Lagrangian and Eulerian Time-Scale Relations in the Daytime Boundary Layer

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ABSTRACT

Lagrangian (neutral balloon) and Eulerian (tower and aircraft) turbulence observations were made in the daytime mixed layer near Boulder, Colorado. Average sampling time was ~25 min. Average Lagrangian time scale is ~70 s and average ratio of Lagrangian to Eulerian time scales ($\beta = T_L/T_E$) is about 1.7. The ratio β is inversely proportional to turbulence intensity i. These data support the formula $\beta = 0.7/i$. Lagrangian time scale for the vertical component of turbulence at heights above ~100 m is given by the formula $T_L = 0.17z_i/\sigma_w$, where z_i is mixing depth. This formula is valid for the horizontal components of turbulence at all heights in the mixed layer. Lagrangian spectra in the inertial subrange are best represented by the formula $F_L(n) = 0.2\epsilon n^{-2}$.

1. Introduction

One of the fundamental problems in turbulence and diffusion is the relation between the Lagrangian and Eulerian frames of reference for measuring turbulence. A Lagrangian measurement is one made by following a small air parcel as it is carried by the winds. Since diffusion is caused by the spreading from one another of these small air parcels, it is most efficiently described in the Lagrangian framework. An Eulerian measurement, on the other hand, is one made by an instrument whose position is fixed in one way or another; e.g., an anemometer on a tower or a pitot tube on an airplane. Usually we must predict Lagrangian diffusion using Eulerian measurements. The purpose of the experiment described in this paper was to simultaneously measure Lagrangian turbulence (using neutrally-buoyant balloons) and Eulerian turbulence (using fixed anemometers and aircraft observations) and develop relations between them.

This approach is not new; Gifford (1955) and Angell (1964) both used basically the same experimental procedure. However, the data obtained here are somewhat more accurate and plentiful than that gathered in previous studies, since we had the benefits of using Doppler radar and of being associated with a comprehensive, month-long convective boundary-layer experiment (Project PHOENIX). As a result high quality Eulerian measurements were available up to heights of 300 m, and large amounts of useful supplementary data such as mixing depth (z_i) were often available.

While it may seem intuitively plausible that Lagrangian and Eulerian turbulence are simply related, it is not easy to prove this theoretically (Lumley, 1962). Instead, it is necessary to rely on "reasonable assumptions" in order to relate the two types of turbulence. Gifford (1955) and Hay and Pasquill (1959) independently made the very useful assumption that the Lagrangian and Eulerian autocorrelograms R were similar in shape but were displaced by a scale factor, called β. The same assumption is valid for the energy spectra, F. This concept is illustrated by Fig. 1, taken from Hay and Pasquill's (1959) article. Mathematically, this assumption can be stated as

$$nF_L(n) = \beta nF_E(\beta n), \quad (1)$$

$$R_L(\beta t) = R_E(t), \quad (2)$$

where β is formally defined as the ratio of the Lagrangian and Eulerian time scales, i.e.,

$$\beta = T_L/T_E. \quad (3)$$

The time scale T is the integral time scale of the autocorrelogram function

$$T = \int_0^\infty R(t)dt. \quad (4)$$

In reality, the infinite limit on this integral must be replaced by the sampling time t_s.

Corrsin (1963) compares the shapes of Lagrangian and Eulerian spectra in the inertial subrange, where the following similarity laws hold:

$$F_E(n) = Au^{2/3}\epsilon^{2/3}n^{-5/3}, \quad (5)$$

$$F_L(n) = B\epsilon n^{-2}. \quad (6)$$

The universal constant A for the Eulerian spectrum is well known ($A = 0.2$ for v and w, $A = 0.15$ for u), but the constant B for the Lagrangian spectrum is quite uncertain. One of the purposes of our experiment is to determine a value for B and to verify the $F_L(n) \propto n^{-2}$ relation implied by Eq. (6).

Corrsin (1963) used Eqs. (5) and (6) and the assumption that the total turbulent energy σ_v^2 was identical in the Lagrangian and Eulerian systems to arrive at the theoretical prediction

$$\beta = \frac{C}{\sigma_v/U} = \frac{C}{i}\,. \quad (7)$$

The denominator, $i = \sigma_v/U$, is the intensity of turbulence, which can also be thought of as a stability index. It is not obvious whether the lateral turbulence intensity σ_v/u also should be used in estimating the vertical (z) and along wind (x) components of β, or whether the appropriate wind component should be used (σ_w/u for the z component, σ_u/u for the x component). In this paper the individual component intensities are used. The intensity i increases as stability decreases or roughness increases. An exception to this rule is very stable, light wind conditions, where meanders lead to relatively large values of σ_v/u. Other theoretical derivations summarized by Pasquill (1974, p. 92) yield a similar dependence of β on the inverse of turbulence intensity. Theoretical values of the constant C range from ~0.3 to ~0.8.

