Proposed Aerodetic Units of Length

JOHN C. BELLAMY

University of Wyoming

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ABSTRACT

It is proposed that units of length called nautical spans, chains and feet—and defined respectively to be 60, 1/60 and 1/6,000 nautical miles in length—be adopted internationally for use in weather-service and aeronautical operations. It is also proposed that the mandatory reporting levels for upper air charts be at intervals of one nautical mile of barometric altitude.

These proposed units have been selected to provide close numerical correspondences with associated sexagesimal measures of angular position and time; they are shown to interrelate values for time, horizontal angular and linear positions, speeds, accelerations, and vertical barometric and geometric positions as closely as possible within our current knowledge of the earth and atmosphere. The kinds of practical advantages which would accrue from the adoption of such geodetically and aerodetically derived units of length in characteristically world-wide weather-service and aeronautical operations are also indicated.

1. Purpose

The purpose of this paper is to propose some more conveniently useful units of length for weather-service and aeronautical operations. The adjective "aerodetic," standing for "dividing the (entire) atmosphere," concisely reflects the world-wide interests of these operations in large scale physical and geometric subdivisions of the earth and atmosphere. Specifically, these units of length are proposed as being most useful for internationally intercoordinating the numerical values of measures of: time, horizontal angular positions, horizontal distances, speeds and accelerations, vertical barometric positions, and vertical geometric positions.

2. Basis of length units

Until quite recently[1] the primary standard measure of length was defined for just this geodetic or aerodetic purpose. That is, the length of the International Prototype Meter was established so that as nearly 10,000,000 of them as practicable would be contained in the meridional quadrant of the earth's mean sea level surface. This particular number was selected so that numbers of meters of distance on the earth would very nearly correspond to numerical differences of positions identified with angular values of latitude and longitude. The values of latitude and longitude were to be expressed for this purpose with the metric, or 100 grads per right angle, units of angular measure.

[1] At the 11th General Conference of Weights and Measures in Paris, 14 October 1960, the orange-red spectral line of Krypton 86 was adopted as the primary standard of length measure, and the meter is now defined to be that length which contains 1,650,763.73 of these primary standard units.

But then, since the companion metric units of angle have not found favor, the meter is not as useful a unit of length for dividing the earth as it once promised to be. The sexagesimal or degree-minute-second units of angle have largely survived even in those countries which have adopted the metric units of length.

Evidently the survival of the sexagesimal units of angle is closely associated with the ease with which each of 360 degrees, 60 minutes and 60 seconds can be divided into any of 2, 3, 4, 5, 6, 10, 12, 15, 20 or 30 integral subdivisions. This facility of subdivision is especially useful for values of longitude and the very closely related hour-minute-second divisions of time. Indeed the meter-kilogram-second system of units itself somewhat inconsistently retains the sexagesimal second, or the 86,400th part of one revolution of the earth, as its basic unit of time.

The units of length proposed here have thus been selected to provide close numerical correspondences among values of distances between points on the earth and associated sexagesimal measures of angular position and time. Their lengths are established in terms of the familiar size of the meter even though it is no longer the primary standard of length.

3. Proposed units of length

The nautical mile has, of course, long provided the desired numerical correspondence between linear distances and minutes of geodetic arc. It is currently defined internationally to correspond to 1,852 meters, or to 6,076.11549 feet. It corresponds within about 1 part in 7,000 with the average length of minutes of latitude.

TABLE 1. Proposed units of length.

Name	Corresponding geodetic angle	Metric equivalent
Nautical span	1 degree	111,120 meters
Nautical mile	1 minute	1,852 meters
Nautical chain	1 second	$308\frac{2}{3}$ decimeters
Nautical foot	1/100 second	$308\frac{2}{3}$ millimeters

The proposed new units of length, as listed in Table 1, are then simply defined to be 60, 1/60 and 1/6,000 nautical miles. They are thereby very nearly equal to average lengths of degrees, seconds and hundreths of seconds of latitude, respectively. The adjective "nautical" is meant to be used in each case as it is in "nautical mile," including the prerogative of dropping the "nautical" whenever it is not needed for clarity of meaning.

