Comments on "Numerical Modeling of Advection and Diffusion of Urban Area Source Pollutants"

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ABSTRACT

The moment method for accurate Eulerian advection computation, originally suggested by Egan and Mahoney (1972), was developed and tested for problems in two dimensions by Pedersen and Prahm (1973, 1974). A two-dimensional version was discussed by Egan and Mahoney (1972) but their mathematical formulation was incomplete. The necessary corrections and additions are presented here.

Egan and Mahoney (1972) described an interesting and much referred method for accurate advection computations in one dimension, called the moment method. Most practical problems are treated at least in two dimensions. Their provisional two-dimensional mathematical formulas, however, suffer from various shortcomings and, unfortunately, Egan and Mahoney (1972) do not present a complete mathematical formulation of a two-dimensional model.

Pedersen and Prahm (1973, 1974) developed, documented and tested a two-dimensional mathematical formulation and computer code of the moment method. In order to encourage the use of the moment method, comments are given here to the prefatorial formulas by Egan and Mahoney (1972) and the necessary moment-conserving transfer formulas are added (Pedersen and Prahm, 1973, 1974).

The formulas (6), (14) and the expressions for P_x and P_y given by Egan and Mahoney (1972, p. 319) should be slightly changed to the equivalent formulas (4), (18) and (20), (21) given by Pedersen and Prahm (1974). Following the notation of Egan and Mahoney

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(1972) these corrected formulas are

$$R_m{}^2=12\int_{0.5}^{0.5} C(\zeta_m)(\zeta_m-F_m)^2 d\zeta/C_m,$$

$$(R^{T+1})^2=\sum_i C_iR_i{}^2/C^{T+1}$$
$$+12[\sum_i C_iF_i{}^2/C^{T+1}-(\sum_i C_iF_i/C^{T+1})^2],$$

where $i=1,\ldots,I$, and I is the number of single puffs in each cell at the time $T+1$ and

$$P_x=[R_x-1+2\sigma_x/|\sigma_x|(F_x+\sigma_x)]/(2R_x),$$
$$P_y=[R_y-1+2\sigma_y/|\sigma_y|(F_y+\sigma_y)]/(2R_y).$$

In addition to the correct portioning parameters and summation expressions for computation of the zero, first and second order moments, the transfer relations of these quantities between neighboring cells are required. Following a notation equivalent to Egan and Mahoney (1972) the contribution to donor cells and neighboring cells from cell m,n is determined as follows, where the first two indices gives the cell numbers and the third the coordinate direction (Pedersen and Prahm, 1973, 1974). The left-hand side quantities are identical to C_i, F_i and R_i used above:

Contribution to cell m,n from m,n

$$C_{m,n}^{T+1}=(1-P_{m,n,x})(1-P_{m,n,y})C_{m,n}^T$$
$$F_{m,n,x}^{T+1}=(1-R_{m,n,x}^T+P_{m,n,x}R_{m,n,x}^T)\sigma_{m,n,x}/(2|\sigma_{m,n,x}|)$$
$$F_{m,n,y}^{T+1}=(1-R_{m,n,y}^T+P_{m,n,y}R_{m,n,y}^T)\sigma_{m,n,y}/(2|\sigma_{m,n,y}|)$$
$$R_{m,n,x}^{T+1}=(1-P_{m,n,x})R_{m,n,x}^T$$
$$R_{m,n,y}^{T+1}=(1-P_{m,n,y})R_{m,n,y}^T.$$

Contribution to cell $m+\sigma_x/|\sigma_x|$, n from m,n

$$C_{m+\sigma_x/|\sigma_x|,n}^{T+1}=P_{m,n,x}(1-P_{m,n,y})C_{m,n}^T$$
$$F_{m+\sigma_x/|\sigma_x|,n,x}^{T+1}=(P_{m,n,x}R_{m,n,x}^T-1)\sigma_{m,n,x}/(2|\sigma_{m,n,x}|)$$
$$F_{m+\sigma_x/|\sigma_x|,n,y}^{T+1}$$
$$=(1-R_{m,n,y}^T+P_{m,n,y}R_{m,n,y}^T)\sigma_{m,n,y}/(2|\sigma_{m,n,y}|)$$
$$R_{m+\sigma_x/|\sigma_x|,n,x}^{T+1}=P_{m,n,x}R_{m,n,x}^T$$
$$R_{m+\sigma_x/|\sigma_x|,n,y}^{T+1}=(1-P_{m,n,y})R_{m,n,y}^T.$$

