Revised Estimates for Continuous Shoreline Fumigation

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ABSTRACT

A model is developed to describe the dispersion of pollutants in a coastal area when stable air masses are transported inland. It is assumed that a convective mixed layer develops over land, the height of which increases with distance from the shoreline. The vertical distribution of pollutants is assumed to be homogeneous within the mixed layer and the distribution in the stable layer aloft is assumed to be Gaussian. Pollutant concentrations in the mixed layer are obtained by analytical integration of the mass conservation equation. It is shown that the earlier and much higher estimates of continuous shoreline fumigation by Lyons and Cole (1973) and Meroney *et al.* (1975) can be derived from our expressions by simplifying substitutions.

1. Introduction

In shoreline areas a particular need exists for a reliable prediction of ground level concentrations because these sites are by their very geographical situation often highly industrialized and urbanized. The description of dispersion during inland flow, however, is rather complicated because the discontinuity in surface temperature often modifies the atmospheric boundary layer considerably. Models based on numerical solution of the diffusion equation, which might be appropriate for problems of this kind, are still in an experimental stage. Moreover, these models are so complex that large computers are required. However, if care is taken, it is believed that a Gaussian plume-model description might give a reasonable estimate of the ground-level concentration.

Lyons and Cole (1973), for instance, give a description of pollutant dispersion from a tall stack, situated near the coast, during onshore flow. Air with stable stratification is advected over a cold water surface. When it passes the shoreline it is heated by the relatively warm land surface. A gradually growing internal boundary layer is then formed (Fig. 1). Initially the plume spreads in the stable air aloft, according to a Gaussian profile. In the region around the intersection of the plume axis and the internal boundary layer the plume is trapped into the mixed layer. The profile within the mixed layer is considered to be uniform in the vertical. Concentrations are given by

$$c_L=[1/L(x)]\int_{-\infty}^{L(x)} c\,dz, \tag{1}$$

where L is the depth of the mixed layer and c the usual Gaussian dispersion formula. Based on Eq. (1), Lyons and Cole arrive at the following expression for the concentration in the fumigation zone:

$$c_L=\frac{q}{(2\pi)^{\frac{1}{2}}\sigma_{yf}uL(x)}\left[\int_{-\infty}^{p}(2\pi)^{-\frac{1}{2}}\exp(-z'^2/2)dz'\right]\times\exp[-\tfrac{1}{2}(y/\sigma_{yf})^2]. \tag{2}$$

Here H is the effective stack height, $p=[L(x)-H]/\sigma_{z,s}$ and $\sigma_{yf}=\sigma_{y,s}+H/8$. The dispersion coefficients in the stable layer are $\sigma_{y,s}$ and $\sigma_{z,s}$. The integration variable z' results from the substitution $z'=(z-H)/\sigma_{z,s}$. The term $H/8$ has been added to correct for the enhanced horizontal mixing in the layer considered. A factor which accounts for losses due to chemical reactions, which appears in the Lyons and Cole formula, has been omitted here for clarity.

For large values of p, and for $y=0$, Eq. (2) reduces to

$$c_L\approx\frac{q}{(2\pi)^{\frac{1}{2}}\sigma_{yf}uL(x)}. \tag{3}$$

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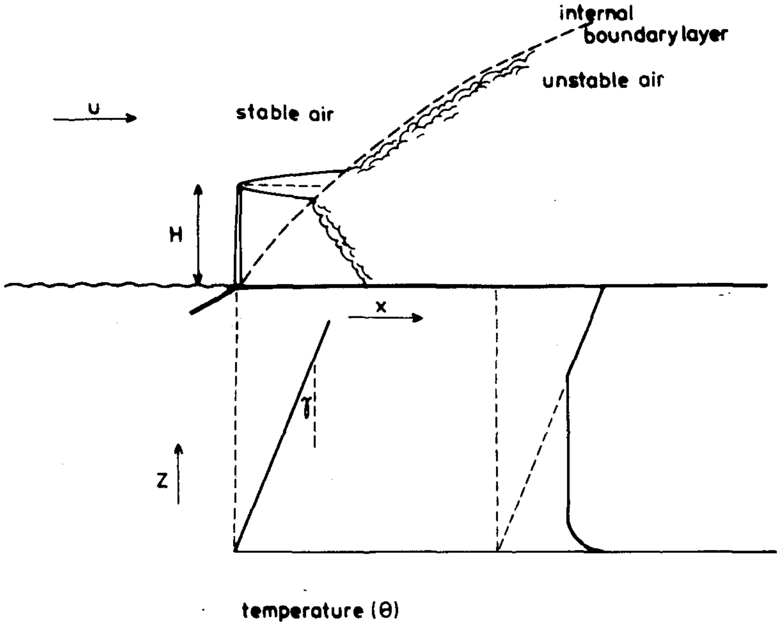

Fig. 1. Schematic view of shoreline fumigation.

