The Effect of and Correction for Different Wet-Bulb and Dry-Bulb Response in Thermocouple Psychrometry[1]

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ABSTRACT

Fast-responding thermocouple psychrometers are often used in atmospheric boundary-layer turbulence measurements for the computation of heat and moisture fluxes. Small size, low cost, ease of interchangeability and the use of the familiar psychrometric equations make this an ideal sensor for many applications at temperatures above freezing. However, a feature of these instruments that is frequently disregarded is that, due to wicking, the wet-bulb sensor has a frequency response that is an order of magnitude slower than the dry-bulb sensor. This difference in response time between the wet and dry sensors causes errors in the variances of humidity in one set of data as large as a factor of 5 and major errors in the shape of the humidity spectrum at high frequencies. We present a known but infrequently applied solution to this problem of sensor response differences in the hope that its simplicity, together with the reminder that a problem exists, will serve to encourage its use in the computation of humidity from fast-response psychrometric sensors.

1. Introduction

A fact often ignored in the psychrometric calculation of rapid humidity fluctuations is that wet-bulb thermocouples are frequently an order of magnitude slower in response than dry-bulb ones. It is essential that the frequency response of the two sensors be matched in order to obtain a proper calculation of the humidity. One can match the response of the two sensors by filtering to either slow the faster sensor or increase the response of the slower one. We have chosen the latter, since it is desirable in turbulence measurements to retain as much information as possible at the higher frequencies. As long ago as 1958, Panofsky and Brier suggested filtering to increase the response of an instrument. [Our first-order equation is equivalent to theirs with our α related to their λ by $\alpha = \lambda/(1 + \lambda)$.]

Many authors do not mention such a correction as being part of the analysis procedure, or they assume that the cutoff frequency of the wet-bulb thermocouple is the limiting frequency for fluctuations in the calculated humidity. For example, Smedman-Högström (1973) states with respect to psychrometric humidity measurements, "... if two instruments have very different time constants ($\tau_1 \gg \tau_2$), the larger time constant will dominate totally." The following data serve to show pitfalls of this assumption in what is a widespread and attractive way of measuring rapid fluctuations of humidity.

We acknowledge that faster measurements of humidity can be obtained from spectral techniques such as the absorption of Lyman-α (Tillman, 1961, 1965; Buck, 1976, 1977) and infrared radiation by water vapor molecules, by the absorption or adsorption of water vapor by various media, and by numerous other techniques. However, many of these techniques suffer from excessive cost, size and reliability problems or are dependent on water vapor-material interactions whose basic physics is not well understood. For temperatures above freezing, psychrometric humidity measurements can be made by wet-bulb and dry-bulb thermocouples with accuracy and reliability and at low cost. Also important in surface layer measurements is the small size of these sensors which enables them to be placed in the immediate proximity of wind sensors for flux measurements without interfering with velocity measurements. These features of thermocouple temperature and humidity measurements make it important to obtain correct measurements of wet-bulb depression at high frequencies.

2. Data acquisition

In the fall of 1976, the University of Washington (UW) participated in the International Turbulence Comparison Experiment (ITCE) sponsored by CSIRO near Deniliquin, Australia. The analysis of humidity fluctuations from these data provided an

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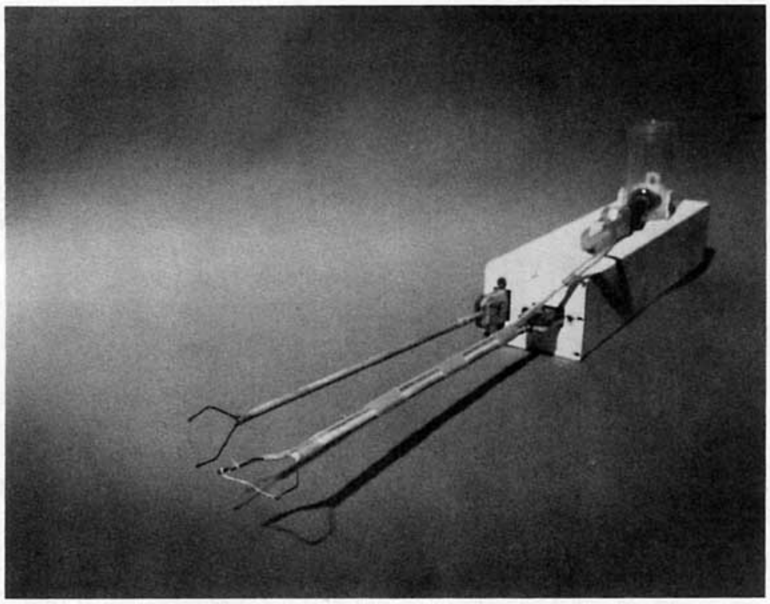