The names, nautical foot and nautical chain, have been chosen since they are so nearly the same length as the common or "statute" foot and the 100-foot engineer's chain. Since 6,000 nautical feet and 6,076.11549 statute feet are contained in one nautical mile, 1 nautical foot or chain equal 1.012 686 statute feet or chains. Or, very nearly, these nautical units are only 1 part in 80, or one and one quarter per cent, larger than their statute homonyms. The decimal (rather than sexagesimal) subdivision of the nautical chain into feet and decimal fractions of feet follows the common engineering practice of denoting a distance such as 1,256.8 feet with a notation such as "station 12+56.8." It also corresponds with the standard geodetic, astronomical and chronometric practices of subdividing seconds of arc and time decimally.

No such homonym was available for naming the proposed 60 nautical mile, or nearly one degree, unit of length. The name, nautical span, has been chosen since it has a strong connotation of largeness, and the span of coverage of many topographic quadrangle sheets and aeronautical charts is one degree of latitude. Also the previous definition of a span as being the nine inch span of an extended hand has a connection, even though remote. The corresponding numbers of degrees separating points on maps are often measured by spanning with the fingers.

4. Units of speed and acceleration

It is significant that the only specific unit of speed now in common use, the knot, is directly associated with the proposed system of units. Its use-demonstrated utility should thus be enhanced even more by the use of other sexagesimally related units of length. For example, a knot is equivalent both to one nautical mile per hour and to one nautical chain (or 100 nautical feet) per minute. Whichever of these interpretations might be more appropriate for the intervals of time and distance involved in a particular problem can then be selected at will. This utility is illustrated especially well by the following equivalents of the 60-knot speed typical of speed limits on through highways, of landing aircraft, and of upper-air winds.

$$
\begin{aligned}
60 \text{ knots} &= 24 \text{ nautical spans/day} \\
&= 1 \text{ nautical span/hour} \\
&= 1 \text{ nautical mile/minute} \\
&= 1 \text{ nautical chain/second} \\
&= 1 \text{ nautical foot/centisecond}
\end{aligned}
$$

Similarly, 600 knots is a very convenient reference for meteorological and aeronautical considerations which involve the speed of sound. It closely corresponds to the speed of sound of dry air at a temperature of −36.2C or 236.90K. This temperature typically occurs at an altitude of about 25,000 feet where near sonic speeds of flight are quite common. Also, since the speeds of sound are proportional to the square root of absolute temperatures, the speeds of sound at other temperatures and with respect to various length and time intervals of potential interest are indicated by:

$$
\begin{aligned}
\text{Speed of Sound}/\sqrt{T°K/236.90} &= 600 \text{ knots} \\
&= 240 \text{ nautical spans/day} \\
&= 10 \text{ nautical spans/hour} \\
&= 10 \text{ nautical miles/minute} \\
&= 10 \text{ nautical chains/second} \\
&= 1 \text{ nautical foot/millisecond.}
\end{aligned}
$$

Such useful correspondences between various kinds of speed ratios of length and time are, however, basically incompatible with the metric units of length. Since lengths and times are then subdivided differently, speeds can usually be coordinated easily only in terms of how many centimeters, meters or other decimal multiples of meters are traversed per second of time. For example, the common metric unit of wind speed is the meter per second in spite of the fact that atmospheric occurrences of interest involve time intervals measured in days or hours rather than even minutes.

This restriction to the use of but one unit of time is especially noticeable in the virtually exclusive use of the centimeter per second per second unit of acceleration in the metric system. In contrast, the knot per second unit of acceleration provides a direct feeling for the meaning of values of accelerations in terms of common personal experiences. For example, the standard acceleration of gravity of 980.665 cm per sec per sec

TABLE 2. International Reference Ellipsoid (Hayford, 1909). (Derived from the number of meters in a, the flattening $(a-b)/a=1/297$, and the equation $x^2/a^2+b^2/y^2=1$)

	Nautical spans	Nautical miles	Nautical feet	Meters
Equatorial radius, a	57.400 90	3,444.054	20,644,324	6,378,388
$a-b=a/297$	0.193 27	11.596	69,577	21,476
Polar radius, b	57.207 63	3,432.458	20,594,247	6,356,912
Equatorial quadrant	90.165 12	5,409.907	32,459,444	10,019,148
Meridional quadrant	90.013 38	5,400.803	32,404,818	10,002,288
Minute of latitude at pole		1.005 208	6,031.247	1,861.646
Minute of longitude at equator		1.001 835	6,011.008	1,855.398
Minute of latitude at equator		0.995 100	5,970.598	1,842.925
Average minute of latitude		1.000 154	6,000.922	1,852.285

(which for completeness is equivalent to 31.7710 nautical feet per sec per sec) is readily interpretable as the acceleration with which an automobile or aircraft changes its speed by 19.0626 knots in one second. Similarly the equivalent values of 1,143.76 knots per minute and 68,625.4 knots per hour clearly indicate the very small fraction of gravitational acceleration involved in changes of atmospheric wind speeds.

