Estimating Hourly Mixing Depths from Historical Meteorological Data

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ABSTRACT

The planetary boundary layer is defined as the layer of the lower atmosphere whose characteristics are directly influenced by the ground surface. In the atmosphere, turbulent mixing forms and maintains this layer; hence, the planetary boundary layer is also a mixing layer. Turbulent mixing can be either convectively or mechanically produced. A simple one-dimensional operational model is proposed to estimate reliable and realistic hourly mixing depths from routinely available upper air and surface data.

The model inputs are 0000 and 1200 GMT temperature soundings from the nearest radiosonde station and the hourly surface wind speeds and temperatures from the nearest representative surface station. The model distinguishes between primarily convective and primarily mechanical mixing regimes. In a primarily mechanical regime, such as during nighttime hours or on cloudy or windy days, the mixing depth can be estimated from the surface wind speed and roughness length. During convective regimes, such as on sunny days, the mixing depth can be estimated from the surface temperature and the morning temperature sounding. The model adjusts the surface temperature for temperature advection. By statistical comparisons with available acoustic sounder and radiosonde data, it is shown that for one month of data at a central Illinois site the proposed model demonstrates more skill than a presently available operational scheme.

1. Introduction

The planetary boundary layer is defined as the layer of the lower atmosphere whose characteristics are directly influenced by the ground surface. Planetary boundary layers are generally turbulent and therefore a simple energy budget can be written (standard notation has been adopted) to describe the time rate of change of the turbulent energy e, expressed as a mean quantity over many parcels, as

$$\frac{d\bar{e}}{dt}=-\overline{u'w'}\frac{\partial\bar{u}}{\partial z}-\overline{v'w'}\frac{\partial\bar{v}}{\partial z}-g\frac{\overline{w'T'}}{\bar{T}}-\frac{\partial}{\partial z}(\overline{w'e'})-\epsilon. \qquad (1)$$

The first two terms on the right-hand side of (1) represent the production of turbulent energy due to extraction of energy from the mean flow by Reynolds stresses. This is generally an important production mechanism since the mean wind speed must go to zero at or near the ground surface, which leads to a wind shear layer near the ground. The third term is the buoyant production or removal of turbulent energy. When the ground surface is warmed by solar heating during the daytime, convective turbulence results from the difference in temperature between the ground and the air above. With the creation of a stable lapse rate at night due to radiational cooling of the ground surface, negative buoyancy damps the production of turbulence. The fourth term is the vertical transfer of turbulent kinetic energy. In the unstable daytime boundary layer, this term transports energy upward, while in the stable nocturnal layer, transport is directed downward and is converted into heat by frictional dissipation at a rate ϵ.

In general, the rate of change of total turbulent kinetic energy in the planetary boundary layer (PBL) is quite slow and a quasi-balance exists between the production and destruction of turbulence. When production is large, such as on a sunny summer day, the PBL is both chaotic and very deep. When the production rate is small, the PBL is shallow and less turbulent.

Turbulent eddies are effective in destroying gradients of any active or passive quantity contained within that turbulent flow. Therefore, the PBL can also be viewed as a mixed layer, representing the vertical extent to which pollutants can be mixed in the atmosphere. The depth of the mixed layer can often be inferred from vertical profiles of quantities such as wind speed, wind direction and temperature—quantities whose profiles are directly influenced by turbulent mixing.

It is apparent that for the air pollution meteorologist, an accurate estimate of the depth of the mixing layer is essential if dispersion of pollutants released in the boundary layer is to be modeled correctly. Environmental Protection Agency (EPA) guidelines for estimating the impact of a polluting source re-

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commend the application of air quality models with one to five years of hourly meteorological data (see EPA, 1978). The need exists for an accurate operational scheme to provide the hourly mixing depths required by these models.

Theoretical investigation of the boundary layer has led to the detailed understanding of two separate mixing-depth regimes; the daytime convective layer and the nocturnal buoyancy-damping shear layer. A long history of one-dimensional entrainment models exists that describe the growth of the daytime convective layer. From the initial work of Ball (1960) and Lilly (1968), Tennekes (1973), Betts (1973), Carson (1973), Stull (1973), and later Stull (1976) and Zeman and Tennekes (1977) have all studied the problem. For the nighttime shear layer, the work of Blackadar (1957) has led to detailed consideration by Deardorff (1971), Delage (1974) and Blackadar (1976).

The daytime convective growth models all require that the surface heat flux be both positive upward and follow some describable time-dependent behavior. Further, the models do not handle the effects of large-scale advection. The nocturnal models require some knowledge of ground surface or soil characteristics and are not applicable in conditions when the nocturnal boundary layer is not cooling. As far as is known, the transitions between the distinctively different daytime and nighttime situations has not been satisfactorily treated by any theoretical model.

