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Estimating Intensity of Atmospheric Ice Accretion on Stationary Structures

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ABSTRACT

The role of various atmospheric parameters in determining atmospheric ice accretion intensity on structures near the ground is examined theoretically, with an emphasis on glaze formation. Methods are presented for calculating the icing rate on cylindrical objects, and estimates of maximum deposition intensities are made. A relationship between meteorological conditions and the type of ice formation (glaze and rime) is given. The lack of adequate experimental data limits verification of the theory, but some comparisons, mainly qualitative, are promising.

1. Introduction

Ice accretion on structures is a major source of financial losses and operating difficulties, and may even constitute a risk to life in some cases. The phenomenon has therefore been examined widely both experimentally and theoretically. Understanding of the ice accretion problem has attained its present level mainly from studies of hailstone growth and aircraft icing. Icing of stationary objects like masts, power lines, trees, automatic meteorological stations, etc., has, however, been the subject of less research—especially as far as the theoretical aspects of the accretion process are concerned. As a result, many of the attempts to describe ice accretion physics have proved unsatisfactory, and hence the inability to predict ice accretion rates from meteorological data currently has been pointed out in the literature (e.g., McKay and Thompson, 1969; Raevskii and Prokhorenko, 1977).

This study examines ice accretion due to supercooled water droplets moving in the air stream. Snow accretion and glaze formation due to rain accompanied by weak winds are not considered. These factors can cause ice loads in some cases, but their contribution to icing on structures seems to be statistically of minor importance in most parts of the world (e.g., Lomilina, 1977; Kemp, 1980). The intensity of direct vapor sublimation is insignificant compared to that of rime and glaze formation. The effect of ice crystals in mixed conditions also is disregarded since recent wind tunnel tests have indicated that it is not as critical as previously assumed (Ashworth and Knight, 1978; Ackley and Templeton, 1979[1]). As icing on structures is for the most part caused by supercooled droplets blown by the wind, it is more significant on vertical than on horizontal surfaces. In fact, it has been found in measurements by the Finnish Meteorological Institute in Lapland (unpublished) that the icing rate per unit area on vertical surfaces may exceed the total precipitation by more than ten times even in one month. According to Wozniak (1975), in some mountain regions ~50% of the available water may originate from rime.

The theoretical considerations presented in studies of hailstone growth (e.g., Macklin, 1978) and aircraft icing (e.g., Cansdale and McNaughtan, 1977)[2] can largely be applied to the problems of atmospheric ice accretion on stationary structures. There are some special features of ice growth conditions near the ground, however, which have to be recognized and which make it possible to simplify certain aspects of the theoretical description of the ice accretion process. These include the smaller liquid water content and relative wind speed, and the lack of rotation and noticeable air pressure variations. In addition, the lower intensity of accretion makes it possible to make some simple analytical estimates of the intensity of accretion instead of the time-dependent simulation that must be used in helicopter rotor blade icing studies (Ackley and Templeton, 1979).[1]

2. Physical nature of the ice accretion process

The growth of ice on a surface due to the accretion of supercooled water droplets occurs in either of two

[1] Ackley, S. F., and M. K. Templeton, 1979: Computer modeling of atmospheric ice accretion. Cold Regions Res. Eng. Lab., Rep. 79-4, 36 pp.

[2] Cansdale, J. T., and I. I. McNaughtan, 1977: Calculation of surface temperature and ice accretion rate in a mixed water droplet/ice crystal cloud. Farnborough, Royal Aircraft Establishment, Tech. Rep. 77090, 29 pp.

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basically different regimes: dry growth and wet growth.

In the dry-growth process all impinging water freezes and there is no runoff from the surface. This means that an impinging droplet freezes completely before another droplet hits the area covered by the first one. The temperature of the deposit surface in dry growth is below 0°C and the ice formed is porous and opaque (see Macklin and Payne, 1968). In the dry-growth process the problem in calculating the icing intensity is how to determine the amount of impinging water. Hence, the intensity I (g cm^{-2} h^{-1}) of accretion can be calculated using the formula

$$I = EvW, \tag{1}$$

where v is the wind speed relative to the surface, W is the liquid water content and E is the collection efficiency, i.e., the ratio of the mass flow of water droplets striking the surface to the mass flow that would have struck the surface if the droplets had not been deflected in the air stream.