Using observations of neutral pilot balloons released from the meteorological tower at Brookhaven National Laboratory, Gifford (1955) found that the ratio $\beta = T_L/T_E$ averaged ~3. Angell (1964) used tetroons to give Lagrangian observations and a tethered balloon system to give Eulerian observations at heights of ~700 m at Cardington, England. His average observed β was about 4.0, which is a frequently quoted value. It is intuitively reasonable that β would be greater than one, since the time required by a balloon to completely pass around an eddy would be longer than the time required by the same eddy to be transported past a fixed anemometer by the mean wind. Angell's data also were plotted as β versus $1/i$, yielding a constant C in Eq. (7) equal to ~0.4. There is much scatter in this plot, however, and a best-fit curve would not be linear. Further observations by Angell *et al.* (1971) of turbulence using tetroons flown past tall towers near Las Vegas also show that β is inversely proportional to turbulence intensity, averaging ~3.

The time scales can also be estimated from planetary boundary-layer theory and observations. Kaimal *et al.* (1976) show that the period T_{mE} at which peak spectral energy occurs for Eulerian horizontal turbulent fluctuations in the convective boundary layer is given by $T_{mE} = 1.5z_i/u$, where z_i is the mixing depth. This relation is based on data obtained over a sampling period of ~75 min. As defined by Eq. (4),

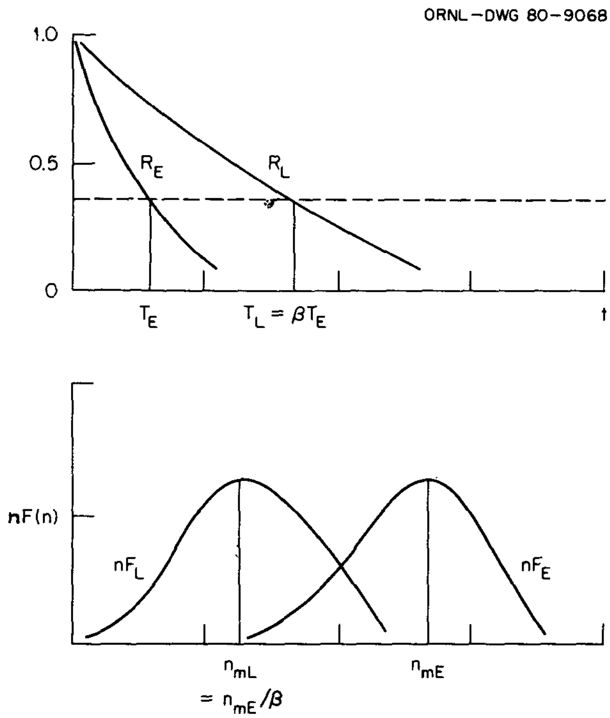

FIG. 1. Schematic drawing of Lagrangian and Eulerian spectra and autocorrelograms (after Hay and Pasquill, 1959).

the period T_{mE} of the spectral peak is related to the Eulerian time scale T_E by

$$T_{Eu,v} = (1/6)T_{mE} = 0.25z_i/u. \quad (8)$$

[The derivation of the relation $T_E = (1/6)T_{mE}$ is given in Section 3c.] For typical values of mixing height ($z_i \approx 1000$ m) and wind speed ($u \approx 5$ m s^{-1}), the Eulerian time scale is predicted by this equation to be ~50 s. The corresponding Lagrangian time scale T_L assuming $\beta \approx 4$, would be 200 s. Kaimal *et al.* (1976) find that T_{mE} for vertical fluctuations is equal to T_{mE} for horizontal fluctuations at heights above ~100 m, but is influenced by the ground at low elevations and becomes inversely proportional to height at heights $\leq$ 100 m. Reid (1979) uses surface layer observations and $\beta = 0.35/i$ to derive the following formula for the Lagrangian time scale for the vertical component T_{Lw} of turbulence, valid at heights $\leq$ 100 m:

$$T_{Lw} = 0.60z/u_*, \quad z < 100 \text{ m}. \quad (9)$$

The above discussion of previous work on this subject leads us to these four basic experimental goals:

(i) Test the $F_L \propto n^{-2}$ relation for Lagrangian spectra in the inertial subrange and determine the universal constant in this relation.