Environmental Protection Agency models currently use an empirical scheme (see, e.g., EPA, 1977) to estimate hourly values of the mixing depth. The scheme is based on a linear interpolation between morning and afternoon mixing depths that are derived from the methods proposed by Holzworth (1967). Holzworth's morning and afternoon mixing depths, however, were approximations primarily intended for application in climatological studies and do not include the important effect of temperature advection. Furthermore, the EPA interpolation scheme does not adequately represent the physical processes accompanying the diurnal and hourly changes in the depth of the mixed layer.

Clearly, the success of an operational mixing depth model is contigent on how successfully the following criteria are met:

1) The model should be able to accurately and realistically estimate the mixing depth on the basis of the time-dependent turbulent characteristics of the boundary layer.

2) The model should be applicable to all meteorological situations.

3) The model should rely on readily available meteorological data only, e.g., that which is available from National Weather Service stations.

4) The model should be simple enough in form so that computational costs are not overly burdensome.

This paper proposes a scheme that attempts to satisfy all of the above considerations. The proposed scheme distinguishes between daytime and nighttime regimes, as well as primarily mechanical or convective situations. The effect of large-scale temperature advection is also considered. In its present form, the proposed scheme is one-dimensional and therefore does not apply when the mixed layer is more appropriately an internal boundary layer associated with two

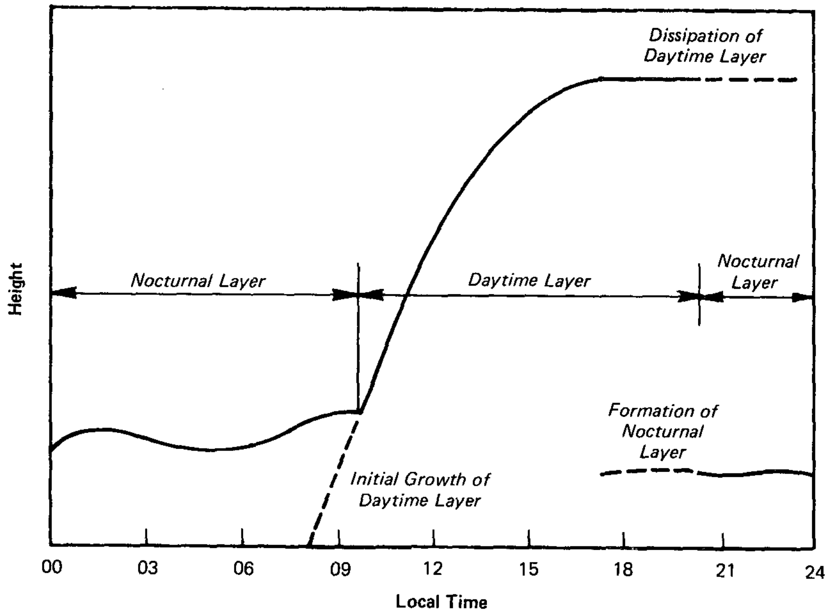

FIG. 1. Schematic depiction of an idealized mixing-depth cycle.

dimensional mesoscale features such as the urban heat island and the sea breeze.

In the next section, the time-dependent progression of the mixed layer, as depicted by Fig. 1, is discussed through consideration of existing theory and physical intuition. This material is used as the basis for constructing (in Section 3) the proposed model. Finally, the model is compared with the EPA scheme by means of a month of hourly mixing depths analyzed from radiosonde and sodar data at a site in central Illinois.

2. The time-dependent mixed layer

Starting at radiational sunrise, insolation induces a vertically directed buoyancy flux. Above the unstable surface layer, the lower atmosphere warms adiabatically as buoyant thermals carry both heat and turbulent energy upward. The depth of the daytime convective layer increases with continued surface heating. Fig. 2 presents daytime profiles of potential temperature, wind speed and Richardson number measured at O'Neill, Nebraska, on 7 September 1953 as part of the Great Plains Turbulence Field Program (Lettau and Davidson, 1957). Potential temperature and wind speed are nearly constant in the mixed layer extending to about 1000 m. Richardson numbers in this layer are less than the commonly accepted critical Richardson number of 0.25 and thus are indicative of a turbulent regime.

Convectively unstable layers are generally capped by a potential temperature inversion (as illustrated just above 1000 m in Fig. 2). The capping potential temperature inversion is created because rising parcels of buoyant air penetrate the potentially warmer stable layer aloft until increasing negative buoyancy slows their motion to a halt. This penetrative convection produces a net cooling at the top of the neutral layer, resulting in the development and maintenance of a capping potential temperature inversion. Heat lost by the capping stable layer is gained by the neutral layer and rapidly mixed downward.

