Measurement of Convective Mean Rainfall over Small Areas using High-Density Raingages and Radar[1]

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ABSTRACT

Techniques of measuring area-mean convective rainfall over small areas (<2000 km^2) are investigated using data from several gage-radar rainfall measurement studies. These data suggest that for low gage densities ($\lesssim 1$ gage per 250–300 km^2) and for some climates (e.g., Illinois) that gage-radar area-mean convective rainfall measurements may be more accurate than gage-only measurements. The same result was not supported by data from another climate, suggesting that changes in raincell size and rain evaporation rate may affect the results. Further studies are needed to verify these suggestions. The data indicate that radar adds little to gage measurement of mean areal convective rainfall for gage densities of $\gtrsim 1$ gage per 100–200 km^2.

Use of a space-variable gage adjustment of radar data is seen to be preferable to use of an averaged single adjustment factor, except in cases involving extrapolations of the space-variable adjustment factor. For such cases, e.g., over large lakes or outside of gage networks, use of the averaged single adjustment factor may be preferable.

1. Introduction

Accurate measurement of rainfall is an elusive but important problem which has been investigated from many points of view. Current techniques of measuring rainfall include the use of raingages, the use of radar, and various techniques of combining radar and raingage data in order to take advantage of the different capabilities of the two instruments. This paper is an evaluation of a technique of combining the raingage and radar measurements in a manner which is designed to utilize the point accuracy of the raingages and the spatial sampling capabilities of the radars. The technique is evaluated using convective rainfall data from two field experiments located in different climates. The analyses give some indication of climatic differences and the potential transferability of the results.

Raingages are frequently regarded as a costly, troublesome, time-consuming method of measuring rainfall. However, they can be shown to be accurate except on very short time ($\lesssim 1$ min) and space (less than gage spacing) scales. The work of Huff (1970, 1971) indicates that networks of many raingages accurately measure mean areal rainfall amounts. Huff considered the accuracy of convective rainfall measurement using a gage network of 49 gages in 1000 km^2 and 49 gages in 1500 km^2 located in Illinois. He compared mean rainfall estimates from various densities, measurement periods, network areas and precipitation intensities. His results are summarized by a regression equation which indicates that the relative errors in mean areal rainfall estimates increase with decreasing gage density, precipitation amount and storm duration. His findings thus support an intuitive consideration of gage rainfall sampling. A surprising and gratifying result of his research is the conclusion that area-mean gage rainfall measurement should be accurate to within about 5% for gage densities >1 gage per 50 km^2 and for rainfall rates >10 mm h^{-1}. In addition, area-mean rainfall measurements should be accurate to within 10% for gage densities >1 per 160 km^2 and for rainfall rates $\gtrsim 4$ mm h^{-1}. These results indicate that despite their problems, raingages at sufficient densities can provide accurate mean areal rainfall estimates. These gage rainfall estimates can be used as a standard of comparison in the evaluation of other techniques of measuring rainfall. Any deviation of other types of rainfall estimates from gage rainfall estimates must be interpreted as a deviation from a standard of known accuracy, but not necessarily as an indication of error. In fact, certain other types of rainfall estimates may be expected to be more accurate in certain cases. This would include, for example, radar estimates of rainfall

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variability on scales smaller than gage spacing. This example, of course, is the impetus for techniques of combining gage and radar data.

The use of radar to measure rainfall has not proved overly successful as a method of replacing raingages. The primary reason for the inability of radar to accurately measure rainfall lies in the uncertain relation between the radar reflectivity factor Z and the rainfall rate R. Average Z-R relations are easily formulated on the basis of theory, observation of raindrop size distributions or empirical Z-R relation; however, these Z-R relations provide mean values which generally do not reflect daily, storm-to-storm or within-storm variability of drop size distributions, particularly in convective rainfall. In addition, there are inevitable slight errors in radar calibration which, when combined with the Z-R relation errors, make the radar-rainfall estimates accurate to only $\sim 50\%$ (Atlas, 1964). Additional errors in radar-rainfall estimation result from the significant effects of attenuation, plus evaporation and advection of the rainfall while falling from the radar's sampling volume to the ground. Vertical air motions induce Z-R errors by changing the vertical rain water flux, and by changing the drop size distribution in vertically inhomogeneous situations (Carbone and Nelson, 1978).

Due to the various difficulties and costs of gage rainfall measurements and the inherent problems with radar rainfall measurements, many investigators have turned to techniques of gage adjustment of radar data to derive mean rainfall estimates. Wilson (1970) determined that the radar estimates of rainfall from Oklahoma storms could be improved by using a raingage to calculate a correction factor. He found that the utility of the correction factor decreased with increasing distance from the gage to the rainfall, and suggested that a field of weighting factors derived from a scattered network of gages would be an improvement. Woodley *et al.* (1975) used a daily areal average correction factor in measuring rainfall from Florida air mass showers, primarily because of the large spatial gradients of rainfall in such storms. In such convective conditions with high rainfall gradients, or similarly with very small rainfall amounts, the gage-measured amount may be unrepresentative of the mean precipitation being measured by the radar, due to frequent occurrence of rain between gages. The technique of deriving one correction factor to apply to an entire field of radar-indicated rainfall has the disadvantage that the final rainfall field frequently does not agree with the gage measurements at individual gage locations.

Brandes (1975) developed a technique of combining gage and radar rainfall estimates. His technique was designed to take advantage of the point accuracy of gage measurements, and the good spatial coverage of radar measurements. The major steps in Brandes' technique are as follows:

1) Gage-radar rainfall ratios (G/R ratios) are calculated at gage locations using evenly weighted radar data from within a fixed radius from the gage. This radius is chosen to be small with respect to gage spacing.

2) Radar rainfall data are thresholded and converted to a Cartesian grid having a grid spacing roughly equivalent to radar data spacing.