FIG. 1. This photograph shows the thermocouple arrangement for one of the psychrometers used during the ITCE. The psychrometer yielding the data for this paper was mounted on a propeller vane anemometer (Gill propellers on a wind vane) but was identical in operation, and the thermocouples were interchangeable. Note the water reservoir on top and the flow rate adjustment.

example of the serious errors that can be incurred when the response differences between the two thermocouples of the psychrometer are ignored.

The thermocouples used by UW (Fig. 1) were type E chromel-constantan sensors, the sensing elements being 25 μm in diameter and mounted 5 cm apart at a height of 5 m. The wet-bulb sensor was fabricated by wrapping one of the thermocouples with wicking obtained by separating the strands of cotton sewing thread. The overall diameter of the sensing elements and wicking is on the order of 200 μm. Water was supplied from a small reservoir while careful control of the flow rate was provided by an "intravenous bag" flow rate adjustment on the tubing. The cutoff frequency (half-power point in the variance spectrum) for the dry thermocouple was determined to be ~4 Hz, while the wet thermocouple was ~0.4 Hz at a wind speed of 7 m s^{-1}. Data were filtered with a 10 Hz analog low-pass filter prior to being digitally recorded at a sampling rate of 26 s^{-1}.

3. Filtering procedure

The full psychrometric equation combined with the Clausius-Clapeyron equation was used to calculate humidity in this analysis. However, to illustrate the effect of differing time constants between dry- and wet-bulb sensors, the expression for humidity fluctuations may be written

$$q' \approx aT_w' - bT', \qquad (1)$$

where a and b are constants, and $a \approx 2b$. (These relationships are derived in the appendix.) The primes indicate deviations from the mean. To see how the variance spectrum of q is affected by response differences between the wet- and dry-bulb temperature sensors, we write the Fourier transform of (1) as

$$\Phi_q(f) \approx a\Phi_{T_w}(f) - b\Phi_T(f). \qquad (2)$$

Then

$$\begin{aligned} S_{qq}(f) &= \Phi_q(f)\Phi_q^*(f) \\ &\approx a^2\Phi_{T_w}(f)\Phi_{T_w}^*(f) - ab\Phi_{T_w}(f)\Phi_T^*(f) \\ &\quad - ab\Phi_{T_w}^*(f)\Phi_T(f) + b^2\Phi_T(f)\Phi_T^*(f). \end{aligned}$$

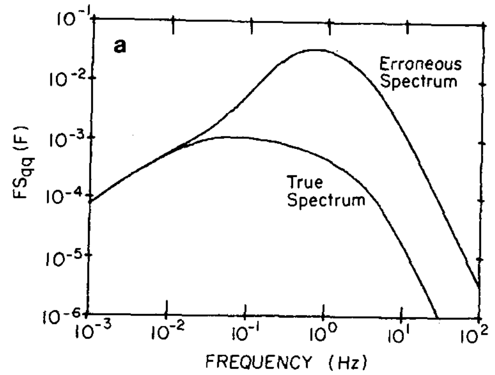
In more standard notation, this is

$$S_{qq}(f) \approx a^2[S_{T_wT_w}(f) - S_{TT_w}(f) + \tfrac{1}{4}S_{TT}(f)], \quad (3)$$

where $S_{xx}(f)$ denotes the power spectrum of $x(t)$ and $S_{xy}(f)$ denotes the cospectrum of $x(t)$ and $y(t)$, and the asterisk denotes the complex conjugate.

If the fluctuations of T and T_w are similar, as was the case in Australia, the behavior of the q spectrum due to the different response times will be as follows: the spectrum will be calculated properly until the T_w response begins falling off; at that point, the first term of (3) rapidly disappears and the second quickly loses its balancing effect on the third, causing a very large erroneous contribution to the variance; above the cuttoff frequency of T, the variance rapidly approaches zero.

