On Estimating Hail Frequency and Hailfall Area[1]

ALEXIS B. LONG[2]

National Center for Atmospheric Research, Boulder CO 80307

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ABSTRACT

Probabilistic and statistical concepts are used to examine how the number of hail observing sites within a region affects the accuracy of estimates of 1) the mean point frequency of hail within the region, 2) the overall regional frequency of hail, and 3) the area covered by individual hailfalls. A practically useful relationship $\bar{\nu} = N\bar{a}/A$ is first derived to connect the mean point frequency $\bar{\nu}$ of hail within a region to the regional hail frequency N, to the mean area $\bar{a}$ of the individual hailfalls and to the area A of the region. The error in estimating the mean frequency $\bar{\nu}$ is found to increase as $n^{-1/2}$, where n is the number of sites placed within a region. If within a region there are proportionally more large hailstorms or if most of the area covered by hail is commonly due to a few large hailstorms, then fewer sites will be needed to estimate the mean point hail frequency. Of the 16 hailfalls detected by the 660 km^2 1976 National Hail Research Experiment (NHRE) network of 603 hailpad sites, it is found, using a simple probabilistic expression, that 12 of the hailfalls still would have been detected using only 50 sites. The smaller hailfalls would have been the first to go undetected. There are diminishing returns in fielding sufficient instruments to detect all, or almost all, of the hailfalls in a region. For hailfalls with the lognormal distribution of areas observed by NHRE, the number of instruments needed increases exponentially with the number of hailfalls to be detected. A formula is derived for a correction to be made to the observed regional frequency of hailfalls of a given size so as to obtain the true frequency. For random networks the coefficient of variation of an estimate of the area of a hailfall is proportional to $n^{-1/2}$. For a hailfall less than one-fifth as large as the instrumented region in which it lies, the coefficient of variation of an estimate of its area approximately equals $n_h^{-1/2}$, where n_h is the expected number of sites within the hailfall.

1. Introduction

A description of the hailfalls in a region usually includes 1) the average point frequency of hail within the region, 2) the frequency of hail for the region as a whole, 3) the ground area covered by individual hailfalls, 4) the total mass or kinetic energy of each hailfall, and 5) the spatial distribution within each hailfall of hail mass, kinetic energy, and other variables. These statements are often based on data from a network of hail instruments distributed over the region and thus should be treated as estimates. This paper shows how the density of a network affects the accuracy of the particular estimates of hail frequency and hailfall area. Another paper (Long, 1978) examined the effect of network density on estimates of total hail mass and kinetic energy. Although the focus of the present paper is on networks of hail instruments, a number of the conclusions will be applicable when hailfall data are obtained from networks of hail observers or from insurance information on the occurrence of crop damage due to hail.

The accuracy of estimates of hail frequency and hailfall area will depend on several factors. Two network characteristics of importance will be (i) the number or "density" of instruments placed in the region and (ii) their spatial arrangement. Three hailfall characteristics of importance will be (iii) the shapes of the hailfall areas, (iv) the location of individual hailfalls within the region, and (v) the true distribution according to size of the ground areas covered by individual hailfalls. The present paper examines the effect of factor (i). Factor (v) will be considered at certain points in the analysis. Certain assumptions will permit us, at least in the present work, to ignore factors (ii), (iii) and (iv).

Each hail instrument in a hypothetical network is assumed to operate perfectly and to sense hail whenever it occurs. Accordingly, whenever data from the 1976 NHRE hailpad network are used below for illustrative purposes inoperative instruments are excluded. How this is done will be shown below. The papers by Towery *et al.* (1976) and Nicholas (1977) give critical descriptions of available hail instruments.

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[2] Present affiliation: Department of Meteorology, College of Geosciences, Texas A&M University, College Station 77843.

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Some previous studies of hailfall bear directly on or are related to the topics covered in the present paper. These studies are reviewed at certain places below within the context of the present analysis. This approach helps to clarify some of the points made.

2. Relation between point and regional hail frequencies

A useful relation exists between the point frequency of hail averaged over all points in a region and the frequency of hail for the region as a whole. This relation applies to the hail itself and is true regardless of whether a network is placed in the region.

"Point" hail frequency refers to the frequency of hail at a geometric point but can operationally be assumed to be that detected by a hailpad of 0.1 m^2 surface area. Point hail frequency as determined by an observer would be approximately the same but would apply to a larger area, perhaps 100–1000 m^2. Regional hail frequency would apply to an area of perhaps 10^8–10^{10} m^2.

We consider a region of area A and assume its shape is generally similar to that of most hail study regions around the world. The region would thus be connected, have dimensions not greatly different in any direction, and would not have indentations much smaller than a typical hailfall. During some time period T there will be N "distinct" hailfalls in this region and each hailfall will cover an area a_i ($i = 1, \ldots, N$). If a hailfall lies only partly within A, then a_i will be that part of the true hailfall area which is inside A. Thus a_i will be that part of the true hailfall area which would be *observable* with a network placed within A, and a distribution of values a_i would be the *observable* approximation to the corresponding true distribution of hailfall areas. The differences between observable and true hailfall areas and distributions of areas can be significant if the true hailfall area is not much smaller than A. Such differences arise because of the finite area A of a region and come under the general heading of "edge effects." These effects are discussed to an extent below.

The concept of "distinct" hailfall is important in the present analysis. It is assumed that only one distinct hailfall will occur in a region during a specified time interval. This interval might be that between servicings of hail instruments. Instruments were serviced every 24 h in the National Hail Research Experiment (NHRE) in 1976. In the case of the NHRE data a distinct hailfall may thus include one or more hailswaths or hailstreaks. The hailswaths or hailstreaks need not be contiguous. They may also cover some points in the region more than once; however, the frequency of hail at such points during 24 h is identified only as unity. Because the 1976 NHRE data are used to illustrate certain results of the analysis the area a_i of a distinct hailfall will be defined in this paper as the sum of the areas of all the hailswaths or hailstreaks occurring in a region in 24 h—but with all areas of swath or streak overlap counted only once. The analysis herein is appropriate for other definitions of a distinct hailfall, however, and distinct hailfalls might be defined as individual hailswaths or hailstreaks. Illustrations of the analysis in these cases would require data with time resolution much finer than 24 h. Hereinafter the adjective "distinct" will be dropped but will be understood to apply to each hailfall.