The collection efficiency E is dependent mainly on the wind speed and on the dimensions of the droplets and the icing structure. Because the dependence of E on these parameters is best known for cylinders, and because many of the objects often affected by icing have a cylindrical form, the following theoretical treatment is restricted to cylindrical structures. It is further restricted to the stagnation line only, because the growth rate of ice on cylinders usually has its maximum value near the stagnation region and since the determination of E is most accurate in that region. Using information on the form of the deposit it is then possible to calculate the rate of increase in the weight of ice. The determination of E at the stagnation line is based on the calculation of the droplet trajectories by integrating the equation of motion in potential flow around a cylinder. The analytical expressions for a calculation of E, given by Cansdale and McNaughtan (1977)[2]—based on the results of Langmuir and Blodgett (1946)[3]—have been used here. In the data of Langmuir and Blodgett the drag coefficient of the droplets as a function of the droplet Reynolds number is very similar to that obtained later by Beard and Pruppacher (1969). For natural fog conditions the collection efficiency can be calculated separately for each size category, and the total value of E is then the sum of E values for each size multiplied by the fraction of the total liquid water content represented by that size.

If the probability of all the latent heat liberated in the freezing of a droplet to be dissipated to the environment before another droplet hits the same spot is small, a water film exists on the ice surface and part of the impinging water runs off from the surface. Then the ice forms by the wet-growth process, and the temperature of the icing surface is 0°C. The glaze thus formed is transparent and its density is 0.92 g cm^{-3} (see List, 1963). In the wet growth process the mass balance is clearly not the only factor governing the icing intensity; the energy balance of the icing surface (water film) is also involved.

It has been found in hailstone growth simulations that the excess water—instead of being shed—may be incorporated into the ice structure, giving a spongy ice deposit. Observations of unfrozen water in rime deposits on stationary structures under natural conditions have, however, not been reported (with the exception of snow accretion), which is probably because the values of the atmospheric parameters in near-ground conditions are quite different from those prevailing during hailstone growth.

3. Intensity of accretion in the wet-growth process

In the wet-growth process, which involves the loss of unfrozen water, the intensity of ice accretion can be formulated as

$$I = E_a E v W, \tag{2}$$

where E_a is the accretion efficiency, i.e., the ratio of the icing intensity to the mass flow of the impinging water. In (2), E_a is assumed to be determined by the heat balance of the water film on the icing surface only. Eq. (2) can also be expressed in terms of the icing efficiency $E_i = E_a E$. Accordingly E_i is the ratio of the icing intensity to the mass flow of droplets that would have struck the surface if they had not been deflected in the airstream, and also must be a function of the factors controlling the heat balance of the icing surface.

The heat balance equation for the water film on the icing surface in the wet growth process is

$$q_f + q_v + q_k = q_c + q_e + q_w + q_r + q_s + q_i, \tag{3}$$

where

- q_f latent heat released during freezing
- q_v frictional heating of air
- q_k kinetic energy of the impinging water
- q_c loss of sensible heat to air
- q_e evaporative heat loss
- q_w heat loss in warming to 0°C the mass of water that does freeze
- q_r heat loss in warming the runoff water to the temperature it has when leaving the area of the surface under consideration
- q_s heat loss due to radiation
- q_i heat loss to the substrate due to conduction.

