On Objective Analysis of Continuous and Discontinuous Parameters

S. PENN, B. KUNKEL

Air Force Cambridge Research Laboratories, Bedford, Mass.

AND W. D. MOUNT

Sperry Rand Research Center, Sudbury, Mass.

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ABSTRACT

The problem of analyzing continuous and discontinuous meteorological surface parameters is investigated. Peculiarities of fitting mathematically a first and second degree surface to observational data are investigated from the standpoint of transforming and constraining the observed values and of the influence of observations as a function of distance. For two data density fields a comparison of results is made between the mathematical surfaces and a simplified procedure of selecting the nearest observation. Verification is based upon observed station values which are withheld from the surface fitting procedures. A significant improvement is derived by transforming the variables to ensure closer fitting in a particular range of the variable. For the area of influence under consideration, no appreciable differences were obtained by the incorporation of a distance weighting factor. It was found necessary to restrain the mathematical surfaces to ensure physical reality. Although the more complex mathematical surface does fit the large-scale nonlinear variations more accurately, its sensitivity to the distribution of data and smaller-scale fluctuations produced errors which were larger than those obtained using a linear surface.

1. Introduction

High speed digital computers have made it possible to consider numerical procedures for analyzing meteorological fields of continuous and discontinuous variables. To date, most of the effort has been directed toward analyzing continuous parameters in the free atmosphere. The need for similar processing of surface weather parameters is a recent requirement generated in the development of automatic weather systems.

Numerical analyses have been obtained using three different approaches. In a statistical approach, Mount (1960) allowed the data to determine the mathematical surfaces by using empirical orthogonal functions and cycled through the data again to obtain the optimum distance weighting function for each station. Although the procedure showed an improvement in accuracy and speed over other methods, it was not suitable for operational use since it could not accommodate missing observations.

Surface fitting or fitting of a polynomial by least squares to the data was first proposed by Panofsky (1949), and modified by Gilchrist and Cressman (1954), Bushby and Huckle (1957), and recently by Corby (1961). In all cases the surface examined was either a quadratic or cubic.

The emphasis on hemispheric analyses with regions of little data led to the development of the method of successive approximations, proposed by Bergthorssen

and Döös (1955) and modified by Cressman (1959). The method is one of applying corrections to a first guess field where each observation within a contracting circle of influence provides a correction to the initial guess as a function of distance from the observation. The corrections are determined by the difference between the observed values and the interpolated value of the guess field.

Of the three methods described above, it was decided to investigate the peculiarities of fitting mathematically a first and second degree surface to continuous and discontinuous surface parameters from the standpoint of transforming and constraining the observed values and of the influence of observations as a function of distance. Attention is directed specifically to the analysis of cloud-ceiling, surface temperature, and sea level pressure.

2. Mathematical surface analysis procedure and data sample

A three-dimensional curve is obtained via least squares. Each grid point is treated individually and a plane surface of the form

$$H_s = ax + by + c, \quad (1)$$

or quadratic surface of the form

$$H_s = ax + bx^2 + cy + dy^2 + exy + f, \quad (2)$$

is fitted to the observations in the vicinity of each grid point. The coefficients are determined by minimizing the expression for E where

$$E = \sum_{i=1}^N W_i (H_s - H_o)_i^2. \quad (3)$$

H_o is the observed value of the parameter, W is the distance weighting factor, and N is the total number of observations within a given distance from the grid point. The minimum number of observations required to fit a plane or quadratic surface is three and six, respectively; however, it was found advantageous to overspecify the plane fitting by requiring at least four observations. No overspecification was demanded with quadratic surface fits.

Verification is always open to question unless observed values are used. For this reason a so-called movable grid was developed. This is one where a station with an observation becomes the point for computation. The station value was omitted in the computation for that point and used for verification. Additionally, any observation within one-half a grid unit from the station was not used in the computation in order to study the surface fitting procedures under more uniform but restrictive conditions. This latter requirement eliminated about five stations within high station density areas such as New York City. The grid unit was defined as equal to one-third of the grid used at the Joint Numerical Weather Prediction unit, Washington, or about 100 km.