5. Shape of the earth

Table 2 illustrates the closeness with which the international definition of the nautical mile provides for the desired numerical correspondences between lengths and angles associated with horizontal positions. For example, the 1.000 154 nautical miles in the average minute of latitude has been calculated as the simple arithmetic average of the lengths, 1.005 208 and 0.995 100, of minutes of latitude at the pole and equator, respectively. Since the lengths of those polar and equatorial minutes of latitude differ by somewhat more than one per cent, the deviation of only about 1 part in 7,000 of their average from an "ideal" value of 1.000 000 is insignificant in any practical sense.

In other words, the current nautical mile is as nearly typical of the length of one minute of great-circle arc anywhere and in any direction on the earth as any other definition is likely to be. Indeed, the comparisons in Table 2 are themselves only approximations of how measured distances between points might correspond with their latitudes and longitudes as determined by astronomical observations. In basic principle, the International Reference Ellipsoid is that simply definable geometric surface which most closely corresponds with world-wide geodetic observations of the shape and size of the earth.

6. Vertical coordinates

In close analogy with the two ways of measuring horizontal position, the atmosphere is divided vertically both barometrically and geometrically. The barometric measure of vertical position in current aeronautical practice is that foot of altitude with which pressure altimeters are calibrated. Alternatively, the millibar is now universally used in weather-services as the barometric measure for identifying the atmosphere's constant pressure levels. Either statute feet or meters are currently used in various weather services to evaluate geometric elevations above mean sea level.

Again by analogy with the horizontal case, major advantages should accrue if these two—barometric and geometric—kinds of vertical measures were defined so that their numerical values correspond as closely as practicable. This kind of correspondence has been achieved by defining values of barometric altitude to be those geometric elevations within the International Civil Aviation Organization Standard Atmosphere at which corresponding intensities of pressure occur. This Standard Atmosphere has carefully been defined so that the altitude numbers associated with any and all constant pressure levels is as typical as possible of the geometric elevations of their occurrences anywhere and at any time in the actual atmosphere.

Values of barometric altitudes are thus indicated as being much more conveniently useful than values of millibars for dividing the atmosphere barometrically. The use of barometric altitude offers the same general kind of numerical correspondence that was sought in the definition of the meter, and which was achieved with the definition of the nautical mile. The advantageous utility of values of barometric altitude has also been well use-proven by long aeronautical practice and, especially, by pressure pattern and optimum flight planning navigational practice. Indeed, they have been demonstrated in several weather station operations in the U. S. Army, Air Force and Navy to be advantageously useful for all phases of weather-service operations.

Significantly the continued use of millibars is closely analogous to a continued use of the English or "statute" units of length for dividing the earth geometrically.

Originally some arbitrary length unit such as the English yard had to be independently defined. Otherwise the correspondences between distances and angular positions on the earth could not have been determined. After such correspondences were established, however, they could be and advantageously were utilized to redefine the units of measure in terms of the meter, the ill-fated grad, and the nautical mile. Similarly it was necessary at the outset to define completely independent measures of pressure (such as millibars) and geometric heights. With them the knowledge was gained that barometric and geometric heights correspond quite closely. That knowledge is then very usefully utilized by defining a more appropriately useful, barometric altitude, measure of atmospheric pressure.

Nevertheless there are good and sufficient reasons for the survival of the millibar measure of barometric position in weather-service operations. They result from the international disagreement concerning which of the two previous contenders, statute feet or meters, should be used as the unit of length for evaluating altitudes. Such an international agreement was necessary since it would determine those mandatory levels for which upper-air observations are to be reported from throughout the world. If those levels were to be designated for the round-value convenience of those who wish to use meters, they would be very inappropriately inconvenient for those who wish to use feet and vice versa. This problem has been solved, in effect, by adopting the millibar as the measure of the basic barometric coordinate of vertical positions of things in and of the atmosphere, and by designating mandatory reporting levels to be at round values of millibars.