3) The gage rainfall data and G/R ratios are transformed to the same Cartesian grid as the radar data using the objective analysis scheme of Barnes (1964). The weighting factor used reflects gage density.

(4) An adjusted radar rainfall field is obtained by a point-by-point multiplication of the G/R ratio and radar rainfall fields.

5) The final rainfall field is constructed using the gage and adjusted radar rainfall fields. The final rainfall estimate is set to 100% of the gage value at gage locations and linearly interpolated to 100% of the adjusted radar value at some fixed radius away from the gage locations.

Although issue can be taken with several aspects of this technique, including the weighting functions and influence radii used in the coordinate conversions and the empirically selected thresholds on radar rainfall data and G/R ratios, the work of Wilson (1970), Wilson,[4] Brandes (1975) and others suggests the technique has promise.

More recently, Eddy and Crawford[5] have developed a technique which makes use of the time-space correlations of the radar and gage data to produce a final rainfall estimate. While conceptually appealing, this technique has not yet been applied in such a manner that its utility can be directly ascertained through comparisons with the known standard of gage measurements.

Cain and Smith[6] have suggested a variation of Woodley's technique in which sequential analyses methods are applied to the radar data. Through an objective process a mean radar rainfall correction factor is developed, based on available gage and radar data. Then for each succeeding rain event (hour or storm) the accuracy of the correction factor is evaluated. When the combined gage-radar rainfall estimate differs sufficiently from the gage measurement over a sufficient duration, an adjustment is made to the correction factor. As is the case with Eddy's technique, this technique has not yet been applied in a situation in which its

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[4] Wilson, J. W., 1975: Radar-gage precipitation measurements during the IFYGL. Rep. 4177-540, The Center for the Environment and Man, Inc., Hartford, CT, 93 pp. [NTIS PB-264 960/6G1].

[5] Eddy, A., and K. C. Crawford, 1977: Rainfall derived from gages and radar: a statistical analysis and evaluation procedure. *Preprints Sixth Conf. Inadvertent and Planned Weather Modification*, Champaign, Amer. Meteor. Soc., 380–383.

[6] Cain, D. E., and P. L. Smith, 1976: Operational adjustment of radar estimated rainfall with raingage data: a statistical evaluation. *Preprints 17th Conf. Radar Meteorology*, Seattle, Amer. Meteor. Soc., 533–538.

utility can be ascertained through comparison with gage data.

The main goal of this paper is to investigate the characteristics of a combined gage-radar rainfall measurement technique which is, with minor modifications, that of Brandes (1975). The gage-radar rainfall estimates will be compared with gage rainfall measurements, which provide a standard of known accuracy. In the process, the effects of gage density on rainfall measurement accuracy will be investigated. The accuracy of a mean rainfall correction factor will be assessed, and a brief consideration of rainfall measurement over large gage voids will be presented.

2. Sources of data

Two sources of data were used in this study. The first was collected as part of the HIPLEX Montana experiment during the summer of 1976. There, 109 recording raingages were operated in a 1500 km^2 network which was centered ~60 km from a 5 cm radar located in Miles City. As a part of this experiment, additional precipitation data were collected using rain disdrometers and instrumented aircraft. Another project, the Chicago Hydrological Area Project (CHAP), provided data from 320 recording raingages in a 10^5 km^2 area near Chicago (Changnon and Semonin, 1978). Radar data were provided by two 10 cm radars located on the edge of the gage network. The CHAP data analyses were based on the averaged results of five subarea analyses, the subareas each having 50–60 gages in 1000–1200 km^2. For both experiments, weighing raingages were used with daily charts which were changed once a week. These raingage charts were digitized in such a manner that 0.25 mm changes in rainfall accumulation were resolved. The gage rainfall values were evaluated every 5 min. The radars for both experiments had similar sampling characteristics, with ~1° beamwidths, pulse depths of 0.5 km for HIPLEX and 0.15 km for CHAP, and volume scan times of 3–5 min.

3. Analysis techniques

Three different gage-radar data processing programs were developed to serve various research goals. All three embody the basic philosophy of Brandes (1975). The three sets of goals dictated that three different sets of analysis decision criteria be applied to the gage and radar data. The three analyses were those of 1) an analysis of HIPLEX data, 2) the CHAP data analyses (Huff and Towery),[7] and 3) an additional analysis of CHAP data (Snell, 1977). The HIPLEX analysis routine was directed at an evaluation of gage-radar rainfall data processing techniques. The CHAP routine was directed at real-time measurement of rainfall and application to urban hydrological problems (Changnon and Semonin, 1978). The second analysis of CHAP data (Snell, 1977) was directed at the problem of measurement of rainfall over large bodies of water and will be referred to as the LAKE analysis. The major reason for consideration of data from these three separate rainfall analysis routines is that through use of the separate and different routines, with their different decision criteria, we are able to rule out the possibility that some of these decision criteria are central to affecting the results. It is the basic Brandes (1975) technique goal of combining the gage and radar data to produce a rainfall measurement that we are testing herein. For the sake of brevity, the HIPLEX analysis routine will be described first, then the significant differences between the HIPLEX and the other routines will be described.

In the HIPLEX analysis routine, radar and gage data were selected for a given time period and the radar data were converted to rainfall amounts using a climatologically appropriate Z-R relation. Radar and gage data time periods were adjusted for the time it took for the rain to fall from the radar observation level to the ground (3–5 min), but not adjusted for advection or vertical air motion. (This was because of lack of knowledge of the true advection of rain near convective cells.) When the radar data for the time period were collected, they were transformed onto a Cartesian grid of 0.5 km spacing using even weighting of the four polar grid points surrounding each Cartesian grid location.