For analyzing discontinuous variables it is desirable to approximate the real curve by a succession of small incremental segments to sharpen the representation of the discontinuity. Observations used to define a surface were selected from successive scans of increasing radii. Further, if sufficient data were not available on the first scan of two grid units to fit a quadratic surface then a plane fit was attempted. If a plane could not be fitted then the scan was increased by one grid unit and the process repeated. In so doing the mathematical surfaces were fitted to the smallest identifiable segment of the discontinuous curve. This process of first attempting to fit a quadratic then a plane surface is referred to as Method Q. This Method can generate, within a data sample, analyzed values derived from either a quadratic or a plane fit. With Method Q, a quadratic surface fit was computed in 77 per cent of the dense data cases, and in 55 per cent of the sparse data cases. Analyses were also produced by two other procedures. One is Method P where observations were fitted solely by a plane even though sufficient observations for quadratic fit were available, and two is Method N where the nearest observation became the analyzed value. It should be pointed out that for an individual grid point the estimates from Method P and Q are derived from the same area and hence from identical data.

The numerical experiments¹ were carried out on a sample of data consisting of twenty sets of observations covering a period from mid-May to mid-June, 1961. A set contains teletype circuit "A" observations for 0000 and 1200 GMT from about 200 stations covering the area east of the Mississippi and including bordering Canadian stations. The more complex patterns of ceiling height were selected as the data sample and a typical pattern, given in Fig. 1, has sharp and complex gradients between clear and overcast skies. The data for each set consisted of ceiling height, sea level pressure, and temperature. The dense data sample uses all the observations with an average stations spacing of 100 km. The sparse data sample was obtained by eliminating observations in a systematic way to increase the distance between stations by approximately a factor of two.

3. Transformation functions

The surface fitting procedure operates on the principle of minimizing the sum of the square of the errors. Coefficients thus derived are most strongly influenced by the square of large errors. Since certain scales of motion or ranges of values are important or critical to operations, it is desirable to minimize the errors in the critical regions and tolerate larger errors where they are more acceptable. Once the critical range of values is defined the problem is to operate upon the initially observed values in such a way that the transformed values have

FIG. 1. Example of a typical distribution of ceiling height, 19 May 1961 at 0000 GMT, contained in the data sample.

¹ Mount, W. D., S. Penn and B. Kunkel, 1962: A surface fitting objective analysis program. Final Report, AFCRL-62-653, AF Cambridge Research Laboratories, Bedford, Mass., 215 pp.

TABLE 1. Comparison of the rms errors (ft) of the cloud-ceiling analyses, using the inverse transformation function (T) and not using it (NT). Method P is the plane surface fit. Method Q contains both quadratic and plane surface fits, and Method N refers to the nearest station method.

Observed ceiling (ft)	Dense data					Sparse data				
	Method P T	Method P NT	Method Q T	Method Q NT	Method N	Method P T	Method P NT	Method Q T	Method Q NT	Method N
≥1000	3814	6118	4152	6381	6642	5105	7292	5608	7594	7500
1100-2000	3489	5862	4259	5970	6750	3830	6332	4868	6308	6576
2100-3000	2918	5242	3848	5242	6178	4384	6784	5569	7929	6809
3100-5000	4097	5832	5291	6250	7188	4222	6051	4946	6190	7442
5100-10,000	4864	4634	5551	5457	6622	5384	5146	5875	5680	6914
>10,000	9084	5328	8324	5719	6448	9271	5626	8716	6093	6944
ALL	7480	5736	7587	6448	8203	8310	6475	8466	7173	9048

their largest change in the critical region. This hypothesis was investigated to determine if transformed variables did indeed force the mathematical surfaces to be fitted more closely to the critically important areas.

The cloud-ceiling was the parameter used in this study. For landing and taking off of aircraft, low ceilings are more critical than high ceilings. Various transformation functions were considered and the simple inverse of the initial values was used to obtain the transformed variables. Thus Eq (3) becomes

$$E = \sum_{i=1}^N W_i (T_i - T_o)^2,$$

where

$$T_s = \frac{1}{H_s} \quad \text{and} \quad T_o = \frac{1}{H_o}.$$

Mathematical surfaces were fitted to both the observed and transformed values and the rms error results are presented in Table 1. The transformation function does force the mathematical surfaces to fit the critical low ceilings more accurately throughout the entire range of ceilings up to at least 5000 ft. As expected, for high ceilings (greater than 10,000 ft), use of the transformation function produced large errors which were considered acceptable, *a priori*. The transformation process gives about a 40 per cent increase in accuracy in the critical low range, where it is most needed.