Assuming spatial homogeneity of hailfall probability, that is, assuming a hailfall can occur anywhere within a region of area A with equal probability, it follows that

$$p_i = a_i/A \tag{1}$$

is the probability that the ith hailfall will cover any given point within the region (see also Carte, 1967). The frequency of hail at a point due to the ith hailfall will be either $\nu = 1$ if hail occurred at the point, or $\nu = 0$ if no hail occurred. Taking all N hailfalls into account it follows that during a time period T the population mean or "expected" point frequency of hailfalls will be

$$\bar{\nu} = \sum_{i=1}^{N} p_i \cdot 1 + (1 - p_i) \cdot 0 = \sum_{i=1}^{N} a_i/A .$$

Since the mean hailfall area is given by

$$\bar{a} = \sum_{i=1}^{N} a_i/N,$$

it follows that

$$\bar{\nu} = N\bar{a}/A . \tag{2}$$

Eq. (2) relates the mean point frequency $\bar{\nu}$ of hail in a region of area A during a time period T to the regional frequency of hail N during the same period T and to the mean hailfall area $\bar{a}$. The range of validity of (2) is limited by the assumption made that all points within a region are equally likely to have hail. Should the mean point frequency vary greatly between subregions because of orographic effects, for example, then (2) should be applied separately to each subregion.

Hail frequency data collected by the 1976 NHRE dense hailpad network can be used to illustrate Eq. (2). The dense network had the shape of a rectangle 20 km wide and 33 km long, and in this region hailpads were placed at 612 sites. Important features of the network were its large area (660 km^2) and high density (on average 1 pad per 1.1 km^2). These features allowed us to detect and measure reasonably well the largest as well as the smallest

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hailfalls, and to develop a fairly complete picture of the distribution, according to size, of the ground areas covered by the hailfalls. Setup of the network began in April 1976, and on each day from 29 May through 30 July on average 98.5% of the pads were in operating order. This high percentage, and the careful data-editing procedures used, contributed to the high quality of the final data set. Table 1 lists the number n_i of pads operating and the number m_i of pads that sensed hail, for each of the $N = 16$ days when hail was recorded by the network. (No hail is considered here which may have fallen in the "holes" between network sites either on the days listed in Table 1 or on other days. No effort was made to find such hailfalls using tools such as radar.) Table 1 provides estimates a_i^* of the area covered by hailfall each day. These estimates are based on the formula

$$a_i^* = \frac{m_i}{n_i} A \quad (3)$$

with A set equal to the 660 km² area of the network. Eq. (3) does not take into account local variations in the spacing of instruments and will be subject to large errors when few instruments record hail and m_i is small. Despite these drawbacks (3) will suffice for the purposes of illustrating Eq. (2). It should be noted that estimates of hailfall areas could also be made from planimetric analyses of the hailfall patterns.

Averaging the values of a_i^* in Table 1 yields an estimate for $\bar{a}$, *viz.*, $\overline{a_i^*} = 102.4$ km². Substituting $\overline{a_i^*}$ for $\bar{a}$ in (2) yields

$$\nu^* = N\overline{a_i^*}/A = 2.483 \quad (4)$$

This is an estimate for the mean point hail frequency

TABLE 1. Some characteristics of 1976 NHRE daily hailfalls.

Date	Number of operating pads n_i	Number of pads with hail m_i	Estimated hailfall area $a_i^* = m_iA/n_i$ (km²)
29 May	592	77	85.8
30 May	597	354	391.4
2 June	587	15	16.9
4 June	591	79	88.2
21 June	606	7	7.6
22 June	601	551	605.1
30 June	610	7	7.6
2 July	609	48	52.0
5 July	607	163	177.2
7 July	605	62	67.6
18 July	604	3	3.3
21 July	608	25	27.1
22 July	609	1	1.1
25 July	608	75	81.4
27 July	602	17	18.6
30 July	608	7	7.6

TABLE 2. Observed point hail frequencies for 29 May–30 July 1976.

Observed point hail frequency ν	Number of pads with this frequency	Fraction of all 612 pads with this frequency
0	10	0.0163
1	104	0.1699
2	214	0.3497
3	194	0.3170
4	74	0.1209
5	15	0.0245
6	1	0.0016
≥7	0	0

$\bar{\nu}$ that would have been observed had all ground area in the dense network been instrumented. From the data in Table 2 on the observed point frequencies of hail at all 612 hailpads one obtains an observed mean point frequency

$$\hat{\nu} = 2.436. \quad (5)$$

The close agreement of the two estimates ν^* and $\hat{\nu}$ for $\bar{\nu}$ provides a good example of the relation in (2) between the mean hailfall area and the expected point frequency of hail. The example is especially good since both ν^* and $\hat{\nu}$ are based on the data of a rather large number of instruments. A small difference is observed between ν^* and $\hat{\nu}$ but partly because (4) allows for inoperative hailpads [the number operating n_i appears in the denominator of (3)], whereas (5) does not. A +1.5% correction to $\hat{\nu}$ to allow for inoperative pads would bring it closer to ν^*.

Eq. (2) can be reexpressed as

$$\bar{a} = A/(N/\bar{\nu}) \quad (6)$$

and used to find the mean hailfall area in a region of area A from the ratio $R = N/\bar{\nu}$ of regional to mean point hail frequencies. Note that the mean area $\bar{a}$ applies only to those observable portions of hailfalls which lie inside the region and does not include portions which lie outside the region. The quantity $\bar{a}$ will be a poor approximation to the true mean area if A is not much larger than most hailfalls. Edge effects must then be considered.

Edge effects may be negligible, however, and (6) may be a good approximation to the true mean hailfall area if one does not consider an entire hailfall but rather only *hail cores*. These are regions within hailfalls where the hail is more intense, e.g., hail mass density >3 kg m^{-2}. A hail core covers only a small fraction of a hailfall and will be less likely to cross the boundary of a region. The area of a hail core is usually found by measuring the area within certain isohyets (e.g., lines of constant hail mass density) for the overall hailfall. Eq. (6) provides a rather speedy way to determine the mean area of a

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number of hail cores without having to make tedious hand analyses of all the hailfalls. One simply divides A by the ratio of the regional frequency of hail intensity values defining a hail core to the mean point frequency of these values. By varying the hail intensity one can determine the fractions of an "average" hailfall covered by hail of various intensities. A hailfall "model" is thereby constructed. It is important to note that although N in Eq. (6) might be estimated with little statistical bias from field data, because N enters (6) in a nonlinear way the estimate for $\bar{a}$ made with (6) may not itself be statistically unbiased.