Next, gage-radar rainfall (G/R) ratios were calculated at each gage location. At this point the radar rainfall amounts were subjected to a minimum rainfall threshold which was selected in a manner which made the radar and gage rainfall boundaries roughly equal. This threshold had the effect of a crude evaporation correction for the radar data. In determining the value of the radar rainfall at gage locations, the four surrounding radar data points were used with an inverse-linear distance weighting from the gage location. The resulting G/R ratios at the gage locations were then spread over the Cartesian grid using Gaussian weighting with an influence radius slightly larger than the half-mean gage spacing. The typical G/R ratios for HIPLEX were of the order 5–20 with standard deviations roughly equal to the mean values. The values of the G/R ratios were restricted to the range 0.1–100, in order to delete spurious values which, in most cases, resulted from sampling errors and mismatch of gage and radar rainfall patterns.[8] A gage-radar rainfall analysis was

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[7] Huff, F. A., and N. G. Towery, 1978: Utilization of radar in operation of urban hydrologic systems. *Preprints 18th Conf. Radar Meteorology*, Atlanta, Amer. Meteor. Soc., 437–441.

[8] The HIPLEX radar rainfall estimates used herein systematically underestimated rainfall as verified by raingages (e.g., see Table 1) and aircraft. The source of this bias was extensively investigated and no cause was located. Consequently, in the present research the HIPLEX G/R ratios were limited to

then obtained on the Cartesian grid using the radar rainfall estimate at each grid point multiplied by the G/R ratio value of that grid point. For gage locations having nonzero gage and radar values, the gage-radar analysis agreed with the gage values. Consequently, the last step of the Brandes (1975) technique, forcing to the gage values at gage locations, was omitted.

An evaporation correction was made by examining the gage-radar rainfall analysis and comparing it with the gage-only rainfall data. At locations where the gage-radar analysis had rain, but the gage data showed none, the gage-radar analysis value was set to zero within a radius equal to half of the mean gage spacing. Samples of the four rainfall analyses produced by this computer program are shown in Fig. 1 using data collected over a period of 1 h. Additional examples can be found in Achtemeier *et al.*[9] Fig. 1a is the gage-only rainfall analysis; Fig. 1b is the radar-only rainfall analysis; Fig. 1c is the gage-radar rainfall analysis prior to evaporation correction; and Fig. 1d is the final gage-radar rainfall analysis including evaporation correction. Gage locations are shown by the large dots on Figs. 1a, 1c and 1d. The contours, particularly the zero contour, have been smoothed. The dissimilarity of the gage and radar rainfall data are immediately apparent in Figs. 1a and 1b; the radar shows rainfall patterns which are totally absent from the gage-only analysis, and rainfall rates far below the gage values. In addition, the radar shows areas of rainfall in the center of the grid which are not seen in the gage analysis. Through the work of Huff (1970, 1971) and other investigators, the accuracy of the gage mean areal rainfall amounts must be accepted, although the radar-measured variability could possibly be the more correct.

The combined gage-radar rainfall analysis in Fig. 1c agrees fairly well in amplitude with much of the gage-only analysis, but still misses the gage zero rainfall contour extending through the middle of the grid. In addition, there are other features which are in different locations in different analyses or are totally missing from one analysis or another. In particular, there are features around the edge of the gage-radar analyses (Figs. 1c and 1d) which apparently are the result of erroneous extrapolation of G/R values outside of the area bounded by gages. These features must be regarded as spurious. While such features were not routine, they illustrate the need to limit the value of the gage-radar ratios, the need to limit extrapolation of the ratios, and the need, at least in the HIPLEX analysis, to have a final evaporation correction. The final gage-radar rainfall analysis with evaporation correction (Fig. 1d) agrees much better with the gage-only rainfall analysis (Fig. 1a). Comparison of the final gage-radar and gage-only analyses indicates some fine-scale features which the gages missed, and several gage locations for which the gage-only analysis indicated rain and the gage-radar analysis had been forced to zero. These last differences are due to the smoothing effects of interpolation in the gage-only analysis, and the forcing of the final gage-radar analysis to zero around individual gages having no rain. Since these differences generally occur in areas of relatively low rainfall rate, they have only a small effect on comparisons of mean areal rainfall amounts.

The CHAP analysis routine was similar to the HIPLEX routine except a 2.4 km grid was used; G/R ratios were derived using even weighting of the four radar rainfall values surrounding gage locations; G/R ratios were limited to $0.1 \leqslant G/R \leqslant 10.0$; final rainfall comparisons were made at the gage locations, rather than on the grid; and no final evaporation correction was made.

The LAKE analysis routine was similar to the HIPLEX analysis routine except that a 2.4 km grid was used; G/R ratios were limited to $0.1 \leqslant G/R \leqslant 10$; and no final evaporation correction was made. In the cases of the CHAP and LAKE analyses, inspection showed that a final evaporation correction would not have been productive, for the minimum rainfall threshold appeared to effectively correct for evaporative effects.

For all three analyses (CHAP, LAKE and HIPLEX), the gage rainfall was considered to be the correct or best measurement of rainfall, and the gage-radar analysis techniques were adjusted to approximately equal the gage values at gage locations. A deviation from the Brandes (1975) technique, which was embodied in all three analyses, was the lack of a forcing of the final rainfall analysis to exactly equal gage values at all gage locations. In the case of the HIPLEX analyses, such a forcing would have produced little change for the condition was met directly at gage locations where both the gage-only and the final gage-radar analyses indicated rain. In the case of the CHAP and LAKE analyses, there were only small residual differences.

