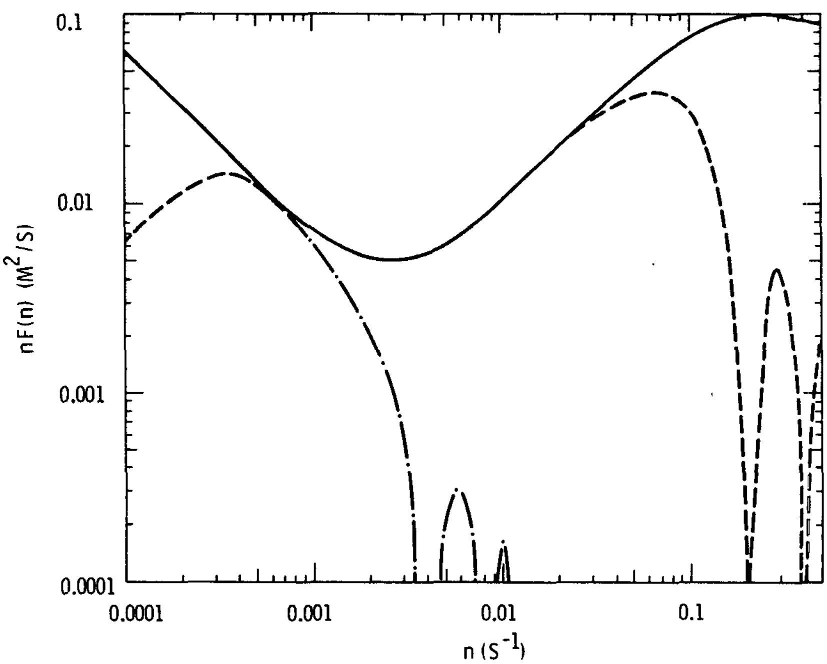

FIG. 2. Variation of $nF(n)$ (solid line) with frequency. Dashed line at left shows effect of high-pass filter while dashed line at right and dot-dashed line show effects of low-pass filters.

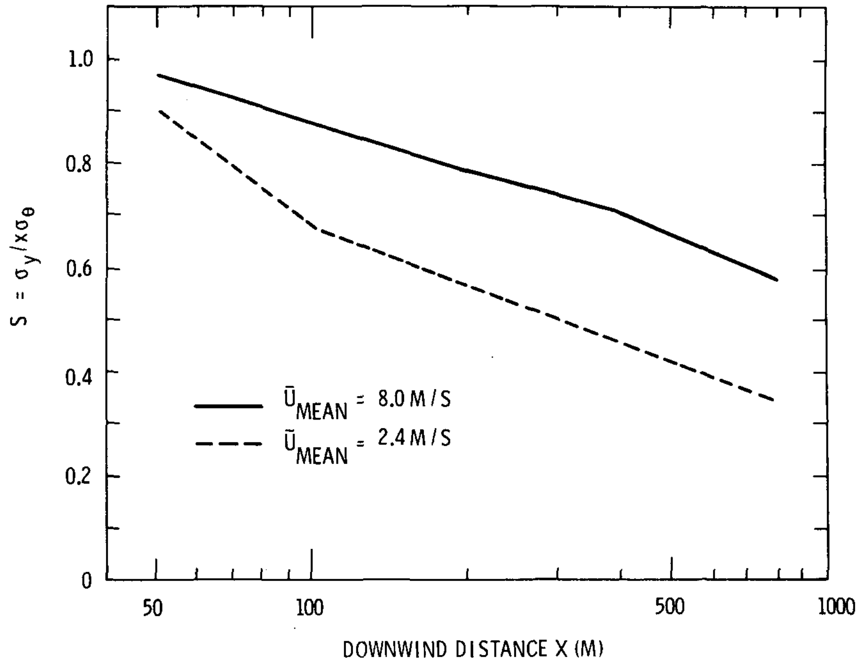

FIG. 3. Variation of $S=\sigma_y/x\sigma_\theta$ with distance for two velocity classes. Data are taken from Project Prairie Grass.