In the ITCE data, the shape of the variance spectra



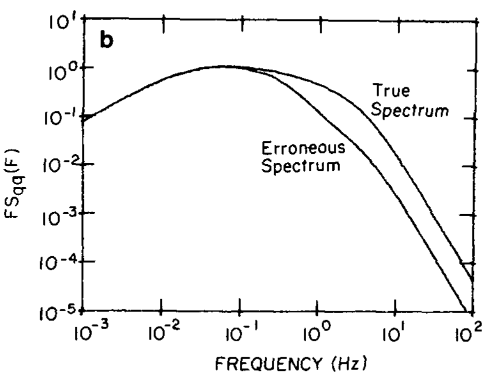


FIG. 2. (a) The upper curve shows the effect of response differences between T and T_w sensors for the case of vertical temperature and humidity gradients of the same sign. The lower curve is the q spectrum if the sensors are matched at 4 Hz. (b) Effect of sensor response difference on the q spectrum if the temperature and humidity gradient are of opposite sign. The upper curve is for matched sensor cutoff frequencies of 4 Hz.

of temperature were virtually identical in shape to those of wet-bulb temperature. To see analytically the effect of response differences on the q spectrum, we can use Kaimal *et al.*'s (1972) spectral shape of temperature for neutral lapse rates (this is permissible since we are interested in the spectral shape at the higher frequencies, which their results show is affected little by changes in stability) which is

$$\frac{fS_{TT}(f)}{T_*^2} = \begin{cases} 53.4f/(1+24f)^{5/3}, & f \leq 0.15 \\ 24.4f/(1+12.5f)^{5/3}, & f \geq 0.15. \end{cases} \quad (4)$$

Assuming T and T_w are perfectly positively correlated so that $S_{TT_w}(f) = [S_{TT}(f)S_{T_wT_w}(f)]^{1/2}$, from the data that $S_{T_wT_w}(f) = 0.55S_{TT}(f)$, $T_*^2 = 0.1$, $f_c = 0.4$ for T_w and $f_c = 4.0$ for T, it is possible to construct a humidity spectrum from (3) and (4). The effect of the cutoff frequencies is obtained by multiplying $S_{T_wT_w}(f)$ and $S_{TT}(f)$ by the transfer function of a first-order low-pass filter [Eq. (5)] having half-power points at 0.4 and 4 Hz, respectively. We see graphically in Fig. 2a the point of the preceding paragraph. The erroneous spectrum is shown compared to the true q spectrum calculated assuming equal frequency response for both temperature sensors. Fig. 3 shows the effect of differing frequency responses on real data. The false variance in some of the runs prior to correction was in error by more than a factor of 5.

In the converse case, where the temperature and humidity gradients are of opposite sign (as over water or land under stably stratified conditions) the spectrum would not show a large bump, but rather would simply fall off too quickly, since the contribution from the second term in (2) would be positive. This is shown in Fig. 2b with the previous assumptions except that $S_{TT_w}(f) = -[S_{TT}(f)S_{T_wT_w}(f)]^{1/2}$ and $S_{TT}(f) = S_{T_wT_w}(f)$. None of the runs analyzed by these authors fell into this category.

To compensate for the differing frequency responses of the wet- and dry-bulb thermocouples, the frequency response of each sensor and its relation to the parameters involved in the measurements must be considered. Marchgraber and Grote (1965) have shown that if one can describe a transducer response by a differential equation, then it may be possible to remove some of the effects of the transducer on the signal. To a first approximation, the time response of such sensors is described by a first-order differential equation characterized by a single time constant τ, i.e., $dB(t)/dt = -\tau^{-1}[B(t) - A(t)]$, where $A(t)$ is the meteorological input and $B(t)$ the sensor output. In the present application, the time constant is a function of several properties of the thermocouple and surrounding fluid governing the heat flow, but the most important is the inverse square-root dependence on wind speed (Sano and Mitsuta, 1968). Results of an earlier test of the wind speed-time constant relation for wet and dry thermocouples with the same diameter and similar fabrication are summarized in Table 1. If the response time is an inverse square-root function of wind speed, the product $\tau U^{1/2}$ would be constant. This is the case for the data of Table 1 and any variations are well within the experimental accuracy.

Except for highly gusty and directionally variable conditions, the dependence on wind speed of the time constant will have little effect in a given run, but may require adjustment from one run to another. Fig. 4 shows the effect on f_c of two wind speeds averaged over 30 min: 5.34 m s^{-1} for Run 8 and 7.83 m s^{-1}

TABLE 1. Response characteristics of wet-bulb and dry-bulb thermocouples versus wind speed U.

