Variability Shown by Hourly Wind Soundings[1]

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ABSTRACT

The time-lag variability of winds at 3, 6, 9 and 12 km, observed at approximately 1-hr intervals over Bedford, Mass., during the first week of April 1960 is proportional to the square root of the time lag. The proportionality factor varies with mean wind speed, and perhaps also with altitude, thus differing from the average value found by previous investigators. Of the other sources of variability present in this unique series, deviations from precise 1-hr intervals were most important; less important, but still significant, were differences in the exact balloon positions when successive measurements were made, and in the exact heights to which they applied.

1. Introduction

With the first dark, drizzling minute of April 1960, an Air Force crew released a rawinsonde from L. G. Hanscom Field, Bedford, Mass. One hour later this same crew started another sounding, and at 2 a.m. a third one. For seven days the 9-man team, of which they were part, proceeded to make soundings every hour, using two AN/GMD-1 ground sets alternately. Of the 168 hourly ascents scheduled, 155 provided information which is being used to study the variability of wind over short time intervals. This paper is a report of some of the results of the study.

Surface weather conditions during the week of observations were quite varied and several frontal systems moved through the area. Strong southwesterly flow existed aloft during most of the week with the jet stream over the area or nearby. Some soundings appear to have penetrated the core of the jet with speeds over 100 m per sec reported.

Wind data obtained in this unique series of soundings have been published[3] in the form of horizontal displacements (by components) of the balloons during 1, 2, 3 and 4 min following each minute of ascent. For this study, the 3-min displacements, expressed in m per sec, have been selected at four levels: 3, 6, 9 and 12 km; in each case, the balloon passed the desired level during the second minute of the 3-min interval. The wind measurements thus applied to a layer somewhat less than 1 km thick, whose thickness actually varied, because of differences in balloon ascent rate, by a few hundred meters.

The difference in the wind vectors obtained from two consecutive wind soundings of the Bedford series has several components:

Measurement and instrumental differences include all errors (m) of instrumental calibration, readout and computation.

Location differences (ℓ) arise because the two reports may not pertain to exactly the same point in the atmosphere: wind changes at lower layers may have caused the second balloon to reach a specified altitude several kilometers away from the corresponding point for the first balloon.

Height differences (h) may be introduced by using 3-min winds for layers whose midpoints are at different heights above sea level.

Thickness differences (k) similarly may be due to using winds averaged for different layer thicknesses.

Time difference (t) is the actual change in the wind from one time (t_1) to another (t_2), from meteorological causes. Study of this time variation of wind is the principal object of this paper.

The square of the modulus of the difference between two wind vectors, W_1 and W_2, obtained at times t_1 and t_2, at heights of h_1 and h_2 meters, through thicknesses of k_1 and k_2 meters, over locations ℓ_1 and ℓ_2, and with instrumental and measurement errors m_1 and m_2, is

$$(W_1-W_2)^2=f_1(h_1-h_2)+f_2(k_1-k_2)+f_3(\ell_1-\ell_2)+f_4(m_1+m_2)+f_5(t_1-t_2) \quad (1)$$

where f_1 through f_5 are functions expressing the contribution of each of the variables to the total difference involved.

[1] Presented in preliminary form by Major Lenhard at the 202nd National Meeting, American Meteorological Society, Washington, D. C., 26 April 1962.

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[3] Court, A., and H. Salmela, 1961: Hourly rawinsondes for a week. Bedford, Mass., USAF Camb. Res. Lab., *GRD Res. Notes. No. 60,* 350 pp.

The chief interest of this study is in the nature of f_5, the way in which wind vectors change with time. Such changes are investigated first, then the contaminating effects of the others are examined.

2. Variability

A series of difference vectors for a given time interval form a bivariate frequency distribution. A set of such frequency tables for various time intervals would provide a description of possible changes. The description can be condensed by assuming a reasonable form for the frequency distribution and presenting the parametric statistics of this distribution.