Eq. (2) can be reexpressed as

$$R \equiv N/\bar{\nu} = A/\bar{a}. \quad (7)$$

The ratio R of regional to mean point hail frequencies is important because it can be used to extrapolate from point to regional frequencies. One simply multiplies a mean point frequency by R to obtain the corresponding regional frequency. A regional frequency can be of greater interest than a mean point frequency. For example, in designing a hail suppression experiment for some region A and for some time period T one is not so much interested in the mean point frequency of hail in the region as in the number of distinct hailfalls N, or potential sample size, for the region as a whole. Eq. (7) shows that for a given region size A the ratio R will depend on the mean area $\bar{a}$ of the hailfalls occurring in the region. If $\bar{a}$ varies from place to place, for example from Illinois to Oklahoma, then one can expect the ratio R also to vary. Early determinations of R were made by Harrison and Beckwith (1951), Beckwith (1957), and Beckwith (1960). They found $R = 4.4$ for a 150 mi² region in and around Denver, Colorado. Changnon (1962), Changnon and Schickedanz (1969), Schickedanz and Changnon (1970),[3] and especially Changnon (1971)[4] have shown from their studies of regional and point hail frequencies that, for A in units of square kilometers,

$$R = sA^t \begin{pmatrix} s = 0.787 \\ t = 0.313 \end{pmatrix}. \quad (8)$$

This relation and the associated values for s and t were based primarily on data for one geographic region, Illinois. Nelson and Young (1979) have shown that this same relation but with different values for s and t applies for hail in Oklahoma and in South Africa (Carte, 1967). The conclusion is that for a given A the ratio R may vary geographically. In view of (7) this implies that $\bar{a}$ varies geographically.

Combining (7) and (8) yields

$$\bar{a} = A^{1-t}s^{-1}. \quad (9)$$

That $\bar{a}$ increases with A is not surprising since each area a_i refers to that part of a hailfall lying inside A. A larger region will sample a larger portion of each of those hailfalls which lie only partly inside a smaller region which it encloses, and these larger portions when combined with the areas of hailfalls entirely within the smaller region will lead to a larger value for $\bar{a}$. In short, $\bar{a}$ will increase with A because of an edge effect. Returning to Eq. (8) it is clear then that it is because of an edge effect that R does not increase as fast as the first power of A. The magnitude of the edge effect and, hence, the values of s and t can be expected to depend on the size distribution, according to area, of the hailfalls in a region. Nelson and Young (1979) show that s and t vary between different geographic regions. The distribution of hailfall areas can therefore be expected to vary.

There is additional support for the statement that R does not increase as fast as the first power of A because of an edge effect. The expression

$$R = A/a + 4(A/\pi a)^{1/2} + 1 \quad (10)$$

can be derived from the work of Carte (1967) if one assumes the region of area A is a square and that all hailfalls have equal areas a and are circular in shape. (In this equation a is the true hailfall area and not the *observable* part of the area located within the region as assumed elsewhere in this paper.) The last two terms in (10) come from consideration of edge effects. They are seen to make the dependence of R on A less than linear. Graphical comparison of (8) and (10) shows they are similar for hailfall areas a of reasonable magnitudes.

3. Point hail-frequency estimates

This Section shows the effect of network density on the accuracy of an estimate $\hat{\nu}$ of the mean point hail frequency $\bar{\nu}$ over a region. Knowledge of the error in $\hat{\nu}$ is important because $\hat{\nu}$ in practice would be substituted for $\bar{\nu}$ in Eq. (6) or used together with values of R to compute the regional frequency N. NHRE hailfall data will be used to illustrate some of the points made. It will be assumed that individual hailfalls are independent events. It also will be assumed that individual hailfalls can occur anywhere within a region with equal probability. This assumption simplifies the analysis, does not limit it to a particular spatial distribution of hailfalls (a random distribution is in effect being considered), and is consistent with there being no spatial gradient of long-term average point hail frequency over a region.

[3] Schickedanz, P. T., and S. A. Changnon, Jr. 1970: A study of crop-hail insurance records for northeastern Colorado with respect to the design of the National Hail Research Experiment. Final Report to the National Center for Atmospheric Research under Contract NCAR 155-70, Illinois State Water Survey, 47 pp. + 3 Appendices.

[4] Changnon, S. A. Jr., 1971: Means for estimating areal hailday frequencies. *J. Wea. Mod.*, **3**, 154–159.

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The population distribution of point frequencies of hail for a region of area A during a time period T will be determined by the distribution of areas a_i of the N hailfalls occurring in the region during T. For any hail detection site within A, p_1 defined in (1) is the probability that the first hailfall of area a_1 will occur at the site. The probability of no hail at the site is $1 - p_1$. Similar expressions hold for each of the N hailfalls. Under the assumption that hailfalls are independent events, the probability that none of the N hailfalls will occur at a site will be

$$\text{Prob (0 hailfalls)} = \prod_{i=1}^{N} (1 - p_i). \qquad (11a)$$

The probability that one, and only one, of the N hailfalls will occur at a site is

$$\text{Prob (exactly 1 hailfall)} = \sum_{j=1}^{N} p_j \prod_{\substack{i=1 \\ i\neq j}}^{N} (1 - p_i), \qquad (11b)$$

and the probability that exactly two of the N hailfalls will occur at a site is

$$\text{Prob (exactly 2 hailfalls)} = \frac{1}{2} \sum_{j=1}^{N} p_j \sum_{\substack{k=1 \\ k\neq j}}^{N} p_k \prod_{\substack{i=1 \\ i\neq j,k}}^{N} (1 - p_i). \qquad (11c)$$

In general,

$$\text{Prob (exactly } \nu \text{ hailfalls)} = \underbrace{\frac{1}{\nu!} \sum_{j=1}^{N} p_j \sum_{\substack{k=1 \\ k\neq j}}^{N} p_k \cdots \sum_{\substack{s=1 \\ s\neq j,k,\ldots,r}}^{N}}_{\nu \text{ summations}} p_s \prod_{\substack{i=1 \\ i\neq j,k,\ldots,s}}^{N} (1 - p_i),$$

$$0 \leqslant \nu \leqslant N. \qquad (12)$$

Eq. (12) explicitly shows how the distribution of areas $a_i = p_i A$ of N hailfalls determines the probability that a certain point frequency of hail will occur at a given site. Eq. (12) takes into account all possible spatial locations of the hailfalls a_i within a region.