The possible effects of radar miscalibration were tested in both the CHAP and HIPLEX analyses. In both cases, mean errors were artificially injected in the radar data, and the analysis technique fully

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$0.1 \leqslant G/R \leqslant 100.0$ rather than $0.1 \leqslant G/R \leqslant 10.0$, which would be applicable if the problem did not exist. Extensive tests of the present data analysis routines indicated that the apparent bias was removed accurately and that the bias appeared to be roughly constant throughout the data. These tests included simulations of radar calibration errors and random errors in the data. Although these HIPLEX radar data are apparently affected by both underestimation of rainfall rates and by severe evaporation below the lowest radar observation level, they serve to illustrate the capabilities of the rainfall estimation technique in an adverse situation.

[9] Achtemeier, G. A., P. H. Hildebrand, P. T. Schickedanz, B. Ackerman, S. A. Changnon, Jr. and R. G. Semonin, 1978: Design and evaluation techniques for High Plains Cooperative Program. Final Report, Division of Atmospheric Water Resources Management, Bureau of Reclamation, Dept. of Interior, Contract 14-06-D-7197, Illinois State Water Survey, Urbana, 269 pp. [Available from the author.]

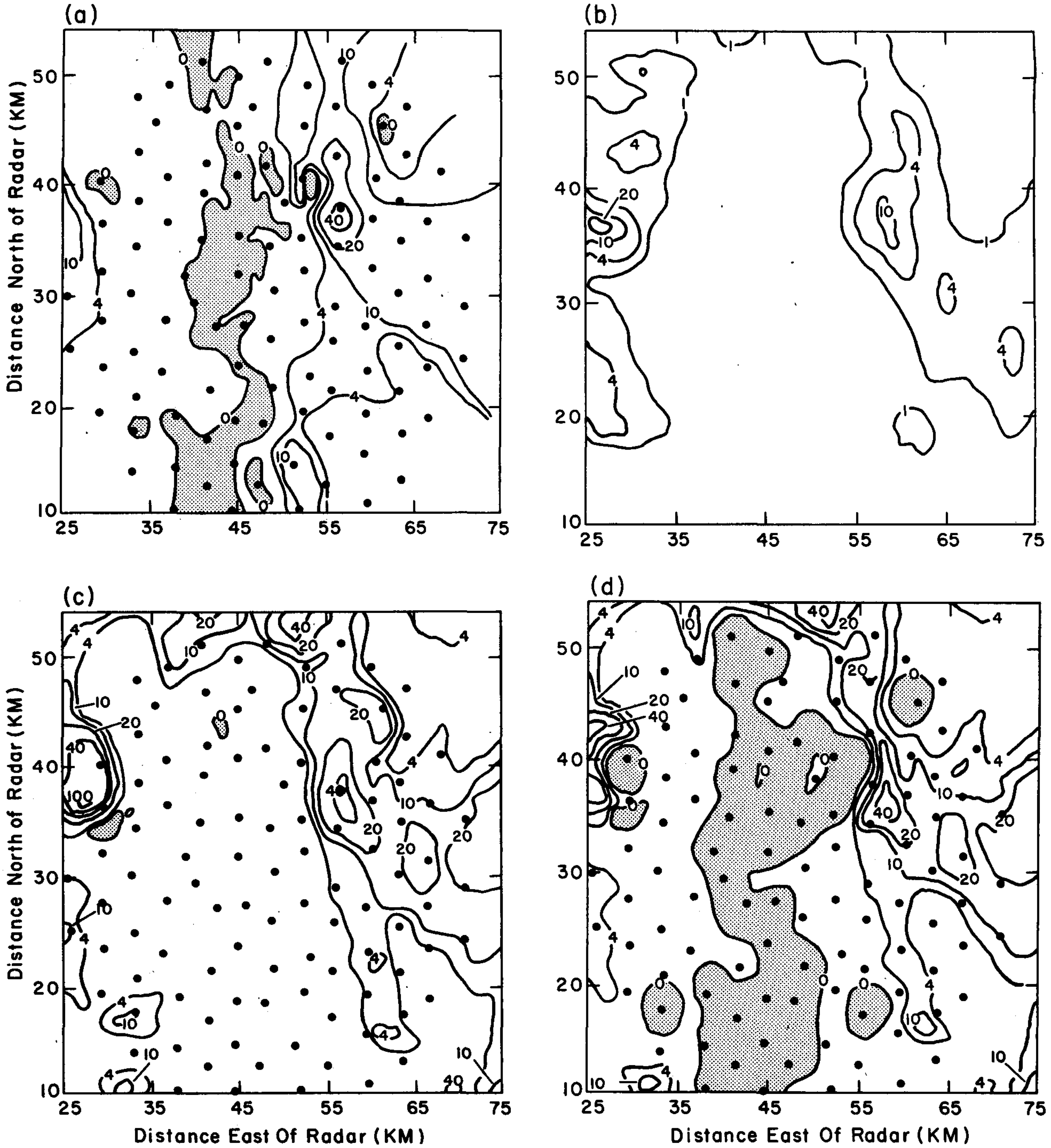

FIG. 1. Sample 60 min rainfall analyses from the HIPLEX data set. Rainfall amounts are in mm h^{-1}. Gage-only (a), radar-only (b), gage-radar without evaporation correction (c), and final gage-radar analyses with evaporation correction (d) are shown. Gage locations are shown by the large dots. Areas with no rain are shaded.

removed the calibration error. In addition, in some HIPLEX analyses, parallel analyses were made using 1° and 2° radar elevation angle data. The differences in 1° and 2° data provide nonuniform errors in the input data. In all cases, the 2° final gage-radar analyses were quite similar to the corresponding 1° analyses. These tests indicate that radar calibration errors are handled correctly by the technique.