The terms in (3) can be parameterized as shown in Eqs. (4)–(10):

$$q_f = IL_f, \tag{4}$$

[3] Langmuir, I., and K. B. Blodgett, 1946: A mathematical investigation of water droplet trajectories. U.S.A.A.F. Tech. Rep. 5418, 65 pp.

where I is the intensity of accretion (mass per unit time and unit area) and L_f the latent heat of fusion;

$$q_v = hrv^2/2c_p, \qquad (5)$$

where h is the heat transfer coefficient, r the recovery factor for viscous heating ($r = 0.9$), v the wind velocity and c_p the specific heat of air at constant pressure. The kinetic energy of the droplets q_k can safely be neglected on stationary objects under natural atmospheric conditions:

$$q_c = h(0°\mathrm{C} - t_a), \qquad (6)$$

where t_a is the air temperature (°C); and

$$q_e = hkL_e(e_0 - e_a)/c_p p_a, \qquad (7)$$

where $k = 0.62$, L_e is the latent heat of evaporation, e_0 and e_a are the saturation vapour pressures over water at 0°C and at t_a, respectively, and p_a is the free atmospheric pressure. The temperature of the droplets in the free stream is very nearly the same as that of air. Hence

$$q_w = Ic_w(0°\mathrm{C} - t_a), \qquad (8)$$

where c_w is the specific heat of water. The temperature of the runoff water leaving the deposit has a considerable effect on the ice accretion, and its determination has been seen as a serious difficulty in estimating the accretion intensity in the wet growth process (e.g., List, 1977). The problem is much simplified, however, if we consider the stagnation area only, because the water is lost within the water film and its temperature is therefore that of the surface, *viz.*, 0°C, unless bouncing of the impinging droplets occurs. No evidence of noticable bouncing has been reported near the stagnation line in the conditions corresponding to atmospheric icing on stationary objects. Hence

$$q_r = c_w(EvW - I)(0°\mathrm{C} - t_a). \qquad (9)$$

The radiation budget at the surface can be estimated, as a first approximation, by neglecting the shortwave radiation in fog conditions, and assuming that the emissivity of the fog in the horizontal direction approaches unity. Linearizing the equation for the difference in the emitted radiation of the icing surface and the fog we obtain

$$q_s = \sigma n(0°\mathrm{C} - t_a); \qquad (10)$$

σ is the Boltzmann constant and $n = 8.1 \times 10^7$ K^3. According to experimental results (see, e.g., Schlichting, 1979) the Nusselt number Nu ($=hD/k_a$) and the Reynolds number Re ($=vD\rho_a/\mu_a$) in a flow around a cylinder are related at the stagnation point by

$$\mathrm{Nu} = \mathrm{Re}^{1/2}. \qquad (11)$$

Using this result the local heat exchange coefficient h in Eqs. (5)–(7) can be expressed as

$$h = k_a(v\rho_a/D\mu_a)^{1/2}. \qquad (12)$$

Here k_a is the molecular thermal conductivity, ρ_a the air density, μ_a the molecular viscosity of air and D the diameter of the cylinder examined. The conductive term q_i is difficult to parameterize, since it is dependent on the thermodynamic properties of the object undergoing icing. The treatment must therefore be limited to cases, where the conductivity of the structure undergoing icing is low, e.g., it is insulated or hollow, or to cases where icing has been going on for a sufficient time for an ice deposit several centimeters thick to develop.

The relative magnitude of the heat balance terms is largely dependent on the environmental conditions. In general, it can be established that the term q_f is the major gain of heat and that q_c and q_e are usually the dominating heat loss terms $-q_w$ becoming, however, more important with increasing liquid water content, and q_s with decreasing wind speed. The term q_v can be neglected except when the wind speed is very high and the air temperature close to 0°C.

Using the parameterizations in (3) and neglecting q_k and q_i an analytical expression for the intensity I is found:

$$I = k_a\left(\frac{v\rho_a}{D\mu_a}\right)^{1/2} L_f^{-1}\left[-t_a + \frac{kL_e}{c_p p_a}(e_0 - e_a) - \frac{rv^2}{2c_p}\right] - L_f^{-1}(EvWc_w + \sigma n)t_a. \qquad (13)$$

The values of k_a, ρ_a, μ_a, c_p and e_a in (13) are dependent on the air temperature and can be