The depth of the convectively unstable layer can be estimated as a function of time by use of any of the models referenced in Section 1. However, since all one-dimensional inversion rise models require that the sensible heat flux be positive and follow some describable time-dependent behavior, their routine use is not advisable. As an alternative, if temperature advection is negligible, one can estimate the time-dependent mixed-layer depth by drawing a dry adiabat upward from the hourly surface (screen height) temperature to the intersection with the morning temperature sounding (see Miller, 1967). At first glance this method might seem to overestimate the mixing depth since the surface potential temperature is measured in the superadiabatic surface layer and will be higher than the average potential temperature of the mixed layer. However, this overestimation is at least partially offset by the fact that penetrative convection and the maintenance of a capping inversion result in a deeper mixed layer than would be indicated by the intersection of the average potential temperature of the mixed layer with the morning sounding.

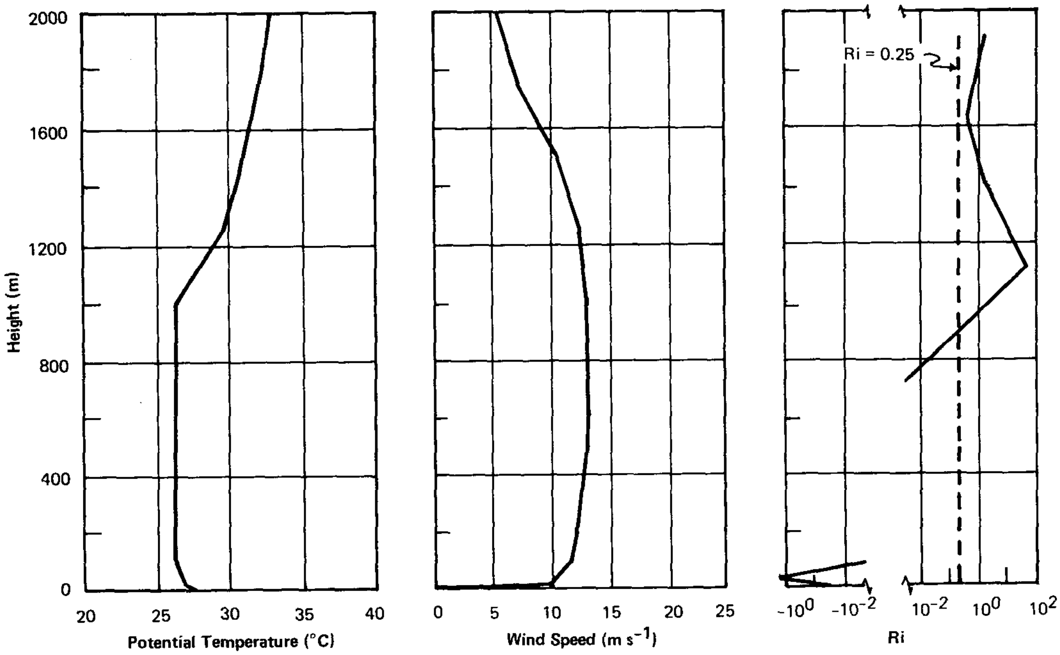

FIG. 2. Profiles of potential temperature, wind speed and Richardson number at O'Neill, Nebraska, 1435 LST 7 September 1953.

The maximum depth of the daytime layer generally occurs when the upward directed sensible heat flux vanishes at radiation sunset and further input of energy by buoyant production ceases. Radiation sunset usually occurs after the time of maximum surface temperature since the surface layer must cool somewhat before the lapse rate changes from superadiabatic to neutral.

As a result of the strength of buoyant production of turbulence, the daytime convective layer often reaches 1000 m or more in depth. On the other hand, the depth of the much more weakly forced nocturnal boundary layer is typically of the order of a few hundred meters. Thus, it must take some time for the mixing layer to adjust from a deep convective layer at radiation sunset to a shallow shear-driven layer at night. If the mixed layer is, for example, 1000 m deep at radiation sunset and 200 m deep when firmly established at night, it follows that turbulent eddies in existence between 200 and 1000 m in height must be dissipated before the nocturnal layer can be considered a 200 m deep mixed layer above which ground-induced turbulent mixing is insignificant. Since both the kinetic energy and dissipation rate of a turbulent flow are dependent on the size of the largest eddy in the system (Tennekes and Lumley, 1972), the time it takes the turbulence in the afternoon mixed layer to decay is proportional to the dissipation time of a convective eddy the size of the mixed layer. When buoyant production ceases at radiation sunset, the energy budget of the system, described by (1), reduces to an approximate balance between the rate of change of kinetic energy and the dissipation rate:

$$\frac{de}{dt} = -\epsilon. \tag{2}$$

Adopting standard scaling relations (see, e.g., Tennekes and Lumley, 1972), this becomes

$$\frac{d}{dt}k_1 u'^2 = -k_2\frac{u'^3}{H}, \tag{3}$$

where U is the mixed-layer depth, $k_1=\frac{3}{2}$ when u' is a one-dimensional turbulent velocity fluctuation in a system which is assumed to be isotropic, and k_2 a constant of proportionality, about equal to 4 (H. Tennekes, personal communication). Solving this expression yields

$$\frac{1}{u'} - \frac{1}{u_0'} = \frac{4}{3}\frac{t}{H}. \tag{4}$$

Thus, the characteristic velocity will decrease hyperbolically. Once the velocity becomes significantly smaller than the initial velocity u_0' it follows that the dissipation time becomes independent of the initial characteristic velocity. If it is assumed that $H=1000$ m and that the large convective eddy is totally dissipated when its velocity is reduced to within the typical range for synoptic-scale vertical motion ($u=5$ cm s^{-1}), then approximately 4 h is necessary for turbulence within a typical convective mixed layer to completely dissipate after radiation sunset.

Just as it takes convective turbulence a certain amount of time to dissipate, so does it take a certain amount of time for the nocturnal boundary layer to become established. Nocturnal boundary layer simulation by the numerical model of Delage (1974) suggests that it takes up to 6 h for final nocturnal equilibrium to be reached, although after 3 h the eddy diffusivity approaches zero at the top of the evolving boundary layer. Thus the scaling argument and numerical modeling results provide a consistent estimate of the time necessary for the planetary boundary layer to adjust from a deep convective layer to a shallow nocturnal mixed layer.

In the nocturnal mixed layer, the buoyancy flux is directed downward as the earth's surface cools. The mechanical production of turbulence through Reynolds stresses remains as the only significant mechanism by which turbulence is produced. However, these motions are to at least some degree damped by negative buoyancy. In the nocturnal boundary layer, momentum is extracted from the mean flow and transported downward by turbulent eddies to be dissipated near the ground. This turbulent transfer is also effective in transferring heat downward where it can be very effectively radiated to space by the ground surface. Because of the stable temperature stratification, the flow above a few hundred meters ceases to be influenced by the drag of the ground surface. Above this height, the unhindered flow assumes the free stream velocity. Vertical wind shear is then confined to a shallow layer above the ground. Frequently, the mean flow undergoes an inertial oscillation, resulting in the formation of a nocturnal jet stream at the top of the nocturnal layer, further increasing low-level wind shear. The height of the wind maximum has been used by Clarke (1970) to define the top of the nocturnal boundary layer, since this is the maximum height to which shear-induced turbulence can be generated as a direct consequence of the ground surface. Melgarejo and Deardorff (1974) conclude that the height of the wind maximum and the height to which significant cooling has extended are intrinsically interrelated, and they can be used simultaneously for best estimation of the mixed layer depth.

It cannot be overemphasized that the nocturnal boundary layer, although stably stratified with respect to temperature, is still turbulent because of the wind shear. The Richardson number can be used to quantify the stability of the nocturnal surface layer. Frequently, the combination of shear and buoyancy is such that the Richardson number is less than critical and the