4. Data constraints

In general, it was necessary that a computed grid point value be constrained in some manner to remain within specified limits. Where a surface has to be extrapolated to a grid point, the estimated value at the grid point can become unrealistic. This difficulty arises because the observed values may be obtained from a highly skewed distribution of stations or may reflect discontinuities, instrumental and data errors, or local

conditions which should not be extended in space. Furthermore, the computed mathematical values could be negative in a physically positive domain. To overcome this difficulty, any computed value not within the range of the values used in the computation was replaced by the value at the nearest station. This restriction becomes essential for a discontinuous variable, such as ceiling height, which experiences large variations over short distances. The same restriction was applied to temperature. Thus a mathematical surface was allowed only if the computed value lay within the range of the observations. In addition, in order to keep the mathematics tractable, the ceiling height observations were bounded by 100 and 20,000 ft. Observational values outside this range were adjusted to the nearest limit. The only limits imposed on the relatively smooth surface pressure field were such that the pressure gradient did not exceed a wind value greater than 70 knots. No attempt was made to constrain the errors produced by a highly skewed distribution of stations such as is found along the coastline and at the edges of the area containing the observations.

5. Distance weighting factors

In a preliminary experiment the relative effectiveness of the following distance weighting factors (W) was compared

$$W = \frac{R^2 - D^2}{R^2 + D^2}, \quad (4)$$

$$W = \frac{2^7}{1 - 10^{82 - PE} D^4}, \quad (5)$$

where R is the outer radius of the scan, D is distance from observation station to grid point, and PE is an integral exponent, entered as initial data. No advantage was found in using the more complex weighting factor; hence W from Eq (4) was applied in all tests.

The effectiveness of W was tested on one set of ceiling height data (107 cases). The inclusion of W resulted in an improvement in 49 cases, a deterioration in 31, and little change in 27. Also in another comparison for these 107 cases, the rms error was 2444 feet with W included, and 2537 ft with W omitted. These results show that the incorporation of a distance weighting factor produces a small but only insignificant change. There is considerable noise in the small-scale variability which is not reduced by emphasizing the nearer observations through a distance weighting factor.

6. Comparison of results

Results from three objective procedures, with sparse and dense data, are presented in Tables 1, 2 and 3 for ceiling height, temperature, and sea level pressure, respectively. In general, both surface fitting procedures improve upon the simple nearest station technique for all three parameters. Of the surface fitting procedures, Method P is better than the more complex Method Q. The overall differences in the rms error between the various objective analysis procedures are not meaningful in the case of ceilings, since the errors are not normally distributed. Comparisons must be made on the basis of the actual distribution of errors within categories.

Distribution of the average ceiling errors for the plane fitting and nearest station methods is given in Table 4 for the dense data sample. Although the rms error given in Table 1 shows that the surface fitting method without a transformation function does better than the nearest station, this is not the case when considering the average errors given in Table 4. Here the average analysis errors clearly demonstrate the importance of transforming the initial data to force the mathematics to depict more accurately those values to be emphasized. Without a transformation function, plane fitting to the observed ceiling values produces significantly better results only for ceilings greater than 10,000 ft. Conversely, using an inverse transformation function not only reduces the rms error by 40 per cent but also reduces the average error by a half for the lower ceilings.

Hales and Sleater² found the standard deviation of the difference between observations taken at four stations separated by an average distance of only 12 miles. For ceiling height and temperature, these are 4,100 ft and 1.8F, respectively, for the San Antonio, Texas, area. These errors can be considered as a combination of those introduced by the observational procedures and the peculiarities of the meteorological parameters for this geographical location. Comparing these results with those for ceiling height and temperature given in Tables 1 and 2, respectively, shows that the analysis

TABLE 2. Temperature analyses—cumulative per cent of the errors equal to or less than specified values, and the rms error.