An illustration of (12) can be developed with the estimated hailfall areas a_i^* obtained in NHRE in 1976 and listed in Table 1. Substitution of these a_i^* into (12) and calculation of the probabilities yields the population distribution of point frequencies listed in column 2 of Table 3. The distribution of point frequencies actually observed in 1976 is shown in column 3 of Table 3 (see also Table 2). The agreement between columns 2 and 3 is good, because of the large number of widely dispersed sites in the 1976 network and apparently also because the NHRE hailfalls were dispersed almost randomly through the study region. Table 3 is a clear example of the interconnection in (12) between a distribution of hailfall areas and a population distribution of point hail frequencies. The deviation of observed and population distributions of point frequencies in a particular case will depend on the particular spatial locations of the hailfalls and on the number and location of network sites.

TABLE 3. Probability distributions of point hail frequency as predicted by Eq. (12) using the estimated hailfall area a_i* in Table 1, and as observed, for the period 29 May–30 July 1976.

Point hail frequency ν	Probability from Eq. (12)	Observed probability
0	0.0118	0.0163
1	0.1606	0.1699
2	0.3650	0.3497
3	0.3004	0.3170
4	0.1257	0.1209
5	0.0310	0.0245
6	0.00484	0.00163
⩾7	0.000531	0

The Monte Carlo method (Metropolis and Ulam, 1949) can be used to show how network density affects the accuracy of an estimate $\hat{\nu}$ of mean point hail frequency over a region. The Monte Carlo method as applied here randomly draws samples from the artificial population distribution of point frequencies discussed in the previous paragraph and developed by the application of (12) to NHRE data. (Although some, but not all, of the results will depend on this particular population distribution, the Monte Carlo method is general and can be applied to hailfalls in other geographic regions.) Drawing a sample of size n from the population distribution is equivalent to setting up a fictitious hail sampling network of n sites. Each site would be placed entirely at random according to a uniform distribution inside the region occupied by the 1976 NHRE network. The mean of the n frequencies in a sample is denoted as $\hat{\nu}_i(n)$. The subscript i distinguishes one particular sample mean from others obtained by repeatedly extracting samples of size n from the population distribution. In the present work, 500 independent samples of size n were obtained for each selected value of n. From the cumulative distribution of the 500 values of $\hat{\nu}_i(n)$, the 5th, 25th, 75th, and 95th percentile values were selected. These are plotted in Fig. 1 as a function of n along with the population mean point hail frequency $\bar{\nu}$. Because of the particular way in which the population distribution was constructed in the present work, the value of $\bar{\nu}$ is given by ν^* in (4). The right-hand ordinate in Fig. 1 makes it also a plot of the percentiles of the percentage error of estimate

$$\epsilon_i(n) = 100[\hat{\nu}_i(n) - \bar{\nu}]/\bar{\nu}. \qquad (13)$$

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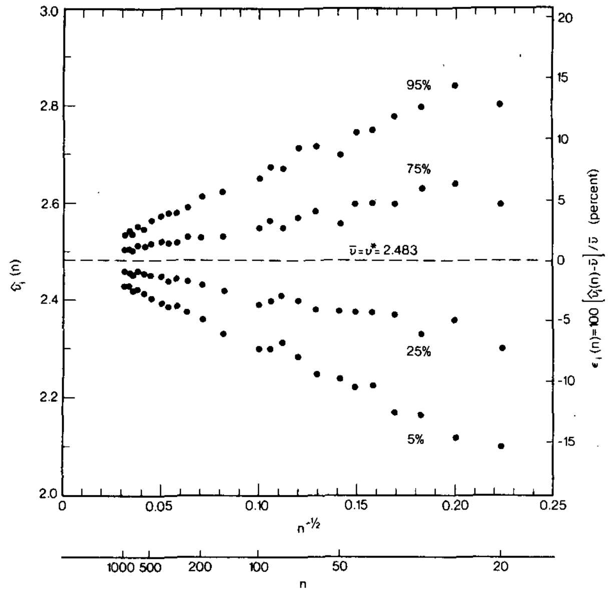

FIG. 1. Percentiles of the sample mean point hail frequency $\hat{\nu}_i(n)$ and of the percentage error of estimate $\epsilon_i(n) = 100 \times [\hat{\nu}_i(n) - \bar{\nu}]/\bar{\nu}$. The percentiles are based on 500 samples of point hail frequency, each of size n where n is the number of sites in a fictitious network, drawn at random from the population probability distribution of point frequencies given in column 2 of Table 3.

The error in $\hat{\nu}$ is 3% or less for ~90% of the networks with ~600 sites and is a good measure of what can be obtained with a network of the density used in 1976. Had only 200 sites been used in 1976 there still would have been a 90% chance the error in $\hat{\nu}$ would have been no larger than ±5%. With 50 sites the error likely would have been only ±10%. The error $\epsilon_i(n)$ increases approximately as $n^{-1/2}$ and is thus proportional to the spacing of sites defined as $(A/n)^{1/2}$. The $n^{-1/2}$ dependence is a well-known feature of the "standard error" of $\hat{\nu}$. The standard error together with the central limit theorem could have been used to determine approximately the percentiles of the error of estimate $\epsilon_i(n)$ shown in Fig. 1. The determination would have been increasingly poor as the population distribution of point frequencies became increasingly non-normal. The Monte Carlo method used here for finding $\epsilon_i(n)$ on the other hand preserves the non-normal features of a distribution. This is especially true when small sample sizes n are considered.

The magnitude of the error of estimate $\epsilon(n)$ will depend on the particular distribution of areas that hailfalls assume. If all hailfalls have equal area the probability will be equal that any of the hailfalls will occur at a point, *viz.*, $p_i = p = a/A$. If there are N such hailfalls the probability that exactly ν of them will occur at a point is given by the binomial "population" distribution (Feller, 1968)

$$\text{Prob (exactly } \nu \text{ hailfalls)} = \binom{N}{\nu} p^{\nu}(1-p)^{N-\nu}. \quad (14)$$

The coefficient of variation of $\hat{\nu}$ associated with (14) for a random network of n sites is

$$\frac{\sigma(\hat{\nu})}{\bar{\nu}} = [(1-p)/Npn]^{1/2}. \quad (15)$$

This can be compared with the coefficient of variation calculated using the population distribution of values for ν developed from the NHRE hailfalls and listed in column 2 of Table 3. In the comparison one first sets $N = 16$ in Eq. (15) so the same number of hailfalls are considered in both cases. One also requires $p = \bar{\nu}/N$. This ensures the same mean point frequency for both distributions. [The value of $\bar{\nu}$ is again given by ν^* in Eq. (4) because of the way the population distribution is constructed.] One then finds that the coefficient of variation of $\hat{\nu}$ in (15) for the distribution in (14) is 35% larger than that for the population distribution in column 2 of Table 3. This result is not surprising since the NHRE probability distribution for ν is dominated through Eq. (12) by the three large hailfalls on 30 May, 22 June, and 5 July. These hailfalls by themselves act to fix the point frequency of hail at $\nu = 2$ for more than half the sites, and at $\nu = 1$ or 3 for most of the remaining sites. When the 13 remaining smaller hailfalls are included, the distribution of ν shifts slightly to the right but maintains its relatively narrow profile. The distribution in (14), on the other hand is not fixed by just a few hailfalls, and each additional hailfall is as important as all previous ones and can act appreciably to spread the distribution of point frequencies.