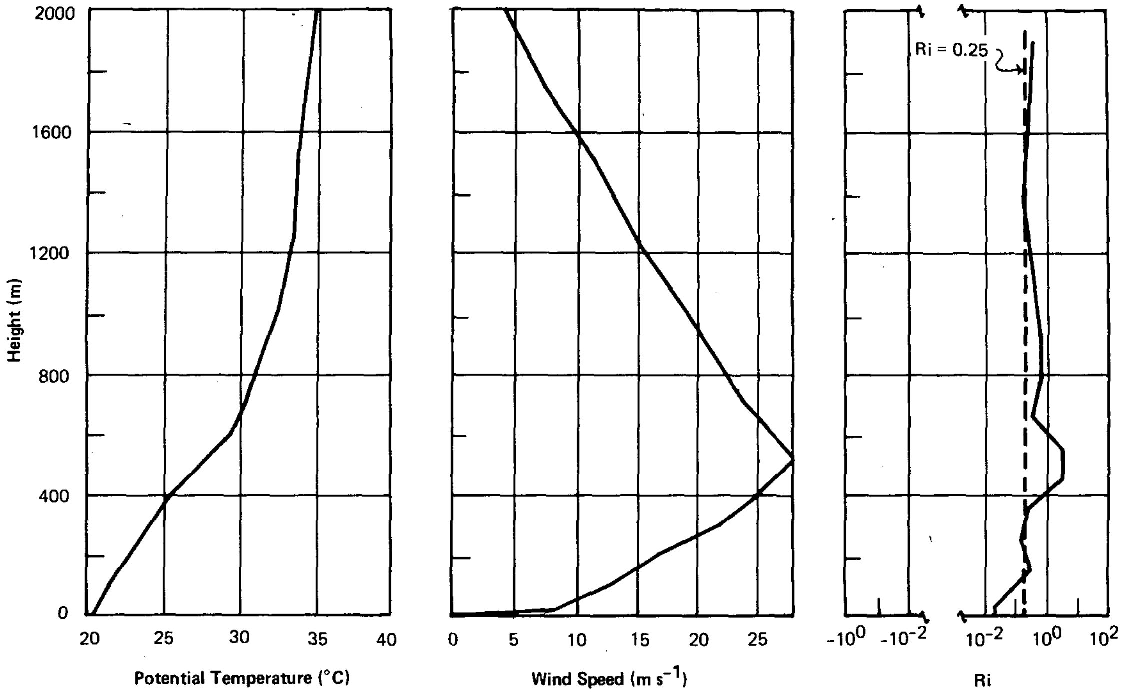

FIG. 3. As in Fig. 2 except for 0435 LST 13 August 1953.

nocturnal layer is continuously turbulent. At other times, in an initially laminar flow, the shear increases (as momentum is extracted near the ground) until flow breakdown and turbulent bursting occur, as first described by Durst (1933). Thus, the shear layer or cooled layer may not be continuously turbulent, but it still remains the maximum depth through which a pollutant can be mixed at night. Smith and Hunt (1978) described an episode where sulfur dioxide trapped within a developing and cooling marine layer (whose characteristics are very similar to the nocturnal boundary layer over land) was completely removed during transport from England across the North Sea. Sulfur dioxide above the sheared, cooled layer remained generally untouched in transit.

Fig. 3 presents nighttime profiles of potential temperature, wind speed and Richardson number measured at O'Neill, on 13 August 1953. Despite a ground-based temperature inversion through 600 m, the very large vertical wind speed shear, culminating in a wind speed maximum of almost 28 m s^{-1} near 600 m, produces Richardson numbers that indicate turbulent mixing through at least 400 m.

Once developed, the nocturnal boundary layer is likely to remain roughly constant in depth unless external forcing (e.g., the free stream velocity) changes significantly. The depth of the mechanically mixed nocturnal layer can be deduced from numerical simulation by models such as those proposed by Deardorff (1971), Delage (1974) and Blackadar (1976). But these models are generally too sophisticated and difficult to apply for routine operational use. Simple scale relations based on easily obtainable parameters seem better suited. Yu (1978) has provided a review and validation study of such formulations. The correlation coefficients he generates emphasize that the simple parameterization $H=cu_*/f$ (e.g., Monin, 1970; Clarke, 1970), where u_* is the friction velocity and f the Coriolis parameter, is as successful as formulas which also include the Monin-Obukov length (e.g., Deardorff, 1972; Businger and Arya, 1974) and certainly better than prognostic scaling equations (e.g., Deardorff, 1971; Zilitinkevitch and Monin, 1974). Yu (1978) did not look at any forms incorporating the nocturnal cooling rate, such as that of Kellermeyer (1976), but it should be pointed out that neither Kellermeyer's form nor forms explicitly incorporating the Monin-Obukov length are valid in the limiting case of neutral conditions. Therefore, it appears that the form $H=cu_*/f$ is most general and most attractive. Values cited for the constant c in this formula for a shear-driven mixing depth range almost an order of magnitude from about 0.05, suggested by the numerical modeling results of Delage (1974), to 0.30, mentioned by Deardorff (1971). Clarke (1970) used $c=0.20$ for the top of the mixed layer under stable and neutral conditions, while Plate (1971), on the basis of matching theory in a neutral layer, derived $c=0.185$. Plate's value was adopted for the proposed model since his value is most firmly based on theory, and falls near the mean of values reported by the other investigators.

At radiational sunrise, heating once again commences, and the daily cycle begins again. During the initial stage of growth, it is unclear whether convection will heat the nocturnal inversion systematically from below or whether the wind shear will tend to mix this heating uniformly throughout the entire nocturnal mixing depth. One-dimensional convective growth models do not incorporate wind shear.

The mixing depth cycle outlined above took the heat flux to be upward during the day and downward at night. Indeed, it was assumed in the discussion of the daytime situation that the buoyant production of turbulence far exceeded the mechanical production of turbulence. This will not be the case with overcast and windy conditions, when the mechanical production greatly exceeds buoyant production. Here, the mixing depth is better described by mechanical considerations. If overcast, windy conditions are present at night as well, then the nocturnal layer, without loss of heat, will be neutrally stratified, and the day-night transition becomes insignificant.