be written in an alternate form

$$S=\left\{\frac{\int_0^\infty F_L(f)\left[1-\frac{\sin^2(\pi f\bar{u}\tau/z)}{(\pi f\bar{u}\tau/z)^2}\right]\frac{\sin^2(\pi fx/z)}{(\pi fx/z)^2}df}{\int_0^\infty F_E(f)\left[1-\frac{\sin^2(\pi f\bar{u}\tau/z)}{(\pi f\bar{u}\tau/z)^2}\right]\frac{\sin^2(\pi f\bar{u}t/z)}{(\pi f\bar{u}t/z)^2}df}\right\}^{\frac{1}{2}}, \quad (7)$$

where the substitution $T=x/\bar{u}$ has been made in the numerator. Thus, in addition to its dependence on t and τ, S should also vary with the mean wind velocity in the same way it varies with τ.

Fig. 3 shows the results for eight tests from Project Prairie Grass with a mean velocity of 8.0 m s^{-1} and six tests with a mean velocity of 2.4 m s^{-1}. The velocity ranges in the two groups were 7.0–9.4 m s^{-1} and 1.4–2.8 m s^{-1}, respectively. The higher velocity values are associated with larger values of S, in agreement with the behavior predicted by (7). Efforts to corroborate this behavior with results from other field programs were inconclusive because of insufficient data or excessive scatter of the data.

3. Sample spectra

In the discussion thus far, comparisons of measured values of S have been made with the implicit assumption that the spectral behavior in the various cases has been similar. While this may be plausible when comparing ensembles of measurements such as might be collected during an entire field program, it is clear that a more refined characterization of the behavior of S must take possible differences into account. The spectra depend on the stability and the mean wind speed, but their precise behavior in the frequency regions which contribute most strongly to S is not well defined. For values of t and T which are encountered in typical measurements, this corresponds to a range of f from 10^{-4}–10^{-2}. The situation is particularly unsatisfactory for unstable conditions (Kaimal *et al.*, 1972), although considerable progress has been made in this area (Kaimal *et al.*, 1976). Despite these difficulties, it is possible to qualitatively model several features of the behavior of S, in addition to the dependence on t and τ already discussed.

Two sample spectra were constructed, one corresponding to slightly stable conditions and one to slightly unstable conditions. Several assumptions were made: 1) the stable spectrum can be described by the general expression given by Kaimal *et al.* (1972) for frequencies above $f\approx 0.005$; 2) the spectrum for the unstable case can be approximated by an expression similar to that used for the stable case, except that the frequency at which $nF_E(n)$ attains its maximum value $f_{\max}$ is shifted downward; 3) a spectral gap exists at $f_{\min}$, below which $nF_E(n)$ becomes larger as f decreases; 4) $nF_E(n)$ is a function of the dimensionless frequency f even for the unstable case; and 5) the Lagrangian and Eulerian spectra are related in the manner suggested by Hay and Pasquill (1959) over their whole frequency range, *viz.*, $F_L(n) =\beta F_E(\beta n)$ with $\beta=4$.

The rate at which $nF_E(n)$ decreases with frequency as the spectral minimum is approached from below

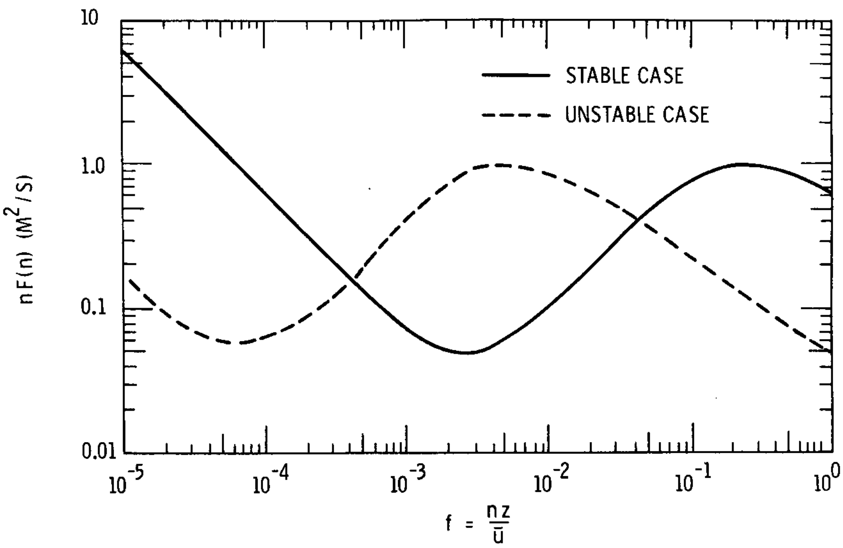

FIG. 4. Sample spectra for stable and unstable conditions. Spectra are not normalized.