(ii) Test the hypothesis that Lagrangian and Eu-

TABLE 1. Summary of Boulder neutral pilot balloon runs.

Run no.	Date (September 1978)	Time (MDT)	Mean elevation (m)	Mean wind speed and direction	σ_w (m s^{-1})	u_* (m s^{-1})	L (m)	z_i (m)
181	18	1645–1710	600	6.0 NNE	0.94	N.A.*	N.A.	N.A.
192	19	1030–1054	600	6.9 SE	0.59	0.49	−47	N.A.
193	19	1125–1154	300	6.7 ESE	1.13	0.49	−47	N.A.
194	19	1340–1359	800	6.6 SE	1.35	0.28	−39	N.A.
195	19	1425–1454	800	3.4 ESE	0.84	0.47	−183	N.A.
196	19	1520–1550	900	4.9 E	1.17	0.48	−46	N.A.
197	19	1606–1636	800	4.0 E	0.92	N.A.	N.A.	N.A.
201	20	1515–1543	800	2.6 SSW	0.72	0.23	−12	N.A.
202	20	1613–1631	600	2.3 WSW	0.84	N.A.	N.A.	N.A.
211	21	1030–1059	200	1.7 NNW	0.85	N.A.	N.A.	300
212	21	1130–1157	300	1.4 SW	0.97	0.16	−2	750
213	21	1300–1321	500	1.8 SSE	1.66	0.32	−10	850
214	21	1400–1425	600	2.9 SSE	1.69	0.41	−25	900
216	21	1530–1554	600	3.8 S	1.43	0.24	−15	N.A.
217	21	1620–1634	120	2.1 NE	0.58	N.A.	N.A.	N.A.
221	22	0930–1013	100	1.3 SSE	0.69	0.19	−4	225
222	22	1100–1153	120	1.5 ESE	0.89	0.25	−9	300
223	22	1212–1242	300	2.8 SE	1.12	N.A.	N.A.	325

*N.A. = not available.

lerian spectra and autocorrelograms are displaced by a constant factor, $\beta = T_L/T_E$.

(iii) Calculate $\beta = T_L/T_E$ from the observations and determine the constant in the relation $\beta = C/i$.

(iv) Determine the Lagrangian time scale T_L for typical daytime conditions.

2. Experimental procedure

During September 1978 a comprehensive study of the convective boundary layer was accomplished by the Wave Propagation Laboratory of NOAA around its Boulder Atmospheric Observatory (300 m meteorological tower) located on the plains 30 km east of Boulder, Colorado. Sophisticated instrumentation on the tower (Kaimal, 1978) gave high-quality Eulerian turbulence observations. Several types of radars simultaneously probed the boundary layer, using chaff, temperature inhomogeneities, particles and moisture as tracers. Aircraft from the National Center for Atmospheric Research (NCAR) obtained another type of Eulerian turbulence observation in the boundary layer. In the presence of all these excellent Eulerian measurements, it seemed desirable to measure the Lagrangian turbulence fluctuations, also.

Two types of Lagrangian measurements were made. In the first method, standard 30 g pilot balloons were made neutrally buoyant and tracked by double theodolites. In the second method, tetroons were inflated to float at tower-top level (300 m) and were manually tracked by two x-band Doppler radars separated by 13 km. During two 5-day periods, useful data were obtained from eight tetroons and 18 pilot balloons.

a. Neutral pilot balloons

Standard 30 g pilot balloons were made neutrally buoyant in a van parked near one of the theodolites, and released at a height of about 100 m using a technique adapted from Longhetto (1971). Two double-theodolite baselines were set up, one 544 m long on the N–S road ~1 km west of the 300 m tower and the other 590 m long on the E–W road ~1 km north of the tower. There was <1 m elevation difference between the theodolites on each baseline. A timing interval of 15 s was used.

A summary of the dates and times of the pilot balloon runs, and the measured mean elevation and meteorological information for each run are given in Table 1. The 300 m tower was operating during all the runs, and on 21 and 22 September the NCAR aircraft were flying patterns overhead. Thirty-minute runs were the goal, although the balloon was lost before this time in some cases; and on 22 September we obtained two 50 min runs. Because of the good visibility and lack of obstructions on the horizon, we could easily follow the balloons to quite low-elevation angles.

b. Tetroons

Tetroons (i.e., superpressured mylar plastic bags) with 1 m³ displacement were tracked during two periods: 29 August–1 September and 18 September–20 September. The Doppler radars used here (Kropfli and Kohn, 1978) give accurate radial velocities of the tetroon at sampling rates up to 10 s^{-1}, although a slower sample rate, 2 s^{-1}, was chosen for this experiment. Components of the wind velocity (u and v) could be calculated from these radial velocities.