3. The proposed model

The proposed scheme, a preliminary description of which appears in a conference paper by Benkley and Schulman (1978), uses hourly wind speeds and surface temperatures from the nearest representative National Weather Service surface station. Temperature soundings (0000 and 1200 GMT, daily) are required from the nearest radiosonde station. It should be noted that because of the standarized times of radiosonde soundings, the scheme is primarily applicable for longitudes intersecting the contiguous United States.

The mixing depth at all hours is based on the maximum of mechanical value and a convective value. The convective value, however, is set equal to zero during all nighttime hours. Both mechanical production and convective production usually occur simultaneously in the daytime layer, but to a rough approximation, the more important production method can be considered dominant.

The model calculates the mechanical value from

$$H_m = 0.185\frac{u_*}{f}. \quad (5)$$

If the wind profile is logarithmic and the von Kármán constant 0.35, the Coriolis parameter 10^{-4}, and for the purposes of model validation (Section 4) the roughness length 5 cm, this relationship becomes

$$H_m \approx 125u, \quad (6)$$

where u is the wind speed (m s^{-1}) measured at 10 m. It is suggested that the wind speed be center-averaged for 3 h around the time when mechanically driven mixing depths are computed. The wind speed is probably subject to greater variability than the mixing depth. As an example, a 2 m s^{-1} surface wind speed averaged from time $t-1$ hours to time $t+1$ hours implies a mechanically turbulent mixing regime through a 250 m depth at time t hours.

When convective turbulence is dominant during the daytime hours, the mixing depth is estimated as the depth of the neutral layer defined by drawing an adiabat from the hourly surface temperature to an intersection with the morning sounding. Two simple improvements are made to the scheme, however. First, the scheme incorporates advection by a simple adjustment of surface temperatures. In many cases, strong warm or cold advection in the course of the daytime hours significantly modifies the 1200 GMT sounding. Thus, hourly surface temperature alone will not reflect the actual rate at which the sun heats the mixed layer relative to the air above. Second, if the 1200 GMT temperature at the surface station differs from the 1200 GMT surface temperature at the radiosonde station, or if the minimum relative temperature occurs before or after 1200 GMT, the sounding is adjusted to account for this difference. Both modifications are explained below.

The model incorporates the effects of advection by means of the measured temperature change at 700 mb between 1200 and 0000 GMT. The 700 mb mandatory level is recommended since it is nearly always above the top of the mixed layer. (In mountainous locations 500 mb may be necessary.) An advective temperature change computed at 850 mb could be contaminated by the growing mixed layer. It is assumed that the advection measured at 700 mb is typical of the entire lower atmosphere. During daytime hours, the model computes a relative temperature T_r, the surface temperature relative to the 1200 GMT sounding at t hours after 1200 GMT:

$$T_r = T_m - \frac{t}{12}[(T_{700\text{ mb}}(0000\text{ GMT}) - T_{700\text{ mb}}(1200\text{ GMT})], \quad (7)$$

where T_m is the measured surface temperature at t hours after 1200 GMT. If advection is negligible, then $T_r = T_m$.

Use of T_r rather than T_m yields a shallower mixing depth during warm advection situations and a deeper mixing depth during cold advection situations. In fact, with strong cold advection the actual surface temperature may decrease throughout the day, but there may well be a deep convective mixing depth with sensible heating leading to a slower temperature decrease near the surface than at 700 mb.

At 1200 GMT the surface temperature at the surface station and the radiosonde station may be different. The minimum relative temperature at the surface station may also occur before or after 1200 GMT. The sounding should therefore be adjusted to be representative of the surface station location at