4. Comparison of the rainfall estimates

The basic hypothesis on which the gage-radar rainfall estimation technique rests is that combination of the gage and radar rainfall data will produce a rainfall estimate with the point accuracy of the gages and the spatial resolution of the radar. This technique thus requires assuming 1) that the gages measure the point rainfall accurately, 2) that the radar measures the rainfall variability accurately, and 3) that a valid comparison can be made between the gage and radar measurement at the individual gage locations. The effect of gage inaccuracies was noted by Wilson,[4] who pointed out that the final analysis accuracy could be no more accurate than the gage measurement accuracy. The capability of the radar to measure rainfall variations is not well verified, but seems reasonable. The acceptability of the gage-radar rainfall comparisons at individual gage locations is also untested and seems questionable on the basis of sampling considerations. Nevertheless, in the studies of Wilson (1970) and Brandes (1975), the gage-radar mean rainfall estimates were more accurate than the gage-only rainfall estimates for large-area (29 000 km^2), low-gage-density ($\leqslant 1$ gage per 900 km^2), long-duration rainfall analyses. These studies both made use of data from Oklahoma.

The data of Wilson[4] were collected in the Lake Ontario watershed and show that the gage-only and the final gage-radar rainfall analyses were of nearly equivalent accuracy, as verified by two small high-density networks (~ 1 gage per 150–200 km^2). This rough equivalence of Wilson's[4] gage-only and gage-radar analyses is in substantial agreement with the findings of Woodley *et al.* (1975), who concluded that their use of areal mean gage-radar ratios to adjust radar rainfall estimates did not produce highly accurate final rainfall estimates. They also concluded that the technique could produce useful rainfall measurements, given typical experimental constraints. They noted that the combined gage-radar rainfall analysis had the ability to identify changes in rainfall shape, size and location in areas where gages could not be placed.

Thus, there is some question as to the accuracy of gage-adjusted, radar rainfall measurements and of the utility of such measurements as an alternative to dense raingage networks where accurate surface rainfall measurements are needed. The technique of gage adjustment of radar rainfall data, as an improvement on gage-only rainfall measurement, can be investigated through consideration of the following two characteristics of gage-only and combined gage-radar rainfall analyses:

TABLE 1. Comparison of R, GR, GR$_e$ and G mean areal rainfall amounts from HIPLEX analyses of 60 min duration.

		Mean areal rainfall (mm)				Percent differences from G analysis		
Day	Time (GMT)	R	GR	GR$_e$	G	R	GR	GR$_e$
158	0400	0.14	1.84	1.36	1.06	87	74	28
158	0500	0.05	6.15	4.41	4.27	99	44	3
159	0400	0.26	4.22	3.58	5.04	95	16	29
159	0500	0.53	14.47	12.55	13.28	96	9	5
184	1700	0.29	2.66	2.14	2.44	88	9	12
184	1800	0.54	5.86	3.99	3.75	86	56	6
184	1900	0.41	6.96	5.29	3.54	88	97	49
195	0200	0.18	0.74	0.59	0.65	72	14	9
201	0130	0.10	0.64	0.36	0.35	71	83	3
Mean						87	45	16

TABLE 2. Comparison of R and GR mean areal amounts (percent differences from G analysis) for CHAP data of 60 min duration analyses and full density gages.

		Percent difference from G analysis	
Year	Number of sample periods	R	GR
1976	35	49	14
1977	89	32	16

1) For relatively long time periods ($\geqslant 30$ min) the gage-radar and gage-only mean areal rainfall estimates should be equal. If this is not true, we must question the gage and radar rainfall estimate accuracies and the capabilities of the analysis technique.

2) Gage-radar analyses should degrade less than gage-only analyses for decreased gage densities. This is because the radar is assumed to accurately measure between-gage rainfall variability and because the technique preserves gage amounts at gage locations. If this test fails, the gage-radar technique is clearly not productive since, on the basis of the assumptions used in deriving the technique, the combined gage-radar rainfall analysis should produce better measurements of rainfall in locations between the gages without hurting measurement accuracy at gage locations.

These two analysis characteristics can be tested using data from the HIPLEX and CHAP projects and processed with the three analysis routines mentioned in Section 3. The use of the diverse data set and differing analysis routines substantially eliminates the possibility that a particular data set or data processing routine could inadvertently determine the results. Accordingly, the different rainfall estimates will be compared in the next two sections—first to investigate the mean differences between the estimates, then to investigate the effects of changes in gage density on rainfall estimate accuracy. For simplicity of discussion, the gage, radar, gage-radar and gage-radar with evaporation correction (HIPLEX only) analyses will be referred to as G, R, GR and GR$_e$, respectively, in the following paragraphs.

a. Comparison of mean area estimates

Table 1 presents 60 min mean areal rainfall amounts from the HIPLEX analysis routine and percent absolute differences between the R, GR and GR$_e$ mean areal

TABLE 3. Mean percent differences between gage-only and gage-radar rainfall analyses for some CHAP data. Simple size *N* and mean percent difference are indicated. Rainfall amounts are 30 min values using full density gages.

		Rainfall amount (mm)										
Year		0.01–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	>10
1976	*N*	93	23	12	8	4	1	1	0	1	0	2
	Mean	243	25	14	8	15	4	4	0	21	0	12
1977	*N*	97	27	8	9	12	3	3	1	0	0	3
	Mean	74	20	14	9	14	14	14	8	0	0	5

values and the corresponding G analysis rainfall amounts. Table 2 presents similar data (percent differences only) for the CHAP rainfall analyses. The data from both experiments indicate that the GR and G mean areal rainfall amounts agree fairly well (small percent differences), and that the radar rainfall mean estimates, based on climatological *Z-R* relations, were systematically incorrect. In the case of HIPLEX, with the dry Montana climate, correction for evaporation appeared to be an essential portion of the radar rainfall estimation process; however, such a correction is not necessary in Illinois. The CHAP and HIPLEX analyses indicated that uncorrected radar rainfall estimates are basically not usable for accurate rainfall estimation in either of the locations and particularly not in the dry Montana climate. The mean differences of about 15% between GR and G mean areal rainfall amounts are a little larger than is expected, considering the anticipated gage accuracy, which is on the order of 5–10%.