7. Mandatory reporting levels

Clearly this solution of the critical problem of designating mandatory levels is more an avoidance of the problem than a solution. The use of millibars avoids the problem since they are no more closely related to the statute foot nor the meter than the other. But then millibars also have a usefully convenient relationship to neither. For example, it is of virtually no significance that millibar values of mandatory levels such as 700 and 500 are round. The values of their logarithms in the all important hydrostatic equation for coordinating vertical positions are then not so conveniently round.

The solution of this mandatory reporting level problem is thus the primary explicit purpose of the proposed nautical units of length. Specifically it is proposed that mandatory reporting levels be established at intervals of one nautical mile of barometric altitude. This corresponds in qualitative principle to utilizing round values of the logarithm of the pressure. It thereby reduces the all important hydrostatic relationship to a linear relationship involving only additions and subtractions of small numerical differences between values of geometric elevation and barometric altitude.

The values of contemporary statute foot, meter and millibar measures for the mandatory levels at nautical mile intervals are listed in Table 3. Corresponding values of millimeters of mercury are included since they are directly involved in the definition of values of altitude. In this use of the ICAO Standard Atmosphere, it is basically a legal and physical law with which values of barometric altitude are defined mathematically in terms of the primary standard, mercury barometer, way of measuring atmospheric pressures.

Fortuitously these levels divide the lower ten nautical miles of the atmosphere with the same number of intervals as are now in use. As indicated in the listing of current mandatory levels in Table 4, the 70-mb level is only about 250 nautical feet lower than the proposed 10 nautical mile level. Similar near coincidences are:

TABLE 3. Equivalent values of barometric levels at nautical mile intervals. (Selected from the National Advisory Committee for Aeronautics Technical Note 3182, *Manual of the ICAO Standard Atmosphere Calculations by the NACA*, Washington, May 1954.)

Altitude nautical miles	Altitude meters	Altitude statute feet	Altitude nautical feet	Pressure millimeters of mercury	Pressure millibars
0	0	0	0	760.0	1013.25
1	1,852	6,076.12	6,000	607.3	809.7
2	3,704	12,152.23	12,000	480.4	640.55
3	5,556	18,228.35	18,000	375.9	501.25
4	7,408	24,304.46	24,000	290.7	387.55
5	9,260	30,380.58	30,000	221.8	295.70
6	11,112	36,456.69	36,000	166.77	222.37
7	12,964	42,532.80	42,000	124.54	166.07
8	14,816	48,608.92	48,000	93.00	124.00
9	16,668	54,685.04	54,000	69.45	92.59
10	18,520	60,761.15	60,000	51.86	69.14

TABLE 4. Barometric altitudes of round-millibar levels. (Selected and calculated from NACA Technical Note 3182, as in Table 3.)

Pressure millibars	Altitude meters	Altitude statute feet	Altitude nautical feet	Difference of altitude nautical feet	Altitude nautical miles
1000	111	364	360		0.060
850	1,457	4,781	4,720	4,360	0.787
700	3,012	9,882	9,758	5,038	1.626
500	5,574	18,289	18,058	8,300	3.010
400	7,185	23,574	23,278	5,220	3.880
300	9,164	30,065	29,689	6,411	4.948
250	10,363	33,999	33,574	3,885	5.596
200	11,784	38,662	38,177	4,603	6.363
150	13,608	44,647	44,086	5,909	7.348
100	16,180	53,083	52,419	8,333	8.737
70	18,442	60,504	59,747	7,328	9.958

the 1000-mb level is 360 nautical feet higher than the zero altitude level; the 500-mb level is only 58 nautical feet higher than the 3 mile or 18,000 nautical foot level; and the 300-mb level is 311 nautical feet lower than the 5 mile or 30,000 nautical foot level.

These nautical mile intervals between mandatory levels might well be adopted even though the associated nautical foot and chain units were not. Whatever utility round values of millibars might have for some other purposes, they have little if any inherent utility for the altimetric purpose of dividing the atmosphere vertically. Hence rawinsonde observations of temperatures, winds and geometric elevations (in units of either statute feet or meters) surely could be reported and charted as well for the levels at round barometric values of the international nautical mile as for the levels at round values of millibars.

At least the atmosphere would then be divided uniformly in the vertical, and both the statute foot and meter altitudes listed in Table 3 would become useful if only by virtue of their internal consistency. In contrast, as illustrated by the difference of altitude column in Table 4, the current attempt toward uniform vertical division of the atmosphere has met with poor success. Due to their virtually random distribution, the values of feet and meters in Table 4 are numbers which have to be memorized, rather than numbers which aid in coordinating thoughts about positions in the atmosphere.