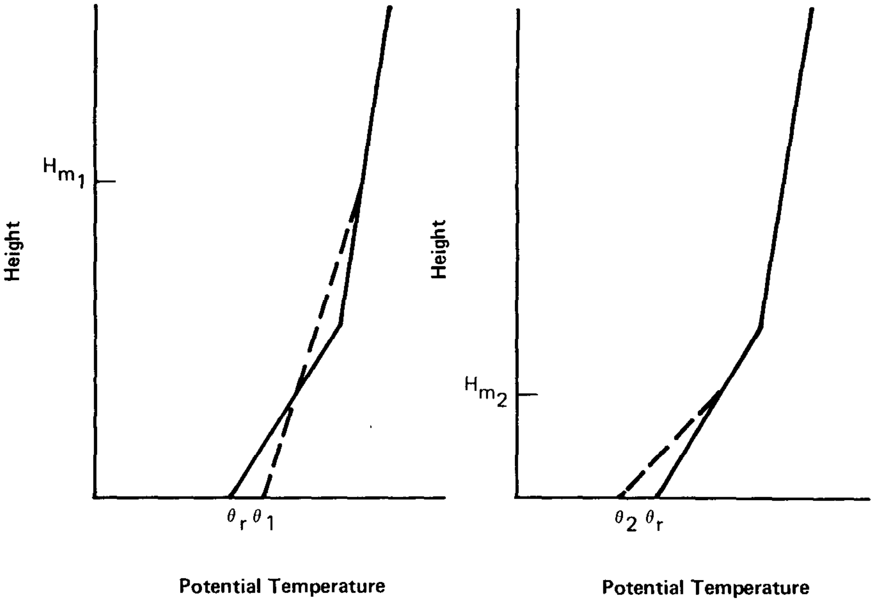

FIG. 4. Adjustment of the morning sounding from ground to the mechanical value H_{m_1} for surface potential temperature θ_1 and radiosonde station surface temperature θ_r for $\theta_1>\theta_r$; and from ground to H_{m_2} for $\theta_2<\theta_r$.

time of minimum relative temperature (the time at which the daytime convective regime begins). The sounding is modified by assuming a linear lapse rate from the minimum relative surface temperature (at the surface station) to the radiosonde temperature at the mechanically mixed height H_m. This adjustment is illustrated in Fig. 4 for two different hypothetical cases.

At all daytime hours the actual mixing depth chosen is the maximum of the convective and mechanical values. In general, with clear skies, light winds and little temperature advection, the convective depth will be the greater of the two. However, some warming after sunrise is necessary for the depth computed by the convective method to exceed the depth computed by the mechanical method. With overcast, windy conditions, the mechanical method will produce the greater mixing depths.

On rare occasions, the relative surface temperature may fall throughout all daytime hours. In these situations, the scheme remains in the nighttime regime, and a mechanical depth is computed at all hours.

If the relative temperature temporarily dips before the hour of maximum temperature is reached during the day, such as during a summer shower, the scheme remains in the daytime regime for that period and the previously occurring maximum convective depth persists.

The model switches from daytime to nighttime regimes when the relative temperature falls a specified percentage of the daytime relative temperature range from the relative daytime maximum. Basing the transition on the daytime relative temperature range is more realistic than requiring the switch from day to night regimes at some fixed time increment after maximum relative temperature. The nighttime regime may begin earlier, for example, on a day with a rapid decrease in relative surface temperature from the afternoon maximum than on another day with the same temperature range but a much more gradual decrease in temperature. From model predictions used in the following section the average time it took the relative temperature to fall 10% was 1 h, 43 min; 20% took a total of 2:43; 30% took 4:11; and 40% took 5:42. Therefore, a percentage of about 25% yields results which are most compatible with the 3–4 h transition time suggested in Section 2.

The maximum afternoon depth is persisted between the hour of maximum T_r and the hour at which the nighttime regime begins.

If by 0000 GMT the relative surface temperature is still rising, or has not fallen 25% of the daytime range from the afternoon maximum, the daytime scheme continues with the next 12 h temperature advection rate (0000 to 1200 GMT) then being used in computing the surface temperature relative to the morning sounding.

4. Model validation

Mixing depths were computed hourly at a smooth, flat central Illinois site (Kincaid, Illinois) for the month of November 1976, with surface data obtained at Kincaid and temperature soundings obtained at Peoria, about 130 km north of Kincaid. The computed mixing depths were compared to values observed by sodar (Aerovironment 300) at Kincaid and by radiosonde at Peoria. Since the two data sets were incompatible in several respects, they were treated as two independent data sets in the model validation rather than merged into one composite data set.

The sodar provided analog measurement of the mixing depth. Because of signal-to-noise ratio problems, the sodar was of limited use in measurement of mixing depths deeper than 325–600 m. However, because of the known structures of both the daytime and nighttime boundary layers as observed by sodar, it was often evident that when the mixing depth could not be determined, it was still possible to categorize the mixing depth as being deeper than some value. Data capture was only 60% for sodar mixing depth measurement, yet much greater for sodar mixing depth categorization (<75, 75–125, 125–175, 175–225, 225–275, 275–325 and >325 m).