TABLE 2. Summary of Boulder tetroon runs.

Tetroon no.	Date (1978)	Times of good data (MDT)	Number of radars operating	Mean elevation (m)	Mean wind speed and direction	σ_v (m s^{-1})	u_* (m s^{-1})	L (m)
T1	8/29	1207–1232	1	300	NE 0.4	1.81	N.A.*	N.A.
T3	8/30	1439–1522	2	350	N 3.6	1.50	N.A.	N.A.
T6	8/31	1335–1405	1	400	SE 5.0	1.34	N.A.	N.A.
T9	8/31	1605–1659	1	800	WSW 7.7	Trend	N.A.	N.A.
T10	9/1	1301–1324	2	2000	NW 4.3	1.55	0.13	−14
T11	9/18	1358–1420	2	300	N 10	0.66	0.56	−113
T12	9/18	1551–1610	2	220	NNE 11	1.45	0.45	−60
T13	9/20	1334–1423	2	300	SSW 1.3	0.77	0.18	−5

* N.A. = Not Available.

Tetroons were inflated as recommended by Hoecker (1975). To aid in radar tracking, a pinch of 1.5 cm metallic chaff was blown into the tetroon. They were released a distance of ~10 km from the radars, so that they would pass reasonably close to the 300 m tower. A mean flight elevation of 300 m was intended, but convective conditions and inflation problems resulted in a wider range of heights than desired.

The dates and times of the tetroon runs, the number of radars operating, and measurements of mean elevation and meteorological information are given in Table 2. During the August runs the tower was not operating, but during the September runs it provided continuous data. No NCAR aircraft runs coincided with these periods. Run durations ranged from about ten minutes to about 50 min, and all data were recorded on magnetic tape. The data of interest were one-half second averages of radial velocities and corresponding signal intensities. This data rate was considered sufficiently fast because the inertia of the tetroon smoothed out the highest frequencies.

3. Data analysis

a. Neutral pilot balloons

A standard double theodolite computer program was used to calculate balloon position (x, y, z) and velocity components (u, v, w). Total number of data records per run ranged from 80 to 200. It was found that if the difference between the azimuth angles of the two theodolites was ≤3°, as usually occurs at the end of a 30 min/run with strong winds (e.g., runs 181–197), significant error was introduced into the data. Schaefer and Doswell (1978) have derived theoretical formulas which give the expected errors in these observations. For runs with light winds of ~2 m s^{-1} perpendicular to the baseline, Schaefer and Doswell's formula predicts that the position error is less than 5 m for travel times < 20 min, assuming a theodolite reading accuracy of 0.01°. This gives an expected speed uncertainty of <0.4 m s^{-1}. Some of the high-frequency wind fluctuations recorded by this system may not be real, but the low-frequency fluctuations, and hence the spectral peak, are expected to be realistic.

Plots of balloon positions were drawn for all runs; an example is shown in Fig. 2. The bouncy behavior of the balloon was quite remarkable on convective days, when it would rise and fall between the ground and an elevation of several hundred meters with a period of 5–10 min. In some cases, the balloon would sit on the ground for several seconds, then suddenly get caught by a thermal and rise at a rate of up to 5 m s^{-1}.

An FFT algorithm was used to produce plots of

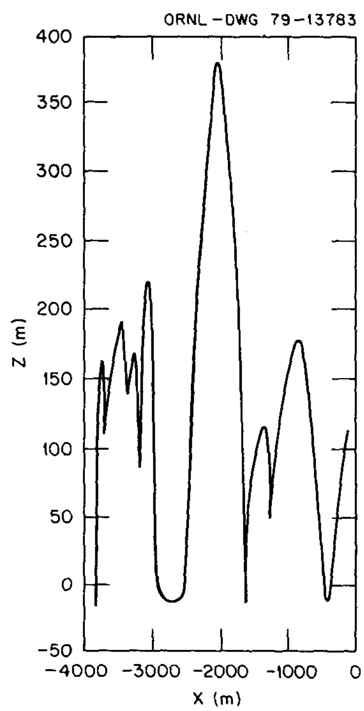

FIG. 2. Observed height variations of neutral pibal 222 as a function of distance.