It is possible that the gage-radar technique could be measuring between-gage rainfall variability accurately enough to surpass the accuracy of high-density gage measurements, particularly in cases of light rainfall. This was tested using a limited sample from the CHAP data set. Table 3 presents the mean absolute percent differences between 30 min GR and G rainfall analyses for various mean areal rainfall amounts. These data show that above 1–2 mm in 30 min the rainfall amount had little effect on the analysis accuracy, but that for mean areal rainfall amounts of less than 1 mm in 30 min the analyses were quite different. Inspection of such cases showed that the large percent differences usually were the results of advection and the consequent mismatches between gage and radar data. It appeared that in many cases the radar was sampling the rainfall better than gages.

On the basis of these data, it appears that the gage and final gage-radar rainfall analyses are comparable.

b. Effects of gage density on rainfall analysis accuracy

In order to evaluate the effects of reduced gage density on analysis accuracy, mean areal rainfall amounts for reduced density analyses were compared with the corresponding full density amounts. This was done for the individual analysis 30 or 60 min time periods and for the gage-only and gage-radar analyses separately. This procedure enabled evaluation of the effects of gage density on rainfall measurement accuracy without making any *a priori* assumption about which rainfall value was the best estimate. As noted in the previous section and in the discussion of gage-only rainfall measurement accuracy (Section 1), it is not known at this point which analysis is actually the more nearly correct. It should also be noted that this technique does not account for the inherent correlation between full and reduced density analyses of the same type. The effects of this correlation should be minimized, however, since the relative gage-radar and gage-only

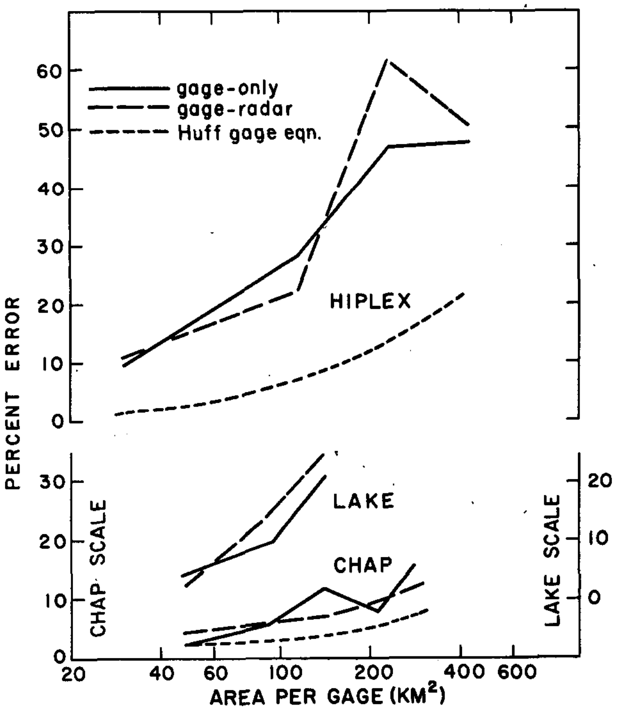

FIG. 2. Plots of gage-only and gage-radar analysis accuracy (percent) versus gage area (km^2) for the HIPLEX, LAKE and CHAP programs. The various analyses are verified against the corresponding full gage density analyses. The solid lines are for gage-only analyses, the long dashed lines are for gage-radar analyses, and the short dashed lines are for the gage accuracy estimated from the Huff (1970) regression equation using a rainfall rate of 10 mm h^{-1}, an analysis duration of 1 h and a 1500 km^2 gage network.

Table 4. Effect of gage density on gage-only and gage-radar rainfall analysis accuracy. Number of samples is indicated by N, the mean absolute percent difference between corresponding analyses based on the reduced and full network densities by percent difference, and the standard deviation of percent difference by σ. Sixty or 30 min analyses were used with 1–6 h of data coming from any one day.

Gage density		Gage-only			Gage-radar		
Fraction of full density	Gage area (km² gage⁻¹)	N	Percent difference	σ	N	Percent difference	σ
CHAP 1976							
1	23	19	0	0	21	0	0
1/2	47	19	4	3	21	6	4
1/4	93	19	16	18	21	11	7
1/6	140	19	15	12	21	14	8
1/9	210	19	33	26	21	22	20
1/12	280	19	37	24	21	22	17
CHAP 1977							
1	23	59	0	0	56	0	0
1/2	47	59	5	6	56	7	9
1/4	93	59	9	8	56	11	10
1/6	140	59	16	13	56	16	16
1/9	210	59	17	16	56	19	31
1/12	280	59	28	26	56	19	17
LAKE 1977							
1	23	7	0	—	7	0	—
1/2	47	7	4	—	7	2	—
1/4	93	7	10	—	7	15	—
1/8	186	7	21	—	7	31	—
HIPLEX 1976							
1	14	8	0	0	8	0	0
1/2	29	8	9	10	8	11	21
1/4	57	8	18	20	8	16	33
1/8	114	8	29	28	8	22	19
1/16	228	8	47	54	8	61	38
1/30	428	8	48	49	8	50	45

differences from the respective full gage density analyses are only being compared, and are not being used in any absolute sense. Values of the percent difference (absolute values were averaged) for the different gage densities, for the gage-only and gage-radar rainfall analyses, and for the three different analysis routines are presented in Table 4 and Fig. 2. For the HIPLEX data the G and GR_e values were used; for the CHAP and LAKE projects G and GR values were used. For CHAP data in Fig. 2 the values for the two years were averaged, taking sample size into account. Table 4 also contains the sample sizes and standard deviations of the percent difference values about their means.