A convenient form to adopt is the circular normal vector distribution with zero mean. It can be specified by a single parameter, the root-mean-square (rms) change in wind, which is an easily interpreted measure of wind variability. Sixty-three per cent of the difference vectors will not exceed the rms in magnitude. The assumption also appears to be a valid one. Ericksson (1961) developed the concept from the empirically established circular normal distribution of vector winds (Brooks *et al.*, 1946) and found the approach to be reasonably applicable for Swedish data. Ellsaesser[4] considered the assumption sufficiently accurate for practical use, although noting that the distribution of difference vectors is slightly elliptical.

The assumption that the resultant difference vector is zero implies no net change in wind over the selected lag interval t during the period of record. This should be true for appropriate periods of record. The magnitude of the mean resultant difference vector, D_t, for each lag t is obtained from

$$[nD_t]^2=[\Sigma(W_i-W_{i+t})]^2 = [\Sigma(u_i-u_{i+t})]^2+[\Sigma(v_i-v_{i+t})]^2. \quad (2)$$

In a continuous series, the component sums are simply the differences between the first and last observations, whose importance decreases as the sample size, n, increases. The Bedford series, however, although adequate in size, is not continuous, and the mean differences over the various usable segments are not, in general, close to zero. To compensate for this difficulty, the rms value was estimated by the standard vector deviations, S, calculated about the observed means:

$$(nS)^2=n\Sigma(u_i-u_{i+t})^2-[\Sigma(u_i-u_{i+t})]^2 +n\Sigma(v_i-v_{i+t})^2-[\Sigma(v_i-v_{i+t})]^2. \quad (3)$$

3. Regressions

The observed variability S was calculated for the four levels of interest for lags from 1 to 48 hr (Fig. 1). As expected, variability increases with increasing lag at all levels; the traces are similar in form. Ericksson (1960) and Arnold and Bellucci[5] following developments from Taylor's statistical theory of turbulence, expressed the wind variability in the form $S=bt^p$ where p is between 1/2 and 1. Arnold and Bellucci, using data from many investigations, determined the proportionality constant empirically as 4 mi per hr and that $p=1/2$ for lags of 1/2 to 12 hr. This becomes $S=1.8t^{1/2}$ for S in m sec^{-1} and t in hours; this curve is plotted in Fig. 1.

Such a square root relation may be valid if all the other effects (the first four functions of Eq 1) were absent. All may have been present in the Bedford data, however, so a constant term to represent them was used: $S=a+bt^{1/2}$.

Values of the coefficient b range (Table 1) from 2.1 to 5.6, and hence are substantially larger than the value found by Arnold and Bellucci. Some of this difference may arise from the lumping into a constant term

[4] Ellsaesser, H. W., 1960: Wind variability. Scott AFB, U. S. Air Wea. Ser., Tech. Rpt. No. 105-2, 91 pp.

[5] Arnold, A., and R. Bellucci, 1957: Variability of ballistic meteorological parameters. Ft. Monmouth, U. S. Army Signal Eng. Lab., Tech Memo No. M-1913, 27+lxxiii pp.

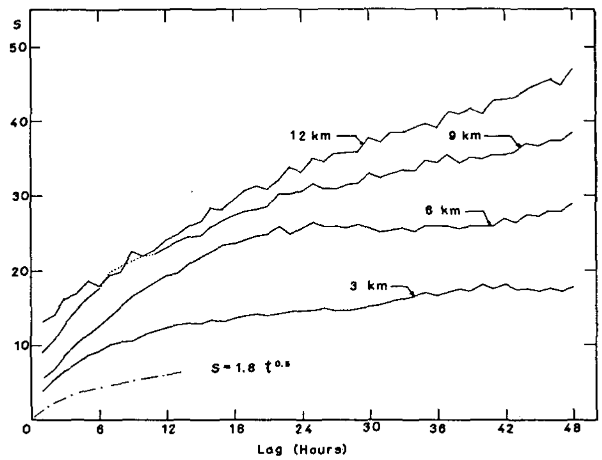

FIG. 1. Lag variability of wind over Bedford, Mass., 1–7 April 1960.