is not well known. Kaimal *et al.* (1976) suggest a −2 power dependence, while Hess and Clarke (1973) found values closer to −1.5. Panofsky and Van der Hoven (1955) and Smedman-Högström and Högström (1975) show a variety of spectra with various slopes. In several cases, a −1 power law seems to fit the data reasonably well. The actual slope chosen depends on the frequency range below $f_{\min}$ one wishes to represent, since the curve becomes flatter as the minimum is approached. A value of −1 was adopted for our sample spectra.

As a working hypothesis, we therefore postulate the following forms as plausible approximations:

For the stable case

$$nF_E(n)=\frac{6.25\times10^{-6}}{f}+\frac{f}{1+15.2f^{5/3}} \tag{8}$$

For the unstable case

$$nF_E(n)=\frac{3.6\times10^{-9}}{f}+\frac{f}{1+10^4f^{5/3}}. \tag{9}$$

Eq. (8) applies to a spectrum with values of $f_{\max}$

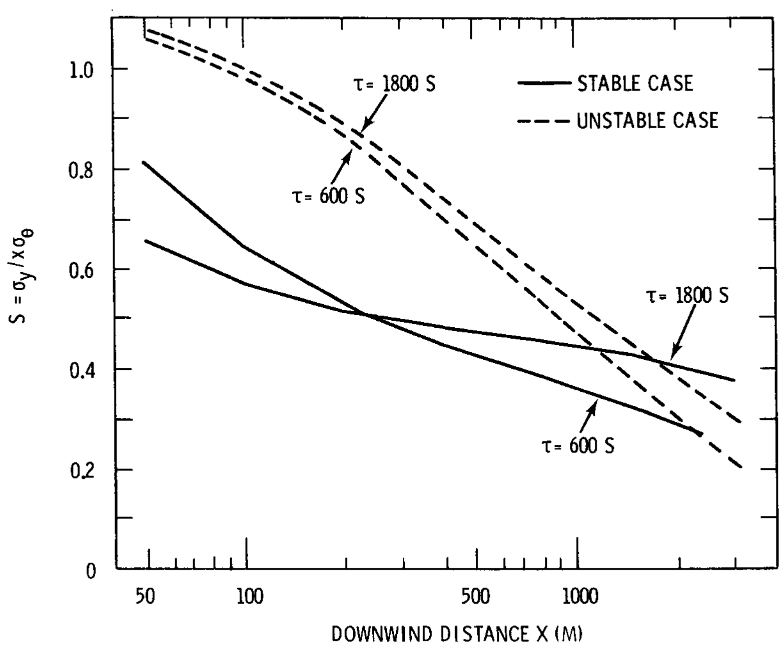

FIG. 5. Variation of $S=\sigma_y/x\sigma_\theta$ with distance. Values are computed from sample spectra for two values of sampling time τ.

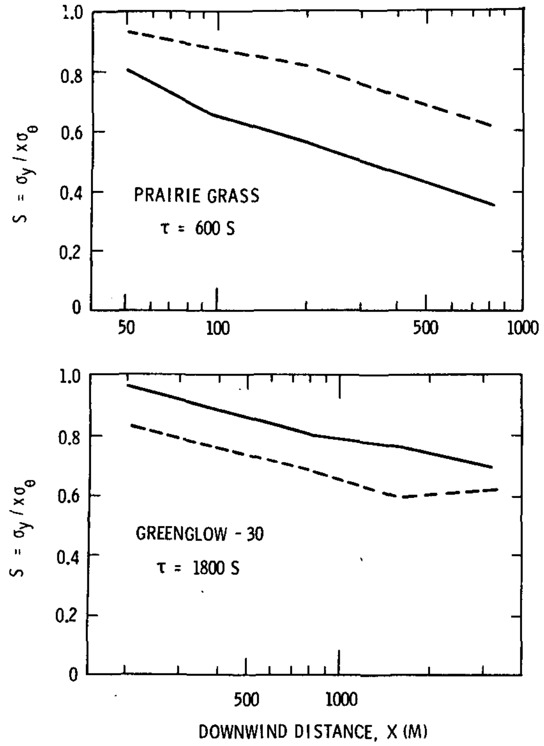

FIG. 6. Variation of $S=\sigma_y/x\sigma_\theta$ with distance for stable (solid line) and unstable (dashed line) conditions.