It may be concluded that if during a hail season a few relatively large storms occur in a region the observed average point frequency $\hat{\nu}$ will be closer to the true frequency $\bar{\nu}$ than if such storms did not occur. The provisos are that the total number of storms be the same in both cases and that the true frequency $\bar{\nu}$ also be the same.

4. Regional hail-frequency estimates

The accuracy of an estimate of the number of hailfalls occurring in a region depends on the number of sites for observing hail within the region and on the distribution according to size of the areas covered by individual hailfalls. These factors enter an expression derived in this Section for the probability of detecting specified numbers of hailfalls. The derivation assumes each hail observing site is placed at random within a region with equal probability given to every possible site location. This assumption of a random network simplifies the analysis. It is also of relevance to networks of volunteer hail observers, which are often fairly random, and to networks of hail instruments in those regions where

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the road system is irregular. The assumption of a random network is perhaps of less relevance to networks set up along a regular grid. It should be noted that the assumption here of a random network is not intended as a statement that a random network is more effective than a uniform network in sampling hailfalls. Further work is needed to answer that question.

Once again N hailfalls are assumed to occur during a time period T within a region of area A and to have a distribution of areas a_i. The probability that an instrument will detect the ith hailfall is

$$p_i = a_i/A.$$

Because the instruments have random locations detection of the ith hailfall by different instruments can be treated as statistically independent events. This will be true regardless of the number of instruments. The probability that none of the instruments will detect the ith hailfall is then

$$\text{Prob (non-detection)} = (1 - p_i)^n. \quad (16)$$

The probability that at least one instrument will detect the ith hailfall is

$$\begin{aligned}\text{Prob (detection)} &= 1 - (1 - p_i)^n \\ &= 1 - [1 - (a_i/A)]^n = \xi_i. \quad (17)\end{aligned}$$

As expected ξ_i increases as the hailfall area becomes larger and as the number of sites increases.

The probabilities ξ_i of detecting the individual hailfalls will determine the probability of detecting exactly M of the N hailfalls. Postulating a new randomly selected network prior to each hailfall it follows that

$$\text{Prob (detecting exactly } M \text{ hailfalls)} = \frac{1}{(N-M)!} \underbrace{\sum_{j=1}^{N} (1-\xi_j) \sum_{\substack{k=1 \\ k\neq j}}^{N} (1-\xi_k) \cdots \sum_{\substack{s=1 \\ s\neq j,k,\ldots,r}}^{N} (1-\xi_s)}_{N-M \text{ summations}} \prod_{\substack{i=1 \\ i\neq j,k,\ldots,s}}^{N} \xi_i. \quad (18)$$

The reasoning used to obtain (18) is similar to that used to obtain Eq. (12) for the probability of certain point hail frequencies. The primary difference between (18) and (12) is that, while both equations show the connection between the distribution by size of hailfall areas and the population distribution of the frequencies being treated, the probability (18) of certain *observed regional* frequencies depends through Eq. (17) also on the number n of sites in the region. On the other hand, the population distribution (12) of *point* frequencies is determined only by the hailfalls themselves and, hence, does not depend on n.

Eq. (18) can be used to determine the adequacy of random networks of various densities for detecting the hailfalls observed by NHRE in 1976. The procedure is to substitute into (17) the NHRE data on hailfall areas in Table 1, to assume each hailfall occurs in a region of area $A = 660$ km^2 (the size of the NHRE study region), and to select n—the number of sites in a network within this region. Eq. (17) is then used to calculate the 16 values ξ_i and these are substituted into (18). This yields the probability that exactly M of the $N = 16$ NHRE hailfalls would have been detected by N randomly selected networks with n sites. Table 4 shows the results. Random networks of 1000 sites would more assuredly have detected all 16 hailfalls than would random networks with the same number of sites, 603, as were operating on average in the NHRE network. On the other hand, random networks of 500 sites would not have been much worse than networks with 603 sites since there still would have been a better than even chance that all 16 hailfalls would be detected. Use of only 200 sites more than likely would have meant the loss of one or two hailfalls, but still 14 or 15 hailfalls likely would have been detected. With only 50 sites, 12 hailfalls would have been most likely detected. It is apparent then that a significant reduction in the density of the 1976 network could have been made with only a relatively small reduction in the number of hailfalls detected.

The actual reduction in number of hailfalls detected with random networks of reduced density will depend in every case on the particular size distribution of the hailfalls that are to be detected and may deviate from that seen with the NHRE data. It is clear, however, that the particular hailfalls that would first go undetected with sparser networks would likely be those covering the smaller areas. Sparse networks would more likely detect some specified number of large hailfalls than the same number of large and small hailfalls. This would be true provided the number of hailfalls required to be detected is not so high that there are insufficient large hailfalls to satisfy the requirement of numbers and some of the smaller, more numerous hailfalls have to be included.