TABLE 1. Relation of lag variability of winds over Bedford, Mass., to wind vector and to square root of the lag.

Altitude (km)	Wind vector ($msec^{-1}$) Resultant	SVD	Regression: $S=a+bt^{1/2}$ a	b	Std. Er.	Corr.
12	36.9	27.5	4.73	5.90	0.49	.999
9	31.8	25.2	6.70	4.66	0.89	.993
6	22.9	19.8	6.08	3.49	2.10	.937
3	13.6	12.6	4.24	2.09	0.67	.981

of all the other functions. Some may be attributable to the restricted nature of the Bedford data (one week at one place), whereas the result of Arnold and Bellucci is an average for all seasons for middle latitudes generally.

The regression coefficients increase with height, as does lag variability (Fig. 1). Loeser (1951) and Gabriel and Bellucci (1951) found *no* significant change in variability with altitude, but Singer[6] found a significant change and Ellsaesser[7] concluded that variability increases up to the tropopause and decreases above that level.

An accompanying variation of wind speed with altitude is well known. Ericksson (1961) concluded that variability increases with wind speed rather than with altitude.

In the Bedford sample, the mean speeds, and the variability of the wind about the mean, increased with altitude, as did the lag variability. The coefficient of variation (variability divided by the mean speed), compensates for the effect of wind speed, and shows (Fig. 2) that the lag variability is related to wind speed rather than to altitude.

Any relation between altitude and lag variability apparently is inverse. For most lags, the coefficient of variation is larger at 3 and 6 km than it is at 9 and 12 km (Fig. 2). At the widest separation of the curves, the relative variability at 6 km is 25 percentage points above that of 12 km. This inverse relationship may not be real, and variability may be independent of altitude within these ranges. The observed dispersion of the relative variability with height could well be due to the effects of differences in height, layer thickness, position, and of measurement errors.

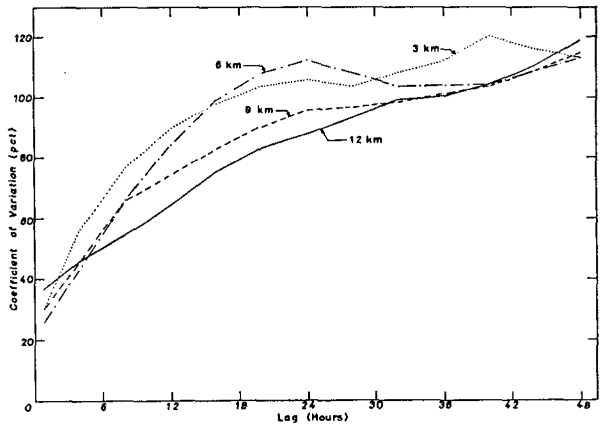

FIG. 2. Relative lag variability of wind over Bedford, Mass., 1–7 April 1960.

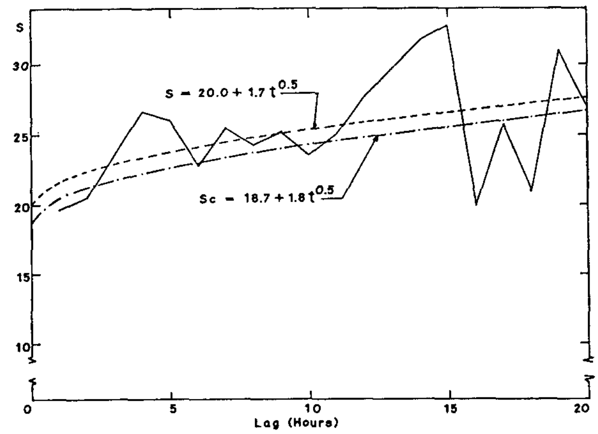

FIG. 3. Lag variability of wind at 12 km over Bedford, Mass., 4 April 1960.