A similar estimate of maximum ground concentration during fumigation conditions was used by Meroney *et al.* (1975). In the present paper a slightly different approach—based on the equation of conservation of mass—is followed in order to derive an estimate of pollutant concentrations in a shoreline region. The geometry of the dispersion (Fig. 1) is identical to that used by Lyons and Cole. The main differences with the Lyons and Cole model are as follows:

1) The fumigation of the plume is not restricted to the fumigation zone but occurs everywhere at the interface between the stable and the mixed layer. Though this will not result in large changes of maximum surface concentrations, it will lead to one consistent formulation of surface concentrations, whereas Lyons and Cole divided the dispersion regime into three zones and used three different equations for their computations.

2) Once the pollutant has entered the mixed layer, it is laterally dispersed according to the corresponding stability class of that layer. This assumption will in general result in a considerably lower prediction of maximum surface concentrations at the fumigation spot than that of Lyons and Cole. They described the enhanced mixing in the fumigation zone by introduction of the dispersion parameter σ_{yf} [cf. Eq. (2)].

2. The internal boundary layer

It is assumed that a stationary internal boundary layer has developed. The dynamics of the formation of this layer has been studied by Venkatram (1977), Peters (1975) and Meroney *et al.* (1975). Theory and experimental data suggest that a square-root relationship between mixed-layer height and distance from the shoreline is an adequate approximation. As the mathematical formulation of the mixed-layer height is not relevant in the present context, we will simply use

$$L(x)=ax^{\frac{1}{2}}, \tag{4}$$

where all the physics of the mixed-layer development is contained in the factor a. Any other expression, however, of arbitrary complexity can be used.

3. The model equation

The model geometry is given in Fig. 1. The wind speed is assumed to be constant with height and independent of the shoreline distance in both layers. The wind direction in the two layers is assumed to be perpendicular to the shoreline. It is assumed that the part of the plume that has entered the mixed layer is immediately homogeneously distributed into the air below. Under stationary conditions the equation of conservation of pollutant mass for a column of air with height $L(x)$ and unit area reads

$$u_1\frac{\partial c_1 L}{\partial x}=Q_e+K_{y,1}\frac{\partial^2 c_1 L}{\partial y^2}. \tag{5}$$

Here c_1 and u_1 are the concentration and wind speed within the mixed layer. The lateral dispersion is represented by the exchange coefficient $K_{y,1}$.[1]

The entrainment flux through the upper lid is given by Q_e and will be specified below. The concentration

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[1] In the Gaussian plume-model formulation the relation between the eddy exchange coefficient K_i and the dispersion coefficient σ_i is given by the well-known expression $K_i=u\sigma_i d\sigma_i/dx$.

distribution c_s in the stable layer is taken to be Gaussian. Hence

$$c_s=\frac{q}{2\pi\sigma_{y,s}\sigma_{z,s}u_s}\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_{z,s}}\right)^2-\frac{1}{2}\left(\frac{y}{\sigma_{y,s}}\right)^2\right], \quad (6)$$

where $\sigma_{y,s}$ and $\sigma_{z,s}$ are the lateral and vertical dispersion coefficients, respectively. The wind speed in the stable layer is given by u_s and the source strength by q.