Contribution to cell $m,n+\sigma_y/|\sigma_y|$ from m,n

$$C_{m,n+\sigma_y/|\sigma_y|}^{T+1}=R_{m,n,y}^T(1-P_{m,n,y})C_{m,n}^T$$
$$F_{m,n+\sigma_y/|\sigma_y|,x}^{T+1}$$
$$=(1-R_{m,n,x}^T+P_{m,n,x}R_{m,n,x}^T)\sigma_{m,n,x}/(2|\sigma_{m,n,x}|)$$
$$F_{m,n+\sigma_y/|\sigma_y|,y}^{T+1}=(P_{m,n,y}R_{m,n,y}^T-1)\sigma_{m,n,y}/(2|\sigma_{m,n,y}|)$$
$$R_{m,n+\sigma_y/|\sigma_y|,x}^{T+1}=(1-P_{m,n,x})R_{m,n,x}^T$$
$$R_{m,n+\sigma_y/|\sigma_y|,y}^{T+1}=P_{m,n,y}R_{m,n,y}.$$

Contribution to cell $m+\sigma_x/|\sigma_x$, $n+\sigma_y/|\sigma_y|$ from m,n

$$C_{m+\sigma_x/|\sigma_x|,n+\sigma_y/|\sigma_y|}^{T+1}=P_{m,n,x}P_{m,n,y}C_{m,n}^T$$
$$F_{m+\sigma_x/|\sigma_x|,n+\sigma_y/|\sigma_y|,x}^{T+1}=(P_{m,n,x}R_{m,n,x}^T-1)\sigma_x/(2|\sigma_x|)$$
$$F_{m+\sigma_x/|\sigma_x|,n+\sigma_y/|\sigma_y|,y}^{T+1}=(P_{m,n,y}R_{m,n,y}^T-1)\sigma_y/(2|\sigma_y|)$$
$$R_{m+\sigma_x/|\sigma_x|,n+\sigma_y/|\sigma_y|,x}^{T+1}=P_{m,n,x}R_{m,n,x}^T$$
$$R_{m+\sigma_x/|\sigma_x|,n+\sigma_y/|\sigma_y|,y}^{T+1}=P_{m,n,y}R_{m,n,y}^T.$$

For $P_{m,n,i}>1$ or <0 the portioning parameter is assigned the value 1 and 0, respectively. In this case, however, special formulas for the center of mass must be applied and the user is referred to Pedersen and Prahm (1973) for these expressions. Our mathematical formulation and computer code were successfully applied for various problems [e.g., regional sulfur transport in Europe (Nordø, 1974), urban and rural air pollution studies in Norway (Grønskei, 1976, personal communication; Sivertsen, 1976), ice drift forecasting in the bay and sea of Bothnia (Udin and Ullerstig, 1976), transport in fluids (Harding, 1974, personal communication), and dispersion computations relevant to nuclear installations (Pepper, 1976, personal communication)].

The moment-conserving principles were developed to minimize the numerical dispersion characteristic for finite-difference methods. The numerical tests (Pedersen and Prahm, 1974) show that the method is especially accurate for single puffs from area sources which are not situated in the same or neighboring cells. The accuracy was compared with other methods (Table 1, Christensen and Prahm, 1976) and found to be inferior to the pseudospectral method presented by these authors. Those who refer to the formulas by Egan and Mahoney (1972) and who do not use the moment conserving formulas given here (Pedersen and Prahm, 1973, 1974), should quantify their numerical errors and specify their method before the models can be considered reliable.

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