4. Errors

To examine the effects of other causes of variability, a subset of data was selected from the sample: the 21 successive soundings on 4 April 1960. The observed variability at 12 km during this period (Fig. 3) was for the most part much larger than the average for the week as was the mean speed, 52.5 m sec^{-1}, compared to 36.9 m sec^{-1} for the entire week. The regression of the observed variability S on the square root of the lag (t, in hours) is (dashed curve in Fig. 3):

$$S=20.0+1.7t^{1/2}.$$

The effect of instrument error (m) was evaluated first, using the method developed in detail by Ericksson (1961). Briefly, the error in a wind observation is given by

$$(dW)^2=2t^{-2}[\cot^2\phi(dH)^2 + H^2\sin^{-4}\phi(d\phi)^2+H^2\cot^2\phi(d\theta)^2],$$

where errors are represented by the differential, W is the wind, H is the altitude, ϕ the elevation angle, and θ the azimuth angle of the balloon. Here t is the time interval between the two positions of the balloon, the time over which the wind is computed.

The error in a difference vector is given by

$$[d(W_{i+t}-W_i)]^2=(dW_{i+t})^2+(dW_i)^2,$$

where W_i and W_{i+t} are the initial and lagged winds, respectively. The observed variability S exceeds the true variability S_c (with instrument error removed):

$$S^2=S_c^2+[d(W_{i+t}-W_i)]^2.$$

Tracking errors are estimated as $d\phi=d\theta=1$ deg. A height error at the 12-km level was estimated from temperature sensing error of 0.7C, pressure error of

[6] Singer, B. M., 1955: Wind variability as a function of time. Bedford, Mass., USAF Cambr. Res. Lab., *AF Survey in Geophys.* No. 72, 20 pp. and Appendix.

[7] *Op. cit.* See footnote 4.

3 mb, and temperature and pressure characteristic of the 12-km level for these soundings. The observing error was nearly constant at all lag intervals, with a mean value of 7.4 m sec^{-1} and a standard deviation of 0.4 m sec^{-1}.

With appropriate allowance for this error, the variability at each lag interval as related to the square root of the lag (solid lower curve in Fig. 3) is $S_c = 18.7 + 1.8t^{1/2}$.

5. Combinations

A second source of measurement error, not mentioned specifically before, is contained in the Bedford data: although scheduled at 1-hr intervals, the rawinsonde flights actually were released as much as 20 min earlier and 30 min later than specified. In addition, variations in ascent rate made the balloons reach the various levels of interest (3, 6, 9 and 12 km) at varying intervals after release. Thus the observations were not at exactly 1-hr intervals, and assuming them to be so introduces another source of variation, the time variation (q).

To examine the effect of this source of variation, as well as of the other three previously mentioned (height, thickness and location), the variability S_c as compensated for instrument error was related by multiple regression to the time lags and these four other sources.

Several combinations of the sources of variation were considered. The greatest reduction of variance, using only one source, was provided by the thickness variation (k). But when four sources were considered, the greatest reduction in variance (97.5 per cent) was offered by a combination of time lag, height, location and time variation, omitting thickness. The 5-variable regression, including thickness, improved this by only 0.1 per cent, showing that the various sources of variation are not independent. If the other sources are used, thickness apparently may be ignored.

When any three sources (except thickness) were used, the further reduction in variance attained by introducing a fourth source of variation was:

Time ($t^{1/2}$)	27
Time deviation (q)	45
Location deviation (l)	18
Height deviation (n)	13

Apparently the observed lag variability is partly due to use of nominal instead of actual time intervals, but also includes some location variability.

6. Correlations

Variability may be studied in another way, through examination of its converse, the correlation. Since winds are represented by two-dimensional vectors, the ordinary correlation methods which apply to one-dimensional variables cannot be used directly, but must be extended dimensionally. Two kinds of extension have been proposed, one from basic statistical principles, the other through superficial analogy with the uni-dimensional case. The first may be called the true vector correlation; the second has two components, called "stretch" and "turn." Formulas for all are given in Table 2.

TABLE 2. Vector correlation formulas for two vectors, W and Z, with orthogonal components u, v and x, y.