The radiosonde record provided measurement of both 0600 CST (nocturnal) and 1800 CST (maximum daytime) depths. Although the sun had set before 1800 CST, measured profiles, except next to the ground, had likely undergone only slight modification since radiation sunset. It was therefore assumed that features representative of the maximum afternoon vertical extent of mixing were still evident on the 1800 CST soundings. One 1200 GMT and two 0000 GMT soundings were missing during the month. The missing 0000 GMT sounding led to the loss of one afternoon measurement, while the two missing 0000 GMT soundings led to the loss of two morning measurements, as well as two afternoon model calculations. Therefore, comparison of computed versus measured mixing depths was not possible on three mornings and two afternoons during the month due to missing radiosonde data. In addition, it was impossible to determine a mixing depth from soundings on an additional three mornings and three afternoons during the month. The three morning cases did not exhibit well-defined shear or temperature features, and the four afternoon cases were either decidedly subadiabatic in temperature or did not exhibit evidence of a mixing layer cap in either the temperature or dew-point soundings.

Radiosonde and sodar measurements were both available at 0600 CST on 17 days out of the month. A small (~10%) difference between the means and a correlation coefficient of 0.67 suggest that 1) the nocturnal mixing depths at Kincaid and Peoria are significantly cross correlated, and 2) radiosonde measurement of the mixing depth agrees with sodar measurement. No such comparison between measurement techniques can be made for the mixing depth at 1800 CST since the maximum afternoon depth is almost always too deep for measurement of the depth by sodar.

Using the chi-square test, the proposed scheme and the EPA scheme were compared to sodar measurements. This yielded $\chi^2 = 145.0$ for the proposed scheme and $\chi^2 = 17.8$ for the EPA scheme. Although one can reject the null hypothesis of no skill for both schemes [χ^2(1%, 4 degrees of freedom) = 13.28], the proposed scheme more closely fits the sodar measurements than the EPA scheme for the data considered.

TABLE 1. Means and standard deviations of mixing depths determined by radiosonde and estimated by the proposed and EPA schemes (μ = mean, σ = standard deviation).

	25 morning cases		23 evening cases		48 total cases	
	μ	σ	μ	σ	μ	σ
Measured	333.4	226.1	1013.6	440.3	659.3	484.5
Proposed	446.5	256.2	1069.0	397.9	744.0	454.3
EPA	229.3	381.0	933.7	428.8	566.8	535.4

The proposed scheme and the EPA scheme were compared to three subsets of the radiosonde measurements: 1) morning radiosonde measurements, 2) evening measurements and 3) the composite set of both measurements. Except for the morning subset, the means of the mixing depths of the proposed scheme fell closer to the observed means than the means generated by the EPA scheme (see Table 1). However, because of the large variance in the data for either model or any of the three subsets, one cannot reject the null hypothesis that the predicted mean is the same as the observed mean. It should be noted that while mechanical mixing depths in the validation of the proposed scheme were estimated *a priori* as $H_m \approx 125u$, the morning case indicates that best agreement between observed and predicted means occurs with $H_m \approx 90u$.

The proposed and EPA-predicted depths were regressed on the depths of each of the three observed subsets. Table 2 presents these results. For all three subsets, the proposed scheme is better correlated with measurements. In fact, the EPA scheme shows no skill in the evening subset; one cannot reject the null hypothesis that the slope of the linear regression is zero since $F = 0.1$ is less than $F(1\%, 21, 21) = 2.72$. This is particularly interesting since in cases of negligible temperature advection, the proposed and EPA schemes are equivalent in predicting the maximum afternoon mixing depth on primarily convective days. Therefore, the inclusion of advection and the recognition of a primary mechanical regime are important to the ability of the proposed scheme to accurately predict daytime mixing depths.

5. Summary

The proposed mixing depth scheme estimates mixing depths on an operational basis. Comparison of both the proposed and EPA schemes with one month of observational data suggest that the proposed scheme is an improvement over the EPA scheme. Three factors

TABLE 2. Results of linear regression of proposed and EPA estimated mixing depths on observed depths (P = predicted, M = measured, r = corelation coefficient, F = F ratio).

	25 morning cases			23 evening cases			48 total cases		
	Regression equation	r	F	Regression equation	r	F	Regression equation	r	F
Proposed	$P=0.91M+142.8$	0.80	42.0	$P=0.68M+376.7$	0.75	28.0	$P=0.83M+200.9$	0.88	167.1
EPA	$P=1.13M-149.2$	0.67	19.1	$P=0.06M+877.5$	0.06	0.1	$P=0.67M+126.5$	0.60	26.5

are important to the improved estimates of the proposed scheme: 1) recognition of a mechanically forced nocturnal boundary layer whose depth is dependent on surface wind speed and roughness, 2) allowance for either primarily mechanical or primarily convective regimes during the daytime and 3) inclusion of the effects of temperature advection. Because of the limited data set available, it would be useful to further validate the model at different locations and for different seasons of the year.

The proposed model can process one year of data in approximately 15 min on a Data General Eclipse C300. Input data are obtained from upper air sounding (5600 format) and surface (TDF 14) tapes, both available from the National Climatic Center.

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