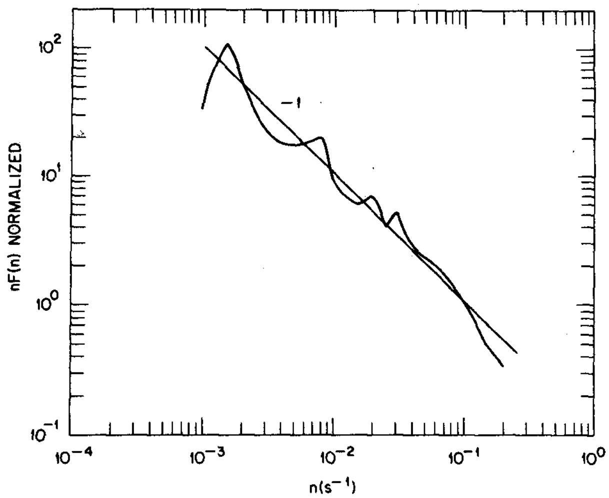

FIG. 3. Observed average energy spectrum for tetroons T3, T6, T10, T11 and T13, where each spectrum was normalized beforehand to $nF(n) = 1$ at $n = 0.1\ s^{-1}$.

the Lagrangian autocorrelogram $R_L(t)$ and the energy spectrum $F_L(n)$. This algorithm removes linear trends from the data. These results were compared with Eulerian autocorrelograms and spectra from the tower and the aircraft. When the mean balloon height was <300 m, the sonic anemometer data from tower levels 100, 200 or 300 m were used. The top tower level (300 m) was used for comparison with balloon runs with mean heights > 300 m. It is assumed that turbulence characteristics in the daytime mixed layer are fairly independent of height (Kaimal *et al.*, 1976), and therefore statistics of balloon measurements at, say, 500 m, could safely be compared to statistics measured at the 300 m level on the tower.

b. Tetroons

Radial velocity, signal intensity, and position information were provided by Wave Propagation Laboratory. After these data were edited to remove obvious errors, the velocity data were then subjected to the same FFT program used for the pilot balloon observations, yielding autocorrelograms and spectra. It was found that most of the balloons reached an equilibrium level within the mixed layer.

c. Determination of autocorrelogram and spectral time scales

It is difficult to integrate the autocorrelograms to give the Lagrangian time scale defined by Eq. (4), since the observed $R(t)$ curves did not generally approach zero at the largest time lags available with these sampling times. It was therefore arbitrarily assumed that T_L corresponds to the time lag at which $R(t)$ first drops to 0.37, which is valid for an assumed exponential correlogram, $R(t) = \exp(-t/T_L)$.

Furthermore, it is not easy to estimate the frequency, n_{mL}, at which the spectral peak, $n_{mL}F(n_{mL})$, occurs, for the spectral peak usually is spread over an order of magnitude range in frequencies, and there can be irregularities caused by statistical uncertainty in the calculated spectrum in this range. Usually these irregularities have little physical significance, and the spectral peak can be estimated by eye.

The inverse of n_{mL} is the time period T_{mL} at which peak spectral energy occurs. It is found that the spectral peak T_{mL} is larger than the Lagrangian time scale T_L. This can be shown by assuming an exponential correlogram and integrating it to obtain a formula for the spectrum. The peak of the resulting spectrum is given by the relation

$$T_{mL} = 6.3T_L \quad [\text{for } R(t) = \exp(-t/T_L)]. \quad (10)$$

On the other hand, a pure sinusoidal velocity fluctuation with period T_{mL} yields $R(t) = 0.37$ at a value of T_L such that the following relation is valid:

$$T_{mL} = 5.3T_L \quad [\text{for } v'(t) = A \sin 2\pi t/T_{mL}]. \quad (11)$$

It is seen that the methods described in this section for determining the time scales of the autocorrelograms and spectra are arbitrary. A basic problem is that there is turbulent energy in the atmosphere at scales larger than the sampling time. Although linear trends are removed from the data, there are still trends present to some degree. Another problem is that atmospheric turbulence energy spectra are nor inherently smooth, continuous and well-behaved.

4. Results

a. Lagrangian spectrum in inertial subrange

In the inertial subrange, similarity theory predicts that the Lagrangian energy spectrum should be given by the formula $nF_L(n) = B\epsilon n^{-1}$.

The tetroon spectra extend to higher frequencies ($n \approx 1.0\ s^{-1}$) than the pilot balloon spectra ($n \approx 0.03\ s^{-1}$), and are therefore more likely to show an inertial subrange. The averaged spectrum from five tetroon runs is plotted in Fig. 3, where each spectrum was normalized so that $nF_L(n) = 1.0$ at $n = 0.1\ s^{-1}$. The other three tetroon runs were not included because of obvious errors in the spectra at high frequencies. The predicted -1 power law is followed over two orders of magnitude of the abscissa, n. The drop off in the spectrum at very high frequencies ($n > 0.1\ s^{-1}$) is probably due to poor response of the 1 m^3 tetroons to eddies smaller than the tetroon.