Wind speed (m s^{-1})		1.0	2.0	5.0	10.0	20.0
Response time (s)	Dry*	0.028	0.023	0.027	0.022	—
	Wet	0.50	0.362	0.279	0.184	0.120
$\tau U^{1/2}$	Wet	0.5	0.51	0.62	0.58	0.54

* The estimated dry-bulb time constant is the upper bound due to the limited response time of the laboratory measurement system. For the higher speeds it is significantly shorter.

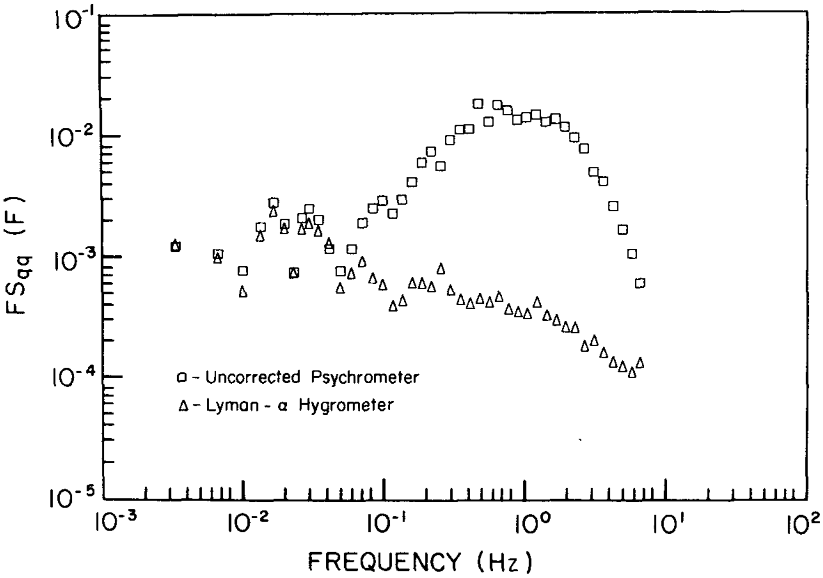


FIG. 3. Variance spectra of humidity from the Lyman-α and the uncorrected thermocouple psychrometer. The high-frequency "bump" in the psychrometric q increases that instruments variance by a factor of 5.

for Run 12. The plot shows a shift in the cutoff frequency which produces a minimum q (at 1 Hz) between the two runs, the true f_c for the wet thermocouple being taken as a minimum in this plot. Fig. 5 shows the relationship of various estimates of f_c in the filter to the total variance of the calculated q trace. The digital analog for the sort of filter described in the above differential equation may be written in recursive form as (Otnes and Enochson, 1978)

$$y_i = \alpha y_{i-1} + (1 - \alpha)x_i, \tag{5}$$

where x is the original time series and y the filtered time series (sensor output).

This has the associated transfer function

$$|H(f)|^2 = \frac{(1 - \alpha)^2}{1 - \frac{1}{2}\cos(2\pi\Delta t f) + \alpha^2} . \tag{6}$$

From this expression it is possible, if the cutoff

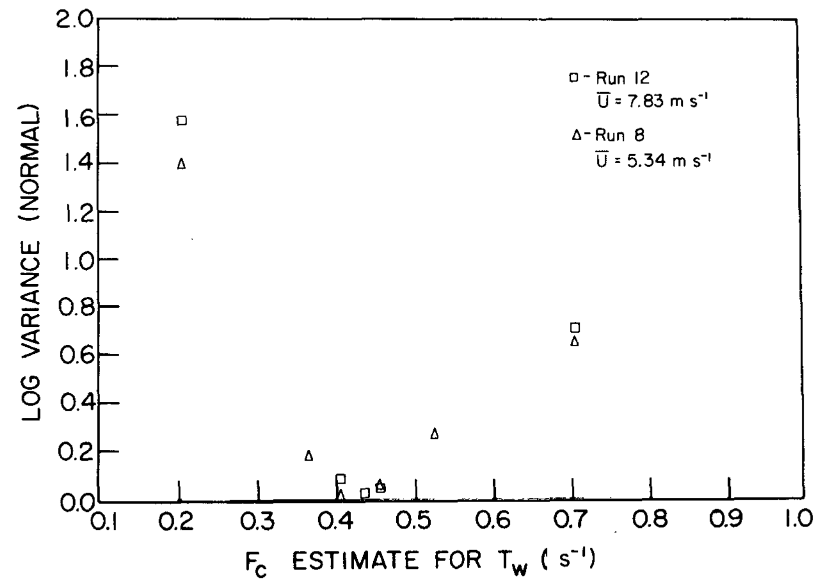


FIG. 4. Effect of two winds speeds on f_c. The abscissa contains the possibilities for the estimated values of f_c input into the correction filter for T_w. The true value of f_c is assumed to be the value for which the variance is a minimum. The ordinate displays the variance of q at 1 Hz. This was chosen as an indicator since the maximum spectral error occurred near 1 Hz.