The entrained flux through the upper lid is given by

$$Q_e=(u_s c_s dL/dx+K_{z,s}\partial c_s/\partial z)_{z\downarrow L}. \quad (7)$$

The requirement that the total horizontal flux be constant leads to the additional second term in the right-hand side of Eq. (7). Substituting Eq. (7) into (5) and using (6), we obtain a differential equation, which can be solved analytically with the boundary conditions

$$c_1(x,y)_{x=0}=0,$$

$$\lim_{y\to\pm\infty}\ \partial c_1(x,y)/\partial y\to 0.$$

The solution is

$$c_1(x,y)=\frac{q}{2\pi L(x)u_1}\int_0^x(\sigma')^{-1}$$

$$\times\exp\left[-\frac{1}{2}\left(\frac{L-H}{\sigma_{z,s}}\right)^2-\frac{1}{2}\left(\frac{y}{\sigma'}\right)^2\right]\frac{d}{dx'}\left(\frac{L-H}{\sigma_{z,s}}\right)dx'. \quad (8)$$

In the integrand L and $\sigma_{z,s}$ depend on the dummy integration variable x'; and σ', which resulted from the integration over y, is given by

$$\sigma'(x,x')=[\sigma_{y,1}^2(x)-\sigma_{y,1}^2(x')+\sigma_{y,s}^2(x')]^{\frac{1}{2}}. \quad (9)$$

In the derivation of Eq. (8) a slight inconsistency is hidden, without which no analytic solution would be obtained. It was introduced by the assumption that the concentration distribution in the stable air aloft is Gaussian. This, of course, is not quite correct, because a Gaussian distribution requires another lower boundary condition. As it is impossible to specify a lower boundary condition without consideration of the physical processes at the top of the mixed layer, the concentration distribution in the stable layer aloft cannot be determined. As a consequence an estimate of the error made by this assumption cannot be given. However, we believe that a slightly different shape of the concentration profile in the stable layer does not affect the surface concentrations seriously.

4. Results

Eq. (8) shows a close resemblance with the expression given by Lyons and Cole [cf. Eq. (2)]. With the substitution $p'=[L(x')-H]/\sigma_{z,s}(x')$ and some rearrangement we obtain from Eq. (8):

$$c_1=\frac{q}{(2\pi)^{\frac{1}{2}}u_1 L(x)}\int_{-\infty}^{p}(2\pi)^{-\frac{1}{2}}(\sigma')^{-1}$$

$$\times\exp(-p'^2/2)\exp[-\tfrac{1}{2}(y/\sigma')^2]dp'. \quad (10)$$

Here $p(x')$ and $p(x)$ are abbreviated as p' and p, respectively. From Eq. (10) it is immediately clear that on replacing σ' by the dispersion coefficient σ_{yf} used by Lyons and Cole, we obtain their expression. We prefer, however, the use of σ', notwithstanding the slightly increased complexity of Eq. (10). Inspection of Eq. (9) shows that the composite dispersion coefficient σ' "depicts" the dispersion process quite realistically. The lateral concentration distribution in the mixed layer at a distance x originates from particles which have traveled in the stable and the mixed layer successively. This is reflected in the dispersion coefficient σ', which contains the lateral dispersion coefficient of both layers. In the stable layer the particles traveled over a distance x', resulting in a spread $\sigma_{y,s}(x')$; the subsequent diffusion in the mixed layer over the distance $x-x'$ is represented by $\sigma_{y,1}^2(x)-\sigma_{y,1}^2(x')$. The integral in Eq. (10) might thus be considered the inverse of an effective lateral dispersion coefficient. Compared with the dispersion coefficient σ_{yf} used by Lyons and Cole [$\sigma_{yf}=\sigma_{y,s}+H/8$], $\sigma'(x,x')$ varies between $\sigma_{y,1}(x)$ and $\sigma_{y,s}(x)$, as x' varies over the interval $[0,x]$. As for common values of x and H, $\sigma_{y,1}(x)$ will exceed $\sigma_{y,s}(x)$ considerably more than $H/8$ m, $\sigma'(x,x')$ will in general be larger than σ_{yf}. According to Eq. (10) this will result in lower estimates of ground level concentrations than those made by Lyons and Cole. (We restrict ourselves from now on to the case $y=0$.)

In Fig. 2 the concentration within the mixed layer is plotted as a function of the shoreline distance. The effective stack height was 300 m. The dispersion parameters in the stable layer were chosen according to class D of Singer and Smith (1966). In the mixed layer stability class B1 was assumed. The boundary-layer growth was given by Eq. (4), where for the factor a the value 4 $m^{\frac{1}{2}}$ was chosen. According to Venkatram (1976) this would correspond to a land-sea temperature difference of 10 K, a ratio of u_*/u of 0.05, and a lapse rate in the stable layer of approximately 6×10^{-3} K m^{-1}. These conditions are compatible with the choice of the stability class B1 in the mixed layer and D in the stable layer aloft.