$=0.25$ and $f_{min}=0.0025$, while (9) corresponds to $f_{max}=0.005$ and $f_{min}=0.00006$ (Kaimal *et al.*, 1972; Smedman-Högström and Högström, 1975). These sample spectra are shown in Fig. 4. They are not normalized either on an absolute scale or with respect to each other, but this is immaterial since any normalizing factors would cancel on application of (6) or (7).

Fig. 5 shows the behavior of S obtained from these spectra, assuming an averaging time $t=5$ s and a mean velocity $\bar{u}=5$ m s^{-1}. Two curves are shown in each case, for sampling times τ of 600 and 1800 s. The curves extend only to a distance x such that $T=x/\bar{u}\leqslant\tau$.

A number of features are immediately apparent. For both stable and unstable spectra, longer sampling times generally imply larger values of S. (An exception to this may be noted for the stable spectrum at small downwind distances.) The values of S determined from the unstable spectrum are larger close to the release point, while the stable spectrum produces larger values farther downstream. The crossover point depends on sampling time; as τ increases, this point moves toward shorter distances. The exact behavior is, of course, dependent on the actual form of the spectra, but a possible indication of this crossover phenomenon may be seen in Fig. 6. For the Prairie Grass results, the higher values of S are associated with those runs taken during daylight hours, while the lower values of S are associated with the more stable night runs. For the Green Glow-30 series, which had longer sampling times, the relative ordering of the curves with stability is reversed. While the actual measured values of S are not reproduced by (8) and (9), their qualitative behavior may be interpreted in terms of the crossover effect.

It is clearly possible to adjust the spectra to produce virtually any desired result, but such an exercise would be pointless. Rather, we have presented two cases which are plausible representations of previously observed spectra to demonstrate the importance of several parameters which have hitherto been neglected in describing the characteristics of plume dispersal.

4. Conclusions

The relationship between $S=\sigma_y/x\sigma_\theta$ and x as proposed by Pasquill is approximately correct for a wide range of conditions, but a more precise formulation of this dependence must take into account a number of factors previously disregarded. Sampling time, mean velocity and stability may all have important effects. In particular, a knowledge of σ_θ is not sufficient to specify σ_y, although in the absence of more detailed information, it is useful as a rough indicator.

The results presented also underscore the necessity for further study of turbulent spectra in the dimensionless frequency range 10^{-4}–10^{-2}. It is the turbulent eddies in this range which tend to dominate the diffusion process, but information about the spectral shapes and their dependence on stability is lacking. While the higher frequency components make important contributions to σ_θ, their effect is relatively less important for σ_y. A complete treatment of the lateral diffusion process cannot be restricted to the frequencies usually treated in micrometeorological studies.

Until additional studies are performed, we recommend some modifications in the values of S originally suggested by Pasquill. For an averaging time of 5 s for σ_θ, the suggested variation of S with x is given in Table 1 for two sampling times (1800 and 3600 s). These values are applicable only for flat homogeneous terrain and for releases near the ground. The results of additional studies indicating the effects of release height will be published elsewhere.

It is interesting to note that the value of S at

TABLE 1. Recommended values of $S=\sigma_y/x\sigma_\theta$ for $t=5$ s.

	x (km)						
	0.1	0.2	0.4	0.8	1.6	3.2	10*
S (τ=1800 s)	0.95	0.85	0.76	0.70	0.64	0.58	0.52
S (τ=3600 s)	1.04	0.98	0.92	0.85	0.77	0.67	0.54

* Extrapolated

$x=0.1$ km is larger than unity for $\tau=3600$ s. Such a value would not be possible if (4) were the experimental quantity actually observed. However, reference to (6) and Fig. 2 suggest that for sufficiently short travel times T, S can attain values larger than 1. In the framework of the Hay-Pasquill theory (1959), this would require that $T<\beta t$.

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