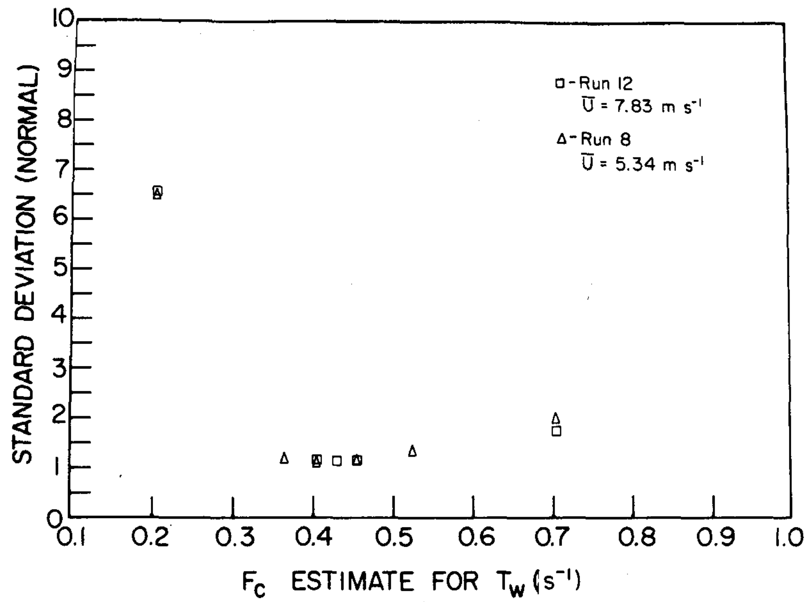


FIG. 5. This figure is analogous to Fig. 4 except the ordinate is now the square root of the *total* variance. One must be careful to obtain the proper f_c. Overcompensating rapidly produces even worse results.

frequency f_c is defined to be $|H(f_c)|^2 \equiv 1/2$, to express α as

$$\alpha = 2 + [3 + \tfrac{1}{2}\cos(2\pi\Delta t f_c)]^{1/2}. \qquad (7)$$

The cutoff frequency for the wet-bulb thermocouple can be determined by a comparison, under conditions of temperature-humidity similarity, of variance spectra of T and T_w. In the ITCE runs, these were found to have virtually identical shapes for the lower frequencies. Assuming their shapes were also identical at higher frequencies in the actual variation of the quantities, it was possible to estimate f_c from the spectra of T and T_w. In other conditions, a laboratory determination is preferable.

If Eq. (5) is written as

$$x_i = \frac{y_i - \alpha y_{i-1}}{(1 - \alpha)}, \qquad (8)$$

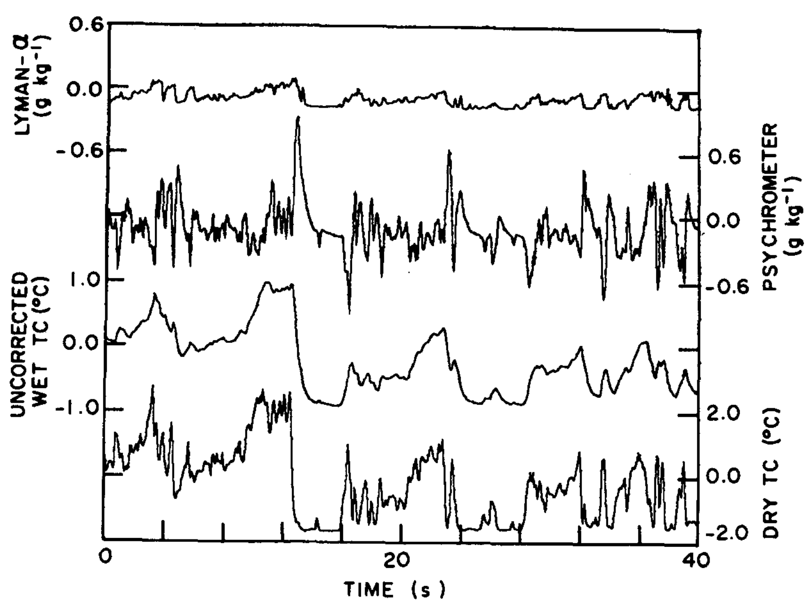


FIG. 6. Raw traces of T, T_w, the resulting trace of q and the Lyman-α humidity trace. T_w is noticeably smoother than T, and the psychrometric q and Lyman-α traces are completely different. Note the large amplitude of the fluctuations in the calculated q.