The concentration distribution according to Lyons and Cole [cf. Eq. (2)] is also depicted. Their estimate of the maximum concentration is about a factor 3 higher in this case. Although this is a fairly large difference, other choices of the dispersion parameter $\sigma_{y,1}$—which does not occur in the Lyons and Cole formula for the fumigation zone—may give smaller ratios. The ratio is also proportional to the distance from the coast to the fumigation spot, so that it has to be considered typical

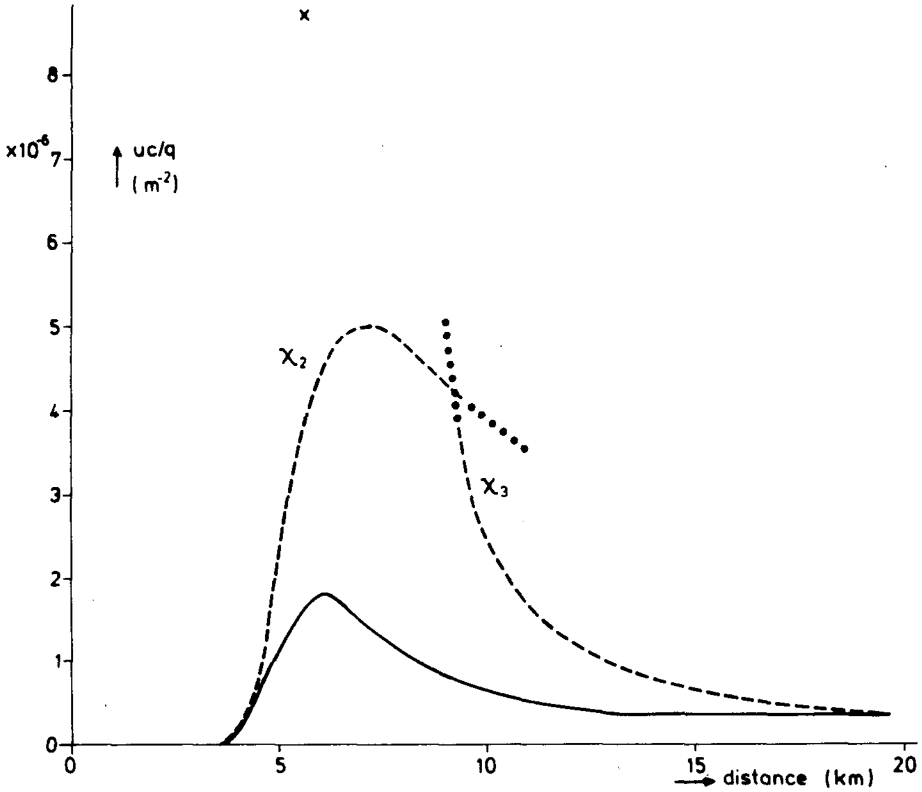

FIG. 2. Concentration in the mixed layer in a plane through the plume axis perpendicular to the earth's surface, according to Eq. (8) (solid line) and the concentration in the mixed layer according to Lyons and Cole (1973) (dashed lines). Here χ_2 is the concentration in the fumigation region and χ_3 the concentration according to their formula for the mixed-layer region. The curves are dotted outside their respective regions of validity. The maximum concentration according to Meroney *et al.* (1975) is shown by an ×.

of the example and not of the model. The estimate used by Meroney *et al.* (1975) is also indicated [cf. Eq. (3)].

The influence of stack height on the maximum concentration is given in Fig. 3. For comparison, the maximum concentrations that would have occurred in a neutral, homogeneous atmosphere are also depicted. It is somewhat surprising to see that the difference between the concentration during fumigation conditions according to Eq. (8) and the concentration under neutral conditions becomes considerable only for relatively small stacks.

Experimental data on shoreline fumigation are unfortunately sparse. One extensive field experiment was carried out by the University of Wisconsin-Milwaukee in conjunction with Environmental Research and Technology, Inc., during the summer of 1974 (Lyons, 1975). The obtained data were used to validate and calibrate the GLUMP[1] model (Schuh, 1975). This is a multiple-source model which is applicable to a shoreline environment. It contains the expressions of Lyons and Cole for continuous shoreline fumigation. It was found that the GLUMP model tended to overestimate surface concentrations, so that a calibration factor of 1.28 was introduced, for an optimum fit with the measurements.