True:

$$r^2_{W|Z} = \frac{s_y^2(s_{ux}^2+s_{vx}^2)+s_x^2(s_{uy}^2+s_{vy}^2)-2s_{xy}(s_{ux}s_{uy}+s_{vx}s_{vy})}{(s_u^2+s_v^2)(s_x^2 s_y^2+s_{xy}^2)}$$

Stretch:

$$r_S = (s_{ux}+s_{vy})[(s_u^2+s_v^2)(s_x^2+s_y^2)]^{-1/2}$$

Turn:

$$r_T = (s_{vx}-s_{uy})[(s_u^2+s_v^2)(s_x^2+s_y^2)]^{-1/2}$$

Stretch-turn-

$$r_{ST}^2 = r_S^2 + r_T^2$$

$$s_u^2 = \Sigma(u_i-\bar{u})^2/n; \qquad s_{uv}^2 = \Sigma(u_i-\bar{u})(v_i-\bar{v})/n; \qquad \bar{u} = \Sigma u_i/n$$

The stretch correlation coefficient has probably received the widest use. Durst (1954) considered it adequate for wind problems, declaring it would be increased only slightly by addition of the turn coefficient.

Fundamental differences exist between these correlation measures, and their sampling distributions are unknown. Empirical comparisons by Charles (1959), based on once-daily winds, showed little difference between stretch and true vector coefficients for large samples of lag correlations, but important differences for space and space-time correlations when the stretch coefficient was less than 0.3. He speculated that this threshold would be higher in smaller samples.

The behavior of all four correlation coefficients is shown for each level of the Bedford data in Fig. 4. In this series, at much shorter time intervals, space variability may be more important than in the once-a-day series used by Charles. The separation between the stretch and true coefficient curves varies from 0.05 to 0.35 when $r_S > 0.3$ or 0.4.

The turn coefficient exhibits a definite progression with height, from large negative values at 3 km through small values, negative at 6 km and positive at 9 km, to large positive values at 12 km. The coefficient is negative for winds that veer with time. A turn coefficient of 0.2 or higher makes an appreciable contribution to r_{ST}; this is a frequent event, contrary to Durst's conclusion.

For lags of less than 18 hours or so, and where the turn coefficient is contributing appreciably, the combination of the stretch and turn coefficients is a fair approximation of the true vector correlation. Otherwise, the approximation can be very poor, e.g.: at 6 and 9 km from 24 to 36 hours lag, the trend of true vector coefficient and of r_{ST} differ, as do the levels.