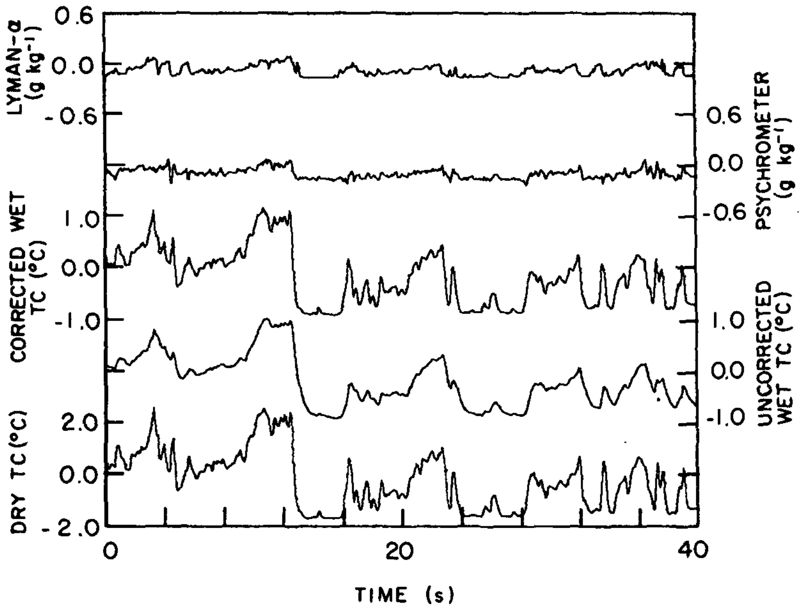


FIG. 7. As in Fig. 6, but with the addition of the corrected T_w trace and the resulting psychrometric calculation of q. The trace now compares well to that from the Lyman-α, although it is somewhat noisier.

one may calculate α from Eq. (7), and use Eq. (8) to actually "speed up" the wet-bulb thermocouple response so that it is compatible with the dry-bulb response. For these data, the Nyquist frequency was 6 Hz after two-point block-averaging. Since the f_c was ~4 Hz for the dry-bulb thermocouple, similar correction for this sensor made little difference and was not necessary. However, if the Nyquist frequency were significantly larger than the dry-bulb thermocouple cutoff, it would have been necessary to correct that sensor also.

4. Results

Fig. 6 shows uncorrected traces of T, T_w and the resulting q compared to the humidity trace from the Lyman-α. Fig. 7 shows the same time traces of the dry- and wet-bulb thermocouples as well as the corrected wet-bulb thermocouple signal and the resulting trace of q. The improvement in response of the corrected signal of T_w is obvious, as is the better agreement between the psychrometric and Lyman-α measurements of humidity. It should be emphasized that this filter does not create new information in the wet-bulb signal; rather, it restores the phases and amplitudes of the signal to nearly their original form before being distorted by the sensor. This technique works well as long as the signal-to-noise ratio is high and the sampling interval is much less than the time constant. If the signal is noisy, some low-pass filtering at the higher frequencies may be desirable.

The same technique may be used on the analog

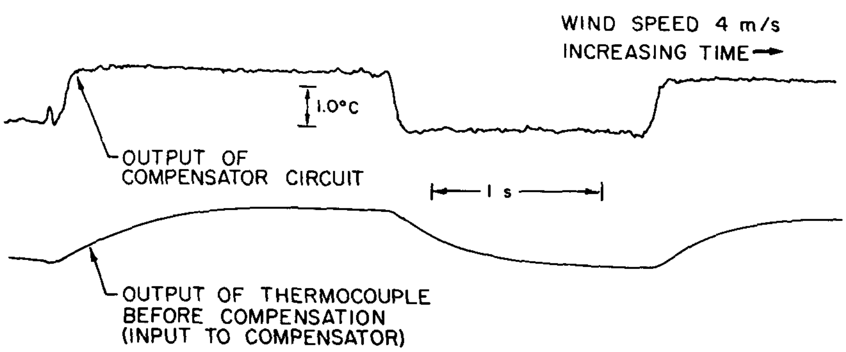


FIG. 8. The raw and the compensated time history of a wet-bulb thermocouple in in response to a stepfunction. The slow rise and decay of the wet-bulb thermocouple is shown by the lower trace while the output of the compensation circuit is shown in the upper trace. The compensation was selected to decrease the time constant by more than an order of magnitude. The fine structure on the compensated output is due to power line pickup as the test was not designed to minimize noise.