Meroney *et al.* (1975) did some wind tunnel experiments in well-defined conditions. They compared some of their measurements with the calculation based on Eq. (3), and found an overprediction by a factor of 3.4, which they mainly ascribed to "shortcomings in the extremely simple analytical models used". Also, Guldberg (1978) reports in a validation study that earlier multi-source Gaussian plume model estimates consistently overpredict maximum 24 h SO_2 concentrations in the coastal environment, by factors ranging from 2–4.

This short review of experimental data strongly favors the present, less conservative model. Unfortunately, the lack of sufficiently detailed information on experimental circumstances hindered a more quantitative verification of the model.

5. Conclusions

1) In an idealized situation it is possible to calculate pollutant concentrations in shoreline fumigation condi-

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[1] Great Lakes University Mesometeorology Project.

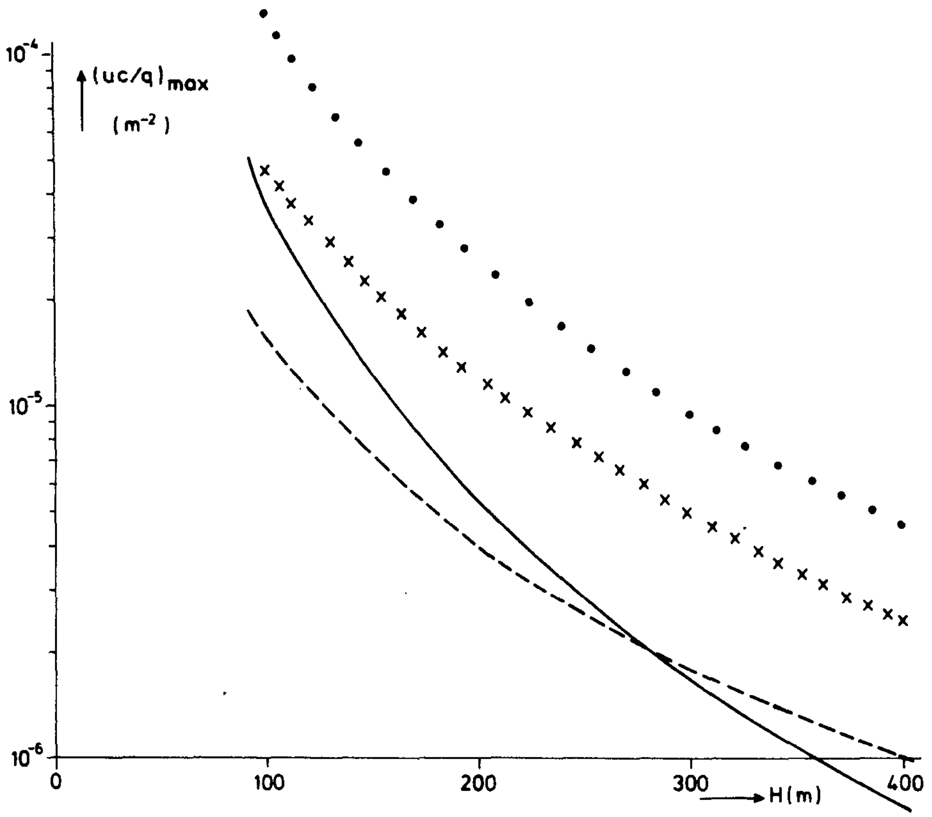

FIG. 3. Maximum concentration as a function of stack height. Dotted line: Meroney *et al.* (1975); crosses: Lyons and Cole (1973); dashed line: neutral case; solid line: this paper.

tions by analytic integration of the equation of conservation of mass.

2) The approximations that the concentration distribution in the stable layer is Gaussian and that the vertical mixing in the unstable boundary layer is instantaneous, still lead to overestimation of the maximum concentration, which thus might be considered an upper limit.

3) Experimental evidence tends to confirm that the present model will give better estimates of maximum ground level concentrations during fumigation conditions than earlier ones. Validation of the model, however, remains necessary.

4) The model can be applied easily, provided that an estimate can be given of the stability of the layer aloft in terms of σ_y and σ_z.

5) The rate of mixed-layer growth has to be chosen with care because it is closely related to the choice of the dispersion parameters in both layers.

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