Error (°F)	Dense data			Sparse data		
	Method P	Method Q	Method N	Method P	Method Q	Method N
≤ 1	43%	40%	36%	35%	30%	29%
≤ 2	65	64	57	55	47	47
≤ 3	79	77	71	70	63	59
≤ 4	87	86	80	80	73	68
≤ 5	93	91	86	86	81	77
≤ 10	99	99	98	97	97	95
RMSE	3.1F	3.3F	4.3F	3.9F	4.4F	5.2F

TABLE 3. Pressure analyses—cumulative per cent of the errors equal to or less than specified values, and the rms error.

Error (mb)	Dense data			Sparse data		
	Method P	Method Q	Method N	Method P	Method Q	Method N
≤ 1	94%	87%	76%	85%	80%	65%
≤ 2	98	95	93	95	94	87
≤ 3	99.6	97	97	98	98	93
RMSE	0.86 mb	1.24 mb	1.64 mb	1.15 mb	1.62 mb	1.95 mb

TABLE 4. Distribution of average ceiling error by categories for the dense data sample. Units are in feet.

Observed ceiling	Method P		Method N
	T	NT	
≤ 1000	1100	4020	2660
1100–2000	1600	4000	3510
2100–3000	2100	4475	3800
3100–5000	2950	4400	4710
5100–10,000	3750	3760	4960
> 10,000	3450	578	1960

errors for both these parameters are about fifty per cent larger than the above values representing the observational uncertainty. The average distance between stations used in the analyses is greater, however, by more than an order of magnitude. The analysis errors are more pronounced for higher than lower ceilings where the best analysis procedure produces results better than this average observational uncertainty.

The sea level pressure observations are more representative of the surrounding conditions than are the temperature observations. The best analysis method has a rms error of pressure less than 1 mb and a 3-mb analysis error is rarely exceeded. For a dense network of grid points a 1-mb differential error between points represents a large geostrophic wind error. Assuming a uniform station distribution, a 1-mb pressure error at 35N latitude for a 100-km (data dense) station spacing represents a wind error of 18 knots. From Table 3, Method N would experience this error 24 per cent of the time whereas Method P reduces this to only 6 per cent.

² Hales, J. V., and J. K. Sleater, Jr., 1962: Spatial variations of various meteorological parameters. Scientific Rept., Met. Dept., Utah Univ., Salt Lake City, Utah, 28 pp.

FIG. 2. An analysis of the sea level pressure field for 26 May 1961 at 1200 GMT.

FIG. 3. Distribution of the surface pressure errors in mb obtained from the plane fitting procedure for 26 May 1961 at 1200 GMT.

Since the pressure field is more uniform than that of the ceilings, Method Q was expected to depict more accurately the nonlinear pressure system such as the troughs and lows. A typical example of the results is shown in Figs. 2, 3 and 4. Fig. 2 is an analysis of the pressure field for 12 GMT 26 May 1961 and Figs. 3 and 4 are the errors obtained using Methods P and Q, respectively. The more complicated mathematical surfaces did fit the large-scale non-linear variations more accurately, but their sensitivity to the distribution of data and small-scale fluctuations produced relatively large errors on the boundaries. The net result was a larger space average rms error for the non-linear method. Eliminating the horizontal boundary values from consideration, the non-linear representation can be obtained without an overall loss in accuracy.

7. Conclusions

Surface fitting analysis procedures can be used to analyze continuous and discontinuous surface meteorological parameters and produce results better than that given by the nearest station. It was found necessary to assign boundary limits to retain mathematically the range of the computed values thus ensuring computational stability and physical significance. Little difference was obtained in the results between using and not using a distance weighting function when applied to all or even one-half of the surface stations covering the eastern half of the United States. The simple plane-fitting

FIG. 4. Distribution of the surface pressure errors in mb obtained from the quadratic fitting procedure for 26 May 1961 at 1200 GMT.

procedure is more effective than the more complex one. The most striking results were found between using or not using a function to transform the observed values so as to fit more accurately those which are deemed most important. Not only does this transformation process produce a 40 per cent improvement in analyzing low ceiling heights but also it appears to be approaching the errors introduced by observational uncertainties.

The results presented in the tables for analysis errors of ceiling height and surface temperature and pressure should be improved upon under operational conditions where observations at or near an analysis point would not be withheld for verification.

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