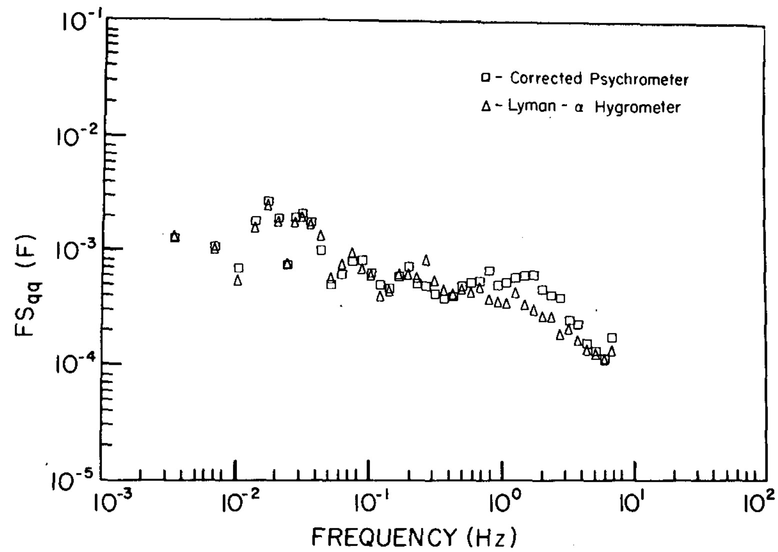


FIG. 9. Spectrum of corrected humidity from the thermocouple psychrometer compared to the Lyman-α spectrum. The remaining bump in the psychrometer is from noise, but the result is now much closer to the Lyman-α.

signal as is illustrated by Fig. 8. The result of selecting the components of an analog filter to compensate for a specific time constant (wind speed) shows the dramatic improvement in response time and, consequently, in frequency response, obtainable with these techniques. It must be remembered that the time constant is wind speed dependent as shown in Table 1. However, the wind speed dependence is mitigated since it varies as the square root of wind speed rather than a higher power. This form of compensation might be especially useful from platforms such as aircraft where the speed variation is minimal.

Fig. 9 shows the spectrum of corrected humidity from the psychrometer plotted with a humidity spectrum from a Lyman-α humidiometer mounted nearby. The agreement is now much better. In this particular run, the signal-to-noise ratio was somewhat low; thus there remained a residual bump resulting from the combination of noise limiting 2 Hz low-pass filter, the noise effect on the spectrum at the higher frequencies, and the proximity of these frequencies to the Nyquist frequency. The noise also appears in the time trace of the corrected q (Fig. 7). However, the trace is now clearly similar to that from the Lyman-α. The values of σ_q from the psychrometer prior to and after correction are 0.19 and 0.085 g kg^{-1}, illustrating the importance of this correction even for accurate estimates of total variance. The value of σ_q from Lyman-α for this run was 0.081 g kg^{-1}.

In conclusion, this discussion presents a clear example of the problems one can encounter if the frequency responses of various sensors jointly used to calculate a quantity are not taken into account. In addition, a known but infrequently applied solution to this particular problem has been presented in the hope that its simplicity, together with the reminder that the problem exists, will serve to encourage its use in the computation of humidity from fast-response psychrometric humidity sensors.

Acknowledgments. We would like to express our appreciation to Fred Weller and Mike Intlekofer, who built and maintained the Lyman-α and the thermocouple psychrometer; to CSIRO, who sponsored the ITCE; and to Al Macklin, who performed many of the early thermocouple response measurements. This work was supported by NSF Grants ATM76-00855 and ATM77-23715, and the majority of the computer work was done on a Raytheon 704 and a PRIME 400 (supported by NASA Contract NAS1-9694) in the department. Bill Guest photographed the psychrometer.