Table 5 shows the potentially diminishing returns that come from designing networks to detect all, or almost all, the hailfalls in a region. (Like Table 4, Table 5 is obtained using Eq. (18), but with the added restriction that the M hailfalls also be the

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TABLE 4. Probabilities of detecting exactly M of the 16 hailfalls observed with the 1976 NHRE network using random networks containing certain numbers of sites n. Probabilities for $n = 603$ are included for comparison because this value of n is the average number of hailpads that operated in the 1976 NHRE network.

	n							
M	10	20	50	100	200	500	603	1000
16	2.99×10^{-9}	1.59×10^{-6}	6.99×10^{-4}	0.0165	0.128	0.509	0.596	0.801
15	3.44×10^{-7}	8.74×10^{-5}	0.0139	0.143	0.448	0.450	0.385	0.198
14	1.34×10^{-5}	1.59×10^{-3}	0.0845	0.337	0.334	0.0403	0.0195	1.34×10^{-3}
13	2.56×10^{-4}	0.0137	0.229	0.323	0.0811	3.45×10^{-4}	5.22×10^{-5}	3.74×10^{-8}
12	2.75×10^{-3}	0.0647	0.320	0.146	8.19×10^{-3}	1.03×10^{-6}	4.79×10^{-8}	3.49×10^{-13}
11	0.0177	0.177	0.239	0.0316	3.23×10^{-4}	1.05×10^{-9}	1.47×10^{-11}	1.10×10^{-18}
10	0.0706	0.286	0.0931	2.90×10^{-3}	2.47×10^{-6}	3.13×10^{-15}	2.76×10^{-18}	

M largest ones. The restriction in Eq. (18) that "exactly" M hailfalls be detected is removed, however, and some one or more of the N-M smallest hailfalls may also be detected.) Table 5 shows there would have been about a 50% chance of detecting the 11 largest hailfalls observed by NHRE using random networks of only 50 sites. To maintain the same chance of detecting the 12, 13, 14, 15 or 16 *largest* hailfalls one must use increasingly greater numbers of sites. In fact, Fig. 2 shows that to maintain a 50% probability of detecting the largest M hailfalls, the largest $M + 1$ hailfalls, and so on by steps of unity, requires, on average and *for each step*, an increase in the number of sites of about 45% over the number at the *previous* step. The number of sites n increases exponentially with M. This relation between n and M ultimately is a consequence of the distribution according to size of the areas of the hailfalls observed in NHRE. That distribution (Table 1) has lognormal characteristics, as may be seen from graphical analysis of the data. Lognormality may be typical of distributions of hailfall areas (e.g., see Crow *et al.*, 1979). The implication is that an exponential increase in n with M may be of rather general validity. A substantial savings may be achieved in numbers of instruments and in their associated costs if one is willing to detect or need only detect the larger hailfalls.

Table 5 shows that once one has selected the number of sites n and has established that these sites will detect with fair likelihood the largest hailfalls of some number there will then be significantly less likelihood that the network will detect the next few largest hailfalls. For example, if $n = 50$ then for the NHRE data there would be a 47.3% chance of detecting the 11 largest hailfalls, but only a 20.9% chance of detecting the 12 largest hailfalls and an even smaller 9.2% chance of detecting the 13 largest hailfalls. This suggests there is a rather sharp limit on the number of the largest hailfalls that a network of given number of sites can detect.

Eq. (18) takes a particularly simple form when all N hailfalls have the same area a. In this case

$$\xi_i = \xi = 1 - [1 - (a/A)]^n. \qquad (19)$$

Substituting ξ into (18) yields the binomial distribution

Prob (detecting exactly M hailfalls)

$$= \binom{N}{M} \xi^M (1 - \xi)^{N-M}. \qquad (20)$$

The mean number of hailfalls that will be detected is simply

$$\bar{M} = N\xi = N\{1 - [1 - (a/A)]^n\}. \qquad (21)$$

TABLE 5. Probabilities of detecting the largest M hailfalls observed with the 1976 NHRE network using random networks containing certain numbers of sites n. Probabilities for $n = 603$ are included for comparison because this value of n is the average number of hailpads that operated in the 1976 NHRE network.

	n							
M (largest hailfalls)	10	20	50	100	200	500	603	1000
16	2.99×10^{-9}	1.59×10^{-6}	6.99×10^{-4}	0.0165	0.128	0.509	0.596	0.801
15	1.83×10^{-7}	4.91×10^{-5}	8.86×10^{-3}	0.109	0.458	0.909	0.948	0.993
14	3.78×10^{-6}	5.18×10^{-4}	0.0402	0.277	0.726	0.991	0.997	1.000
13	3.46×10^{-5}	2.51×10^{-3}	0.0917	0.405	0.806	0.994	0.998	1.000
12	3.17×10^{-4}	0.0121	0.209	0.590	0.894	0.997	0.999	1.000
11	2.89×10^{-3}	0.0586	0.473	0.859	0.991	1.000	1.000	1.000
10	0.0127	0.145	0.652	0.929	0.997	1.000	1.000	1.000

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The mean relative error in the observed regional frequency is then

$$\delta(a,n) = (\bar{M} - N)/N = -[1 - (a/A)]^n. \quad (22)$$

Note that $\delta(a,n)$ will be largest in absolute value when the hailfall area a is small or the number of sites n is small.

A correction to the observed regional hail frequency that allows for the error $\delta(a,n)$ can be obtained by noting that (21) can be rewritten as

$$N = \bar{M}/\{1 - [1 - (a/A)]^n\}. \quad (23)$$

Replacing $\bar{M}$ in Eq. (23) with the observed frequency M of hailfalls with areas in some small size range about a allows one to estimate their true frequency N. On average the estimate would be exact. The sum of such corrected frequencies would be an unbiased estimate of the true regional frequency of hailfalls of all sizes. Table 6 shows the correction to M needed for selected values of a/A and n. The correction is substantial for small hailfalls and sparse networks. But the correction factor represents only a 5% or smaller change in the regional frequency if the total number n of observing sites in the region is such that there are on average about three sites inside a hailfall of area a. The average number of sites inside a hailfall of area a will be $n_h = na/A$,

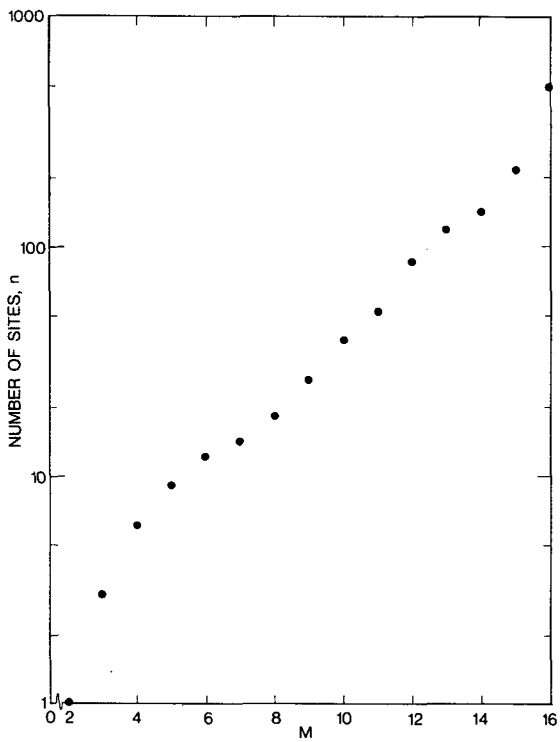

FIG. 2. Number of sites n that would have had to be placed at random in a 660 km² study region in order to obtain a 50% probability of detecting the largest M hailfalls observed by the NHRE network in 1976.