8. Vertical length units

Clearly, though, the full advantage of nautical mile intervals between the mandatory levels of charts would only be realized by also adopting the nautical foot and chain units. Since these units are sexagesimally related to the one nautical mile intervals, each of the miles could then readily be divided into whichever of many finer integral subdivisions might be most convenient for particular purposes. For example, the sixty subdivisions of 1 chain or 100 feet each are appropriately representative of the limit of accuracy of most methods of measuring vertical positions in the atmosphere. Similarly, the 12 subdivisions of 500 feet each might well be assigned as aircraft flight levels. In that case interpolation between charted levels to determine values of conditions to be encountered at any and all flight levels would be exceptionally convenient. The vertical change of conditions over the 500, 1000, 1500, 2000, 2400 or 3000 feet from the nearest chart level would usually be readily apparent as a twelfth, sixth, fourth, third, five-twelfths, or half of the changes of conditions between charted levels.

As indicated by this last example, full advantage also requires that aeronautical operations adopt the nautical foot unit for adjusting the calibrating screws of pressure altimeters. The obvious explicit advantage for flyers would be that weather services' charts would then become directly and conveniently useful aeronautical tools.

Flyers might well wish to convert to the nautical foot for altimetry in any event. Horizontal and vertical positions are now considered by most flyers throughout the world in terms of nautical miles and feet of altitude. Consequently most flyers now think of the nautical mile as containing "about 6000 feet." The conversion from statute to nautical feet would thus, in essence, convert a common lax thinking habit into an exact relationship. No basic change in thinking habits nor in "the feel" of the numbers would be involved since the nautical and statute feet are of so nearly the same length.

Also, of course, the desired advantages could only be achieved in weather-service and aeronautical operations by using nautical foot measures for geometric as well as for barometric vertical positions. The conversion to the nautical foot measures of geometric heights would involve such tasks as: revising published elevations of bench marks, of runways and of the ivory points of station barometers; revising the topographic contours on aeronautical charts; recalibrating weather, navigational and traffic control radars; etc. Although these tasks are by no means insignificant the potential rewards seem well worth the cost.

It is noteworthy in this respect that the commonly cited advantage of merely having to shift decimal points to change units of metric measure is by no means limited to the metric system. This prerogative exists for any unit which one might wish to extend or subdivide decimally. For example, tenths and hundreths of feet are frequently used instead of inches when it is convenient to do so. Similarly, tenths of nautical miles will continue to be more convenient than nautical chains or feet for usual finer-than-one-mile aeronautical subdivisions of horizontal positions. Also it is hardly necessary to convert an altitude value such as 12,570 feet into 2 miles 5 chains and 70 feet just because the mile and chain units happen to be available (and are available!) for the convenience of some other purpose.

9. Conclusion

The preceding discussions reflect the earnest hope that these nautical units of length will prove to be suitable for all nations' adoption and use. It is not intended that they be adopted internationally for all purposes since even the desirability of such complete regimentation is quite questionable. Rather they are intended primarily for use in weather-service and aeronautical operations which are necessarily world-wide in scope.

Optimism that these units might well provide the long-sought solution to a very difficult problem seems quite warranted. The proposed units divide the earth and atmosphere with more usefully interrelated positional measures than their metric predecessors. In effect they interrelate values of all of time, horizontal angular and linear positions, speeds, accelerations and barometric and geometric vertical positions as closely as possible with our current knowledge of the nature of the earth and atmosphere. On the other hand the difficulty heretofore of converting from arbitrary English measures to geodetically and aerodetically derived measures is greatly eased by the coincidental close correspondence of the lengths of statute and nautical feet. Optimism is especially warranted since the basic nautical mile unit is already in extensive international use. In addition the statute foot is used even now for aircraft altimetry in many countries which have adopted the metric system for most other purposes.

Hopefully the potentialities of these units have also been indicated here sufficiently well to initiate actions leading to their world-wide adoption. More detailed discussions of basic related physical concepts could not readily be incorporated in this introductory paper. Neither was it feasible to include usefully complete descriptions of the many practical advantages to weather services of dividing the atmosphere vertically with values of barometric altitude. These basic aerodetic subjects are to be discussed in detail in several companion articles and a forthcoming book.