APPENDIX

Linear Approximation for Fluctuations of Humidity

Following is a justification for Eq. (1). The psychrometric equation can be written as

$$q = q_s - \frac{c_p}{L}\frac{K_L}{K_q}(T - T_w), \qquad \text{(A1)}$$

where q is the specific humidity (g g^{-1}), q_s the saturation specific humidity (q at $T = T_w$), T and T_w are the dry-bulb and wet-bulb temperatures, respectively, c_p is the specific heat of air, L the latent heat of vaporization, and K_L and K_q are the convective heat and water vapor diffusivities, respectively. Since this equation is approximate, (c_p/L)

$\times (K_L/K_q) \equiv A$ may be conveniently treated as a "tunable" constant.

The Clausius-Clapeyron equation is written for this case as

$$\frac{de_s}{e_s} = \frac{L}{R_w}\frac{dT}{T^2}, \qquad \text{(A2)}$$

where e_s is saturation vapor pressure and R_w the gas constant for water vapor. Saturation is assumed to occur at the thermocouple at a temperature T_w. Integrating Eq. (A2) between T_w and a reference wet-bulb temperature T_{ref} yields

$$\ln[e_s(T_{\text{ref}})] - \ln[e_s(T_w)] = \frac{L}{R_w}\left(\frac{1}{T_w} - \frac{1}{T_{\text{ref}}}\right)$$

or

$$e_s(T_w) = e_s(T_{\text{ref}}) \exp\left[B\left(\frac{1}{T_{\text{ref}}} - \frac{1}{T_w}\right)\right], \qquad \text{(A3)}$$

where $B \equiv L/R_w$ and $e_s(T_{\text{ref}})$ may also be regarded as constants "tunable" to a particular temperature range. For the ITCE analysis, T_{ref} was chosen to be 288 K, giving from vapor pressure tables (e.g., CRC *Handbook of Physics and Chemistry*) $e_s(288) = 17.05$ mb. Also from the tables $e_s(283) = 12.28$ mb, yielding a value of $B = 5358$. Thus Eq. (A3) was adjusted for the temperature range 10–15°C. However, errors in e_s remain less than 1% far outside this range.

Reynolds decomposition of Eqs. (A1) and (A3) yields

$$q = \bar{q} + q' = \bar{q}_s + q_s' - A[(\bar{T} + T') - (\bar{T}_w + T_w')], \qquad \text{(A4)}$$

$$e_s = \bar{e}_s + e_s' = B_0 \exp[-B/(\bar{T}_w + T_w')], \qquad \text{(A5)}$$

where

$$B_0 = e_s(T_{\text{ref}}) \exp(B/T_{\text{ref}}) = 2.05 \times 10^9 \text{ mb}.$$

Expansion of Eq. (A5) in a Maclaurin series with T_w' as the independent variable gives

$$e_s = B_0\left\{\exp\left(-\frac{B}{\bar{T}_w}\right) + \frac{BT_w'}{\bar{T}_w^2}\exp\left(-\frac{B}{\bar{T}_w}\right) + \left[\frac{-2B}{\bar{T}_w^3} + \frac{B^2}{\bar{T}_w^4}\right]\tfrac{1}{2}T_w'^2 \exp\left(-\frac{B}{\bar{T}_w}\right) + \ldots\right\}$$

$$\approx B_0 \exp\left(-\frac{B}{\bar{T}_w}\right)\left[1 + \frac{B}{\bar{T}_w^2}T_w'\right] \qquad \text{(A6)}$$

and the fluctuating part is

$$e_s' = \frac{B_0 B}{\bar{T}_w^2} T_w' \exp\left(-\frac{B}{\bar{T}_w}\right). \qquad \text{(A7)}$$

For a wet-bulb temperature $T_w = 283$ K,

$$e_s' \approx 0.82 T_w'. \qquad \text{(A8)}$$

The relationship between specific humidity and vapor pressure is

$$q_s \approx \frac{0.62 e_s}{P} \Rightarrow q_s' \approx \frac{0.62 e_s'}{P}, \qquad \text{(A9)}$$

where P is atmospheric pressure. Substitution of (A8) and (A9) into the fluctuating part of (A4) yields

$$q' = 5.02 \times 10^{-4} T_w' - A(T' - T_w'), \qquad \text{(A10)}$$

assuming a standard pressure of 1013 mb. Combination of the expression for $\bar{q}$ [non-primed portion of Eq. (A4)] and inspection of the pseudo-adiabatic chart yields a value of $A = 3.91 \times 10^{-4}$ so that

$$q' = 8.93 \times 10^{-4} T_w' - 3.91 \times 10^{-4} T'. \qquad \text{(A11)}$$

Therefore, if $a = 8.93 \times 10^{-4}$ and $b = 3.91 \times 10^{-4}$, $a \approx 2b$.

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