The constant B was evaluated by first calculating the eddy dissipation rate ϵ from the inertial subrange of the observed crosswind component Eulerian spectrum [Eq. (5)]. Eulerian spectra were measured using sonic anemometers at the appropriate height on the meteorological tower. These spectra were then compared with observed Lagrangian spectra for the same time period, and the constant B was cal-

TABLE 3. Calculation of constant B in $nF_L(n) = B\epsilon n^{-1}$.

Run	Component	U (m s^{-1})	Mean balloon height (m)	ϵ (m^2 s^{-3})	B
Tetroon 11	v	10.0	300	0.0035	0.20
Tetroon 13	v	1.3	300	0.0025	0.18
Pibal 221	w	1.3	100	0.0034	0.12
Pibal 222	w	1.5	100	0.0084	0.11
Pibal 223	w	2.8	300	0.0032	0.31

culated from Eq. (6). The two tetroon and three pilot balloon runs that were used in these calculations have the longest inertial subrange of any of the runs for which simultaneous Eulerian spectra are available. The important parameters are summarized in Table 3. The range of the parameter B is surprisingly small for these runs, giving an average value of B equal to 0.2.

b. *Displacement of Lagrangian and Eulerian spectra*

Hay and Pasquill (1959) hypothesize that Lagrangian and Eulerian spectra are displaced from each other along the frequency axis by a factor β, as shown schematically in Fig. 1. Examples of Lagrangian-Eulerian spectra comparisons for tetroon 13 and pilot balloon 222 are given in Figs. 4 and 5, respectively. In the inertial subrange of these figures, the Lagrangian balloon spectra are displaced a factor of 3 or 4 to the left of the Eulerian tower spectra. The maximum spectral energies of the balloon and tower spectra are about equal, but the smoothed peaks of the Lagrangian spectra are displaced a factor of 2 to 4 to the left of the peak of the Eulerian spectra. No significance should be given to the minor variations on the spectral curves.

c. *Estimation of* $\beta = T_L/T_E$

There were 18 pilot balloon runs and three tetroon runs (T10, T11, T13) for which simultaneous tower

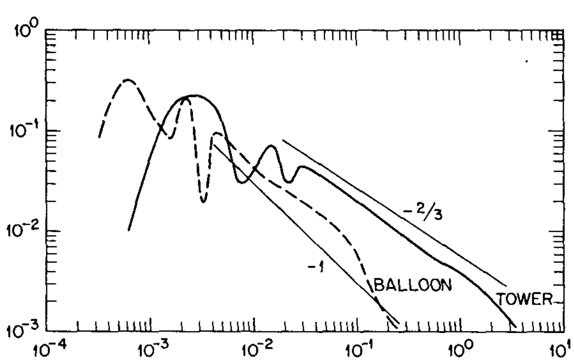

FIG. 4. Lagrangian (tetroon run T13, average of radar 3 and 4) and concurrent Eulerian (300 m tower sonic anemometer v component) energy spectra.

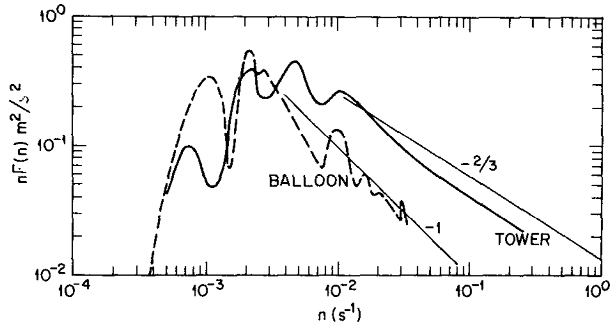

FIG. 5. Lagrangian (pibal run 222) and concurrent Eulerian (300 m tower sonic anemometer) energy spectra for w component.

anemometer data were available. The pilot balloon and tower data could be compared for all three components of the wind, while the tetroon and tower data could be compared for only the v component.

First the autocorrelograms were studied to determine the value of the time lag, T_L or T_E, at which the autocorrelogram first dropped to $0.37(1/e)$. The average time scale and the corresponding average β's are listed in Table 4. There is very little difference among the u, v and w components, as would be expected in daytime conditions, and the average β is ~ 1.6.