These data indicate that there is little difference between the gage-only and gage-radar rainfall values for the various analysis routines with the exception of the CHAP data for a gage density of 1 gage per 280 km². The apparent improvement in CHAP gage-radar over gage-only rainfall measurement accuracy must only be regarded as suggestive due to the lack of adequate supporting data at lower gage densities. However, the suggestion is supported by some limited lower gage density studies of CHAP data (Table 5) and by the findings of Wilson (1970) and Brandes (1975) using gage densities $\leqslant 1$ gage per 900 km². The sample size in Table 5 is small (thirteen 30 min samples), and both analyses are verified against the full density gage-only analyses (a deviation from the data in Table 4). Nevertheless, these data suggest for the Illinois climate, that gage-radar mean areal convective rainfall may be more accurate than gage-only measurements for gage densities $\lesssim 1$ gage per 250–300 km². The absence of a similar finding for the LAKE analysis may be due to the small sample size and to the limited range of gage densities.

A similar improvement of gage-radar over gage-only accuracy is not seen in the HIPLEX data analyses. Inspection of the individual analyses of the HIPLEX study (Achtemeier *et al.*)[9] shows that the HIPLEX findings are not the result of a few bad samples, since

Table 5. Comparison of CHAP 30 min gage-only and gage-radar rainfall analyses for low gage densities, using the full density (1 gage per 23 km²) gage-only analyses for verification. Data are the average of 13 samples taken from 1976 and 1977.

Gage density (km² per gage)	Percent difference	
	Gage-only	Gage-radar
460	20	13
690	24	13
920	38	17
1150	33	29
1380	38	27

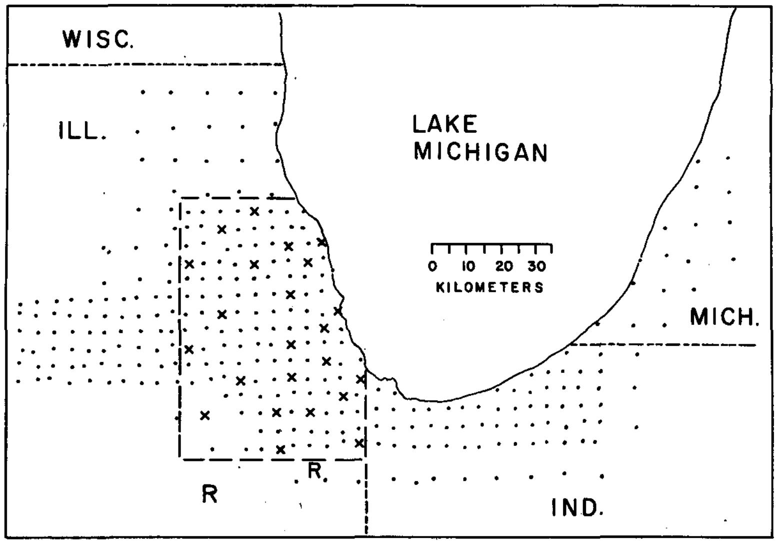

FIG. 3. Map of CHAP (●) and MSD (×) gage locations. Radar site is marked by the symbol R.

individual analyses show a consistent lack of improvement of gage-radar over gage-only analysis accuracy. It therefore appears that the HIPLEX findings may be the result of the dry eastern Montana climate, with its extreme evaporation and relatively large-scale storm systems. The data from all three analysis routines showed large measurement errors when less than three gage-radar ratios were used in the rainfall analyses.

The finding of approximately equivalent gage-only and gage-radar analysis accuracy for high gage densities is further supported by the results of Wilson.[4] In his Table 7, the gage-only and gage-radar analysis accuracies, for a gage density of ~1 gage per 200 km² are roughly equivalent as verified against higher density, 1 gage per 40–150 km² gage networks.

A further test of these findings was provided in the analyses of CHAP data through use of the Metropolitan Sanitary District of Chicago (MSD) raingage network (23 gages in 1750 km² or 75 km² per gage) which is located in an area surrounding and including the metropolitan area of the city of Chicago. The CHAP gage network encompasses the MSD network (Fig. 3). A simulation of real-time rainfall analysis operations was made in which ISWS gages located next to the MSD gage network were used to simulate MSD gage-adjusted radar rainfall estimates. This ISWS/MSD simulation was done using 1976 and 1977 data for both 30 and 60 min data averaging. The radar data were adjusted using the ISWS/MSD gages. The ISWS/MSD gage-only and ISWS/MSD gage-radar rainfall values were verified against the full density (1 gage per 23 km²) CHAP gage network. The results for 60 min data averaging are summarized in Table 6. These tests indicated that the ISWS/MSD gage-radar values were, in general, slightly less accurate than the ISWS/MSD gage-only values. Thus, the only viable reasons for adding radar to the MSD gage network, using the present analysis technique, are to provide rainfall measurements between gages, or at locations outside of the gage network, but within radar range.

TABLE 6. Comparison of low density (1 gage per 76 km²) ISWS/MSD gage-only and ISWS/MSD gage-radar rainfall with full density (1 gage per 23 km²) CHAP gage rainfall. The 60 min analyses are compared. The number of samples (N), the mean absolute percent difference (D) and the standard deviation (σ) about this mean are shown.

	Radar-only			ISWS/MSD gage-radar			ISWS/MSD gage-only		
Year	N	D	σ	N	D	σ	N	D	σ
1976	14	44	20	15	26	16	15	21	14
1977	17	26	17	17	33	49	16	14	11

c. *The use of mean* G/R *value*

As an added test of the utility of simple combinations of gage and radar rainfall data, the mean G/R ratio for various rainfall analysis durations was used with the radar rainfall estimate to produce a combined gage-radar rainfall estimate. This test is comparable to the technique of Woodley (1975), who used a similar technique in evaluations of dynamic cloud seeding effects. This technique also has application to the problem of measuring rainfall in locations, such as Woodley's, where gages cannot be used.