Durst (1954) found that the stretch vector correla-

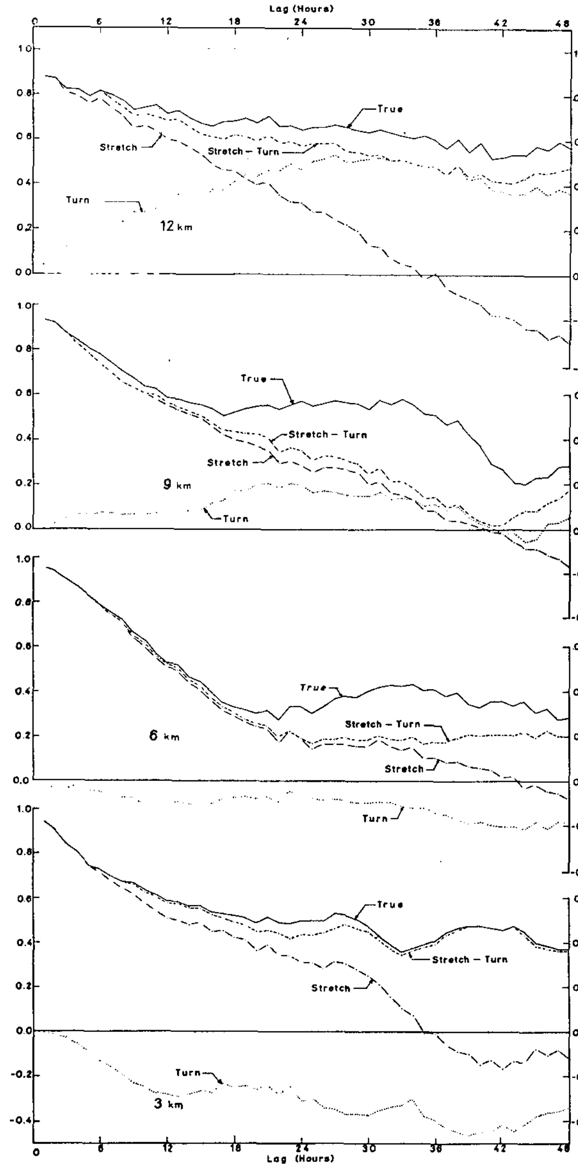

FIG. 4. Vector correlation of lagged winds over Bedford, Mass., 1–7 April 1960.

tion varied but little with height and that its decrease with increasing lag could be expressed by an exponential decay function. The stretch coefficients for all levels are plotted on a single chart in Fig. 5; little variation and no systematic change with height are shown. An exponential decay function would not be an adequate description for the Bedford data, however, as it would not accommodate the negative values of the stretch correlation beyond 36-hr lag.

In contrast to r_S, the true vector correlation varies considerably from level to level, as is shown in Fig. 5, but with no apparent systematic change with height. These curves appear to decay with the characteristics of a damped wave. If such a "correlation wave" exists,

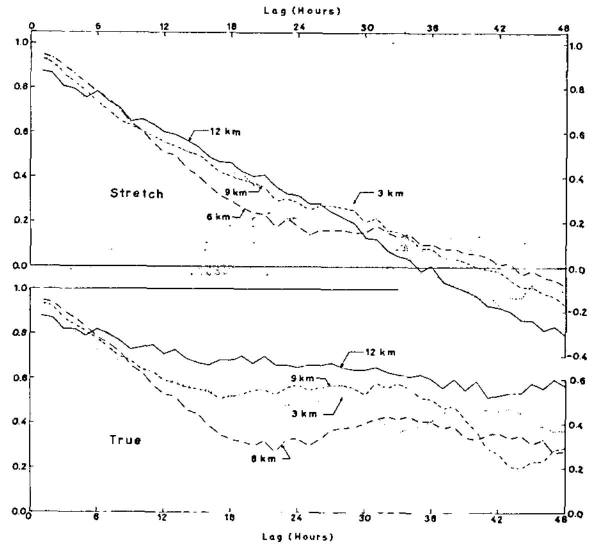

FIG. 5. Comparison of stretch and true vector lag correlations of winds over Bedford, Mass.

its amplitude and length vary with height as does, possibly, its phase angle.

7. Summary

Previous investigations of the dependence of wind variability S on time lag t have yielded expressions of the form $S=bf(t)$; Arnold and Bellucci[8] found that $S=1.8t^{1/2}$ with S in m sec^{-1} and t in hours, for lags up to 12 hours in all seasons and at all altitudes up to about 10 km in middle latitudes.

This study has shown (Fig. 2) that the coefficient of variation, $S/\bar{W}$, tends to be constant with altitude, indicating a dependence of the lag variability on the mean wind speed. A more suitable form for the dependence of wind variability on time would be $S=C\bar{W}f(t)$, where $f(t)$ is apparently $t^{1/2}$. Thus b depends upon mean speed, which varies with height, season and geographic location, and the local departures from the general estimate of Arnold and Bellucci for the regression coefficient observed in the Bedford data, are to be expected.

While the decay of the stretch correlation coefficient is fairly constant with altitude, even for this single week it is not well-described by the exponential function of Durst. Ellsaesser[9] shows how the exponential description varies seasonally and geographically, even when applied to suitable records. In a short period sample, the stretch correlation does not provide a good estimate of the true vector correlation beyond about 18-hr lag. At 21-hr lag, the difference between the 6- and 12-km levels in true vector correlations is three times as great as in stretch correlations.

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[8] *Op. cit.*
[9] *Op. cit.*

Local and seasonal departures from simplified and generalized estimates of wind relatedness and wind change can be quite large. Generalizations may even be misleading when applied to the solution of problems that are severely restricted in time or space.

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