In the next part of the study, the peak periods T_{Lm} and T_{Em} of the energy spectra were estimated and converted to time lags T_L and T_E by dividing by six [Eqs. (10) and (11)]. These data are summarized in Table 5. The average β calculated from the spectra for daytime conditions during this experiment is ~ 1.8. The time scales and β's calculated from the autocorrelograms and spectra are clearly very similar ($\pm 10\%$ agreement) leading to the recommendation that it would be more efficient to determine these parameters from autocorrelograms rather than the much more difficult-to-calculate spectra. Of course, trends should be removed from the data before any technique is used.

The theories mentioned in Section 3 all lead to the prediction that $\beta = T_L/T_E$ is roughly inversely proportional to turbulence intensity, $i = \sigma_v/u$ or σ_w/u. Values of T_L and T_E from the spectra and turbulence intensities calculated from balloon data were used to construct the plot of β versus $1/i$ in Fig. 6. In all cases the turbulence intensity for the speed component being studied is used. There is

TABLE 4. Summary of average T_L, T_E, β from autocorrelograms.

Component	Balloon T_L (s)	Tower T_E (s)	β
Pibal u	67	45	1.5
Pibal v	78	54	1.4
Pibal w	82	52	1.6
Tetroon v	88	50	1.8
Overall	79	50	1.6

TABLE 5. Summary of average T_L, T_E, β from spectra.

Component	Balloon T_L (s)	Tower T_E (s)	β
Pibal u	90	42	2.1
Pibal v	87	43	2.0
Pibal w	80	57	1.4
Tetroon v	62	35	1.8
Overall	80	44	1.8

clearly a lot of scatter, as was the case in similar plots by Angell (1964). The theoretical formula $\beta = 0.4/i$ preferred by others best fits the few points at high $1/i$ (high wind velocities), but underestimates the points at low $1/i$. The linear formula that fits these data is

$$\beta = 0.7/i \tag{13}$$

or

$$\frac{T_L}{T_E} = \frac{0.7u}{\sigma}.$$

This relationship also implies an increase of β as stability increases during the daytime, since high winds (low i) during the daytime reflect nearly neutral conditions and light winds (high i) reflect unstable conditions. From Fig. 6 it can be concluded that the neutral value of β is 4. The values of β found in this study agree roughly with those given by Pasquill (1974).

Most β calculations are made using fixed point Eulerian measurements, which yield spectra as functions of frequency. Aircraft spectra, on the other hand, yield Eulerian spectra as functions of wavenumber. Eulerian spectra from the NCAR aircraft were available only on 21 September, when they overlapped three pilot balloon runs. Because of the light and variable winds on that day it was difficult to transfer the abscissa in the aircraft spectra from wavenumber to frequency. Consequently, the calculated value, $\beta = 0.5$, must be regarded as uncertain within a factor of about 5. This comparison would be more valid on days with steady moderate winds.

d. Estimation of Lagrangian time scale

The results in Tables 4 and 5 show that you can calculate the Lagrangian time scale T_L from the Eulerian time scale T_E in daytime conditions in the planetary boundary layer by multiplying T_E by ~1.7. But suppose Eulerian observations are not available? In this case, Eqs. (8) and (9) can be multiplied by β and (13) used to obtain the predictions:

$$\begin{aligned} T_L &= \beta T_E \\ &= 0.25\beta z_i/u \\ &= 0.17 z_i/\sigma_{u,v,w} \end{aligned} \left(\begin{array}{ll} u: & z < z_i \\ v: & z < z_i \\ w: & 100\text{ m} < z < z_i \end{array} \right), \tag{14}$$

$$\begin{aligned} T_L &= 0.60\beta \frac{z}{u_*} \qquad (w: \quad z < 100\text{ m}), \\ &= 0.42(z/\sigma_w)(u_*/u). \end{aligned} \tag{15}$$

Eq. (14) can be tested using our data, since observations of mixing depth z_i were available on several days. Fig. 7 contains z_i estimated by Kaimal *et al.* (1980) using eight different instruments for the days 21 and 22 September. The line gives the best estimate of z_i which varies from ~200 m at sunrise to ~1000 m at mid afternoon on both days.

In Table 6 calculated and observed Lagrangian time scales are given for nine pibal runs on these two days. The observed T_L are obtained from the autocorrelograms. The only inconsistent run is number 217, which occurs late in the afternoon when the surface layers are starting to stabilize and z_i is not well known. Leaving out run 217, the mean calculated and observed Lagrangian time scales are 80 and 70 s, respectively. The theoretical formula appears to be adequate (usually within a factor of 2) for estimating T_L in these conditions.