TABLE 6. Correction factor $\{1 - [1 - (a/A)]^n\}^{-1}$ by which the observed regional frequency M of hailfalls of area a is to be multiplied in order to obtain an unbiased estimate of the true frequency N [see Eq. (23)].

	n			
a/A	10	30	100	300
0.01	10.42	3.85	1.58	1.05
0.03	3.80	1.67	1.05	1.00
0.1	1.54	1.04	1.00	1.00
0.3	1.03	1.00	1.00	1.00
0.5	1.00	1.00	1.00	1.00

and if $n_h = 3$ then $n = 3/(a/A)$. Table 6 confirms that if this last relation is satisfied the correction to M should be 5% or less. If there is on average only about 1 site in every observed hailfall and $n_h = 1$, then there should be about a 50% correction upward to the regional hail frequency estimates. These two approximate "rules-of-thumb" might prove useful in attempts to establish the correct hail frequency in a region on the basis of network hailfall data. In general, the correction factor to be applied to M increases as $n_h = a/(A/n)$ decreases. The correction factor depends on the relative sizes of the hailfall area a and the mean "representative area" of stations A/n. The correction factor needs to be applied to observed hail frequencies for different regions before they are compared. Regional differences in hail-detection network densities are then allowed for.

An illustration of some of the points of the previous paragraph may be helpful. Held (1973) observed that the frequency of hailfalls in his 2800 km² study region increased by ~50% when the number of hail observers was increased from about 850 up to ~4000. Held noted that the two hail frequencies would become identical if he deleted from the set of hailfalls observed by the dense network all those on which the daily hailfall covered less than five sites. Each of these small hailfalls on average would have covered approximately 0.5 site in the sparse network. But the necessary correction factor to the frequency of these small hailfalls, as observed by the sparse network, would be about 2.5 according to the present analysis. If for purposes of argument these small hailfalls account for perhaps one-half of the total number of small and large hailfalls observed by the dense network then the necessary correction to the *total* number of hailfalls observed by the sparse network is about +43%. This figure is close to the increased regional frequency observed by Held with increased density of sites.

An illustration of the correction needed to regional hail frequencies comes from the work of Changnon (1970). He tabulates the number of hailstreaks detected by networks operated in Illinois in 1967

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and 1968. The tabulation gives the number of streaks in which three or more sites had hail and the number of streaks in which only one site had hail. The earlier paragraph suggests that the true number of streaks potentially covering three or more sites was only ~5% larger than Changnon's number, but that the true number of streaks potentially covering one site was 50% larger than Changnon's number. This shows how sensitive an estimate of the number of hailstreaks in a region is to the minimum number of sites selected as necessary to define a streak.

5. Estimates of hailfall area

This Section shows the effect of network density on the accuracy of an estimate of the area of a hailfall. Site arrangement is assumed to be random.

Hailfall area a is estimated by the product of A/n times the number m of sites with hail. (A and n have their usual definitions.) Thus, a will be estimated by

$$a^* = \frac{m}{n}A. \tag{24}$$

This estimate simplifies the analysis and is that employed in Section 2 and Table 1. For random networks the total hailfall area will be overestimated if the hail happens to occur in a dense part of the network where stations are spaced relatively closely. An underestimate will result if the hail occurs in a sparse part of the network.

The error associated with the estimate a^* can be calculated. Once again, the probability that any one site will lie within the area a of a hailfall is

$$p = a/A. \tag{25}$$

Because all sites have random locations the probability that a particular site is within the hailfall area will be independent of whether another site is within it. The probability that m sites will be found in a is then given by the binomial distribution

$$\text{Prob } (m \text{ sites}) = \binom{n}{m} p^m (1-p)^{n-m}. \tag{26}$$

Associated with each value of m there will be an estimate a^* given by (24). Considering all values of m, the mean of these estimates is

$$\begin{aligned} E(a^*) &= \sum_{m=0}^{n} \frac{mA}{n} \text{ Prob } (m \text{ sites}) \\ &= \sum_{m=1}^{n} Ap \binom{n-1}{m-1} p^{m-1}(1-p)^{n-1-(m-1)} \\ &= a. \end{aligned} \tag{27}$$

Thus, for random networks of hail instruments the estimate a^* of hailfall area is statistically unbiased. The variance of the possible values of a^* is given by

$$\begin{aligned} \sigma^2(a^*) &= E(a^{*2}) - E^2(a^*) \\ &= \sum_{m=0}^{n} \left(\frac{mA}{n}\right)^2 \text{Prob } (m \text{ sites}) - a^2 \\ &= (A^2/n)p(1-p). \end{aligned} \tag{28}$$

Thus, the coefficient of variation of the values a^* as estimates for a is

$$\sigma(a^*)/a = \left(\frac{1 - a/A}{na/A}\right)^{1/2}. \tag{29}$$

Eq. (29) takes into account all possible values of m. Although the coefficient of variation in (29) will be comparable to the normalized error in estimating a in practice, in general the coefficient of variation will not equal the error in a^* associated with a particular random network placed in the field and selected from the ensemble of all possible random networks with n sites. The error in practice will depend on the particular random network selected, on the location of the hailfall within the network, and on the shape of the hailfall.

Eq. (29) shows that the "average" linear spacing of sites $(A/n)^{1/2}$ is important for estimates of hailfall area. Quadrupling the number of sites in a region is necessary to reduce the error by half. Recall that Section 3 showed that the linear spacing of sites had a similar effect on the accuracy of estimates of the *mean point frequency* of hail.

Eq. (29) can be reexpressed in a particularly illuminating form if the substitution $n_h = na/A$ is made where, as in Section 4, n_h is the average number of sites expected to lie within a hailfall. Making this substitution yields

$$\sigma(a^*)/a = \left(\frac{1 - n_h/n}{n_h}\right)^{1/2}. \tag{30}$$

Eq. (30) shows that n_h has an important effect on the accuracy of estimates of the area of a hailfall. In fact, for hailfalls covering <20% of a network one is 90% correct in evaluating the coefficient of variation of a^* from the approximate expression

$$\sigma(a^*)/a \approx n_h^{-1/2} \tag{31}$$

If one replaces n_h in (31) with the number of sites actually observing hail this expression becomes a rather simple, if approximate, means for evaluating the uncertainty in an estimate of hailfall area obtained with a random network and expression (24).