In the LAKE analysis of CHAP data (Snell, 1977), both the Brandes (1975) technique and the mean G/R ratio technique were used to estimate rainfall over simulated lakes. These "lakes" were 800 and 1100 km² areas where gages were withheld from the

Table 7. Comparison of estimates for rainfall over a large area containing no gages. Mean absolute percent differences are shown using 27 h of data from 4 days. The rainfall estimates were verified against a gage-radar estimate which included gages in the gage-void area.

	Percent differences		
Area	Radar-only	Extrapolated gage-radar	Radar corrected with mean G/R ratio
B	54	15	0
C	48	5	9

analysis. A summary of the results is presented in Table 7. These data show that in some cases use of the mean G/R ratio for all gages is preferable to use of an extrapolated G/R ratio, even when viewed in the mean. This point was further amplified upon inspection of individual 1 h analyses which frequently showed very large errors due to extrapolation and smaller errors resulting from use of the mean G/R ratio. This study thus suggests that a preferable means of using gages to correct radar data over gage voids may be use of the mean G/R ratio.

For most cases not involving large gage voids, the mean G/R ratio technique of adjusting radar data appeared to be less accurate than the full objective analysis of G/R ratios as proposed by Brandes (1975). This suggests that it is preferable to restrict use of the mean G/R ratios to cases where extrapolation is likely to be involved. The results of the CHAP, LAKE and HIPLEX analyses suggested that final gage-radar analysis errors became large when G/R ratios $\leqslant 3$ were available. The final gage-radar analyses appeared most preferable in cases where there were uniformly distributed gage data which could be used to correct the radar data.

5. Conclusions

For measurement of area-mean convective rainfall, this study shows that the present combinations of raingage and radar data are no more accurate than gage-only data for gage densities $\gtrsim 1$ gage per 100 km^2. For lower gage densities (1 gage per 250–300 km^2) and for the Illinois climate, there is a suggestion that combinations of gage and radar data may be more accurate than gage-only mean convective rainfall measurements. The point at which radar may possibly begin to improve gage areal mean convective rainfall measurements is not entirely clear, since the large-sample CHAP gage density analyses ended at 1 gage per 280 km^2, the point at which an effect began to be apparent. However, there is some consistency between those results, the limited sample CHAP results for lower gage densities (Table 5), Wilson's (1970) and Brandes' (1975) results, and the typical Illinois summer convective raincell size ($\sim$200–300 km^2). These factors lend credence to the suggestion that, at least in the summer Illinois climate, radar begins to improve mean convective rainfall measurement for gage densities of $\lesssim 1$ gage per 250–300 km^2. The lack of a similar result for the eastern Montana HIPLEX data suggests the possibility that climatic differences in raincell size and evaporation rate may have major effects on rainfall analysis. Further studies are clearly necessary to accurately establish the point at which radar may be a useful addition to gages in the measurement of mean areal convective rainfall.

It is clear from the wide variety of high gage density ($\geqslant 1$ gage per 100 km^2) studies that radar adds little to mean area convective rainfall measurement for high gage densities. The main causes for this lack of improvement for high gage densities are thought to be the large differences in the sampling techniques of radar and the gage, errors in *Z-R* relations, and the inability to correct the radar data for the effects of advection and evaporation. Calculation of the radar rainfall estimate within the sample volume involves assumption of a *Z-R* relation, hence drop-size distribution, which may be inaccurate enough to produce a 50% or larger error in rainfall amounts. This possible error is compounded by the problem of advection and evaporation of the rain between the radar and the ground and by significant changes in the drop size distribution and vertical rainwater flux due to vertical air motions (Carbone and Nelson, 1978). Tests performed in the course of these studies indicate that environmental winds clearly are not adequate for advecting the rain, and hence, no advection is the preferable means of combining the gage and radar data. This could explain why sampling periods $\geqslant 1$ h used in gage-radar rainfall analyses are little better than short ($\leqslant 15$ min) duration analyses. So far, correction for evaporation has only been accomplished through use of radar rainfall thresholds and crude schemes such as used for the HIPLEX analysis. While these techniques can improve the comparisons between the gage-radar and the gage-only analyses, they are not expected to be highly accurate.

For cases in which there are large areas where gages cannot be placed, use of the objective gage-radar technique (Brandes, 1975) can lead to large errors in the rainfall estimate due to the effects of the extrapolation. In such cases, use of the mean gage-radar ratio, as done by Woodley *et al.* (1975), seems preferable. That form of rainfall measurement might be useful over lakes and inaccessible areas for flood warnings and other hydrologic applications where high accuracy is not needed. It is possible that in such applications, use of a carefully selected climatological *Z-R* relation, or a sequential analysis approach similar to that of Cain and Smith[6] would suffice.

This study indicates little hope for simple combinations of gage and radar data which will enable the fine-scale, highly accurate convective rainfall measurements necessary for many studies, including weather modification experiments. While it is possible that more sophisticated statistical techniques may improve rainfall measurements, such techniques will have to be verified against the known standard of raingages.

An aspect of rainfall measurement only touched upon in this study is measurement of rainfall at scales smaller than raingage spacing. Here, only mean areal rainfall measurements were considered. In cases where single-cell rainfall is important and the rainfall rate between gage locations must be monitored, use of a combined gage-radar rainfall analysis technique is probably preferable. The impact of between-gage rainfall variations was shown to affect particularly the light rainfall rate cases. In such cases the final gage-radar rainfall analysis was suggested to be more nearly correct than the gage-only analyses. The data presented herein indicate that routine use of the combined gage-radar technique is not likely to seriously degrade the rainfall analyses and, if carefully applied, may provide small improvements over gage-only analyses.

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