The value of 70 or 80 s for T_L is in agreement with values inferred from diffusion data analyzed by Doran *et al.* (1978) and Draxler (1976). By assuming that Taylor's diffusion equation with an exponential correlogram, $R = \exp(-t/T_L)$, is valid for their data, it is possible to obtain daytime estimates of T_L equal to 70 s.

5. Further comments

The analyses in this paper are all based on data samples ranging from 10 min to 1 h, with the most common sample length being ~25 min. Of course, the atmosphere contains much energy at periods > 25 min, and these larger eddies would influence the Lagrangian time scale T_L for larger sample times. The value of T_L of 70 s that we found is a good esti-

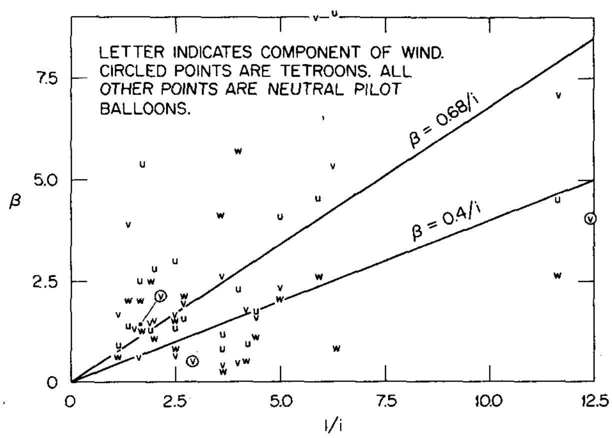

FIG. 6. Observed ratios $\beta = T_L/T_E$ plotted versus inverse turbulence intensity $1/i = u/\sigma_w$ or u/σ_u. Lagrangian time scales were obtained from the autocorrelograms. The letter at each point represents the velocity component. A circled letter indicates a tetroon.

mate for typical micrometeorological and diffusion experiments, where sampling times are usually 10 min to 1 h. For much larger sampling times, say 24 h, a larger value of the Lagrangian time scale T_L should be used, based on a different set of measurements.

A final comment is that uncertainty estimates should be attached to each of the numbers given in this paper. The individual measurements of T_L and T_E are probably accurate within a factor of 2 or 3. On the average, some of these random errors should cancel out, so that final estimates of $\bar{\beta}$ and $\bar{T}_L$ probably have accuracies of ±50%.

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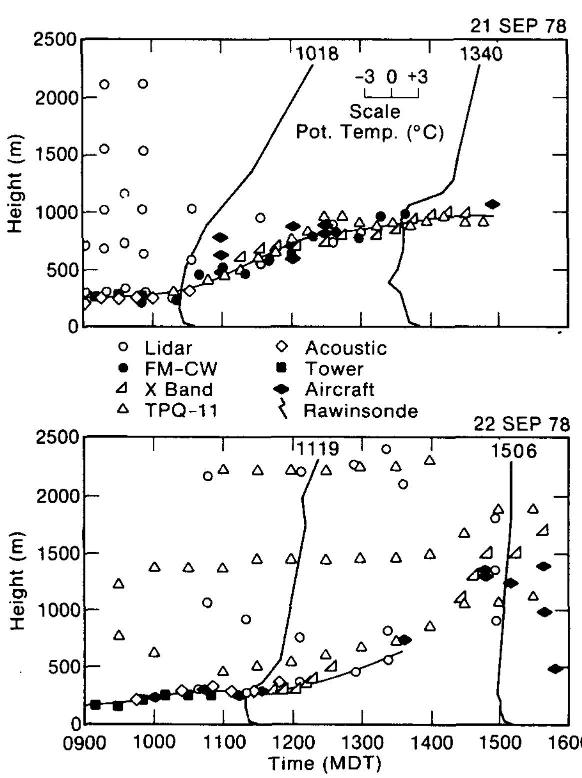

FIG. 7. Estimates of mixing depth z_i from several different instruments for 21 and 22 September (after Kaimal *et al.*, 1980).

TABLE 6. Observed and calculated T_L.

Run	Midtime (MDT)	z_i (m)	σ_w (m s^{-1})	Calculated $T_L = 0.17z_i/\sigma_w$	Observed T_L
211	1045	400	0.85	80	70
212	1145	700	0.97	120	30
213	1310	800	1.66	80	70
214	1410	900	1.69	90	110
216	1540	1000	1.43	120	70
217	1630	1000	0.58	290	25
221	0950	250	0.69	60	60
222	1130	300	0.89	50	70
223	1230	400	1.12	60	110

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