The average number of sites n_h expected within an *individual* hailfall has an effect on estimates of its area as shown above, *and* on estimates of the frequency of occurrence of hailfalls of its size in a region as shown in Section 4. But note that if $n_h = 3$ for a given size rainfall then 95% of such hailfalls will be detected (see Table 6), whereas the coefficient of variation of a^* in (31) is 0.58. Detecting a hailfall is apparently better done than meas-

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uring its area. This is understandable since accurate measurement of the area of a hailfall is equivalent to detecting most subregions of the hailfall, and detecting these subregions will require more sites than simply detecting the hailfall as a whole.

6. A practical application

The results of the previous Sections have a practical application. Suppose one wishes to design a hail suppression experiment for a particular region and wishes to know 1) how big a target area to place within the region so as to obtain some number of hailfalls N deemed sufficient for the experiment, and 2) how many hail-detection instruments to place in the target area, once it is selected, so as to detect some minimum number of the hailfalls that occur and so as to measure their areas adequately. Suppose an independent statistical study was made to determine the number of hailfalls N deemed sufficient for the experiment. Suppose one also has available long-term frequency data for hailfalls at selected points in the region (collected either by climatological stations or by other means) and long-term data on hailfall areas from crop-insurance claims. (In the absence of these data an exploratory hail-data gathering effort would be required.)

The size of target area to use comes from the product

$$A = (N/\bar{\nu})\bar{a}. \tag{32}$$

This is derivable from (2). The mean point hail frequency $\bar{\nu}$ is obtainable from the climatological data, and the mean area of hailfalls $\bar{a}$ is obtainable from the insurance data. The quantity A is then the size of the desired target area provided $\bar{\nu}$ and $\bar{a}$ are the values applicable for a region of area A. This proviso is necessary given the definitions of $\bar{\nu}$ and $\bar{a}$ stated in Section 2. Iterative substitution of the climatological and insurance data can ensure this proviso is satisfied. It is important to note that although $\bar{\nu}$ in (32) might be estimated with little statistical bias from field data, because $\bar{\nu}$ enters (32) in a nonlinear way the estimate for the target area A itself made with (32) may not be statistically unbiased. {We note that if one possesses a relation $R = f(A)$ between the ratio R of regional to point hail frequencies and A [Eq. (8) is such a relation] then the inverse of this relation, *viz.*, $A = f^{-1}(R)$, gives the size of the target area once N is specified and provided $\bar{\nu}$ is known. This is true provided the relation $R = f(A)$ has been developed by considering regional and mean point hail frequencies for each of a sequence of regions ranging from those enclosed within the target area to those enclosing it.}

The number of hail-detection instruments to place in the target area comes from applying Eqs. (18), (21) and (29) to the insurance data on hailfall areas. Eq. (18) tells us the probability of detecting exactly M of N hailfalls having a known distribution of areas, for a given number of instruments n and for a given target area A. Eq. (21) tells us how many hailfalls of a *particular* area will be detected on average, for given n and A. Finally, Eq. (29) is a measure of the error that will be incurred in estimating individual hailfall areas, also for given n and A. The application of (18), (21) and (29) assumes network sites would be randomly located. Other, but perhaps not significantly different, numerical results would be obtained should a regular (not random) network be planned.

7. Summary

This paper has demonstrated the close relationship between the number of hail observing sites within a region and the accuracy of estimates of (i) the mean point frequency of hail within the region, (ii) the overall regional frequency of hail, and (iii) hailfall area. Information of practical use has been presented. The 1976 NHRE hailfall data have been used extensively to illustrate the points made. The main points of the paper are as follows:

1) A practically useful relation $\bar{\nu} = N\bar{a}/A$ exists between the mean point frequency of hail in a region $\bar{\nu}$, the overall regional frequency of hail N, the mean area of the hailfalls $\bar{a}$, and the area of the region A. This relation can be used to find the size of region needed to provide a specified regional frequency of hailfalls. This is of importance in the design of hail suppression experiments where the size of a target area must be selected. The information on $\bar{\nu}$ and $\bar{a}$ required to find the target area can be estimated from climatological data on hail frequencies and from data on insurance claims for crop damage due to hail. If these data are not available, an exploratory hail data-gathering effort would be required. The mean area of the hailfalls in a region is given by $\bar{a} = A/(N/\bar{\nu})$.

2) An expression can be derived to relate the distribution of values of point hail frequency in a region to the distribution according to size of the ground areas covered by individual hailfalls in the region. The error in estimating the mean point frequency of hail over all points in a region is found to increase as $n^{-1/2}$, where n is the number of sites within the region. An analysis of NHRE hailfall data suggests that in regions where there are proportionally more large hailfalls fewer sites may be needed to estimate the mean point hail frequency.

3) Estimates of the regional frequency of hail depend for their accuracy on the number of sites in the region and on the distribution by size of the areas of the individual hailfalls. For the 1976 NHRE hailfall data collected in a 660 km² region it was found that the number of hailpads could have been reduced from 603 to 50 with a reduction only from 16 to 12 in the number of hailfalls detected. As the density of

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sites is reduced the smaller hailfalls are the first to go undetected. There are diminishing returns in fielding sufficient instruments to detect all, or almost all, of the hailfalls in a region. For the lognormal distribution of hailfall areas observed by NHRE, the number of instruments increases exponentially with the number of hailfalls to be detected. A formula is derived for a correction to be made to the observed regional frequency of hailfalls of a given size so as to obtain the true frequency. For those hailfalls, each potentially covering three sites, the frequency should be increased by 5%, but for those hailfalls potentially covering only one site the observed regional frequency should be increased by 50%.

4) Estimates $a^* = mA/n$ of the area of an individual hailfall (m is the number of sites with hail) depend for their accuracy on the number of sites n present for observing hail and on the true area of the hailfall itself. For random networks the coefficient of variation of a^* is proportional to $n^{-1/2}$. A quadrupling of the number of sites in a region is necessary to reduce the coefficient of variation by half. For hailfalls covering $\leqslant$20% of the instrumented region the coefficient of variation approximately equals $n_h^{-1/2}$, where n_h is the number of sites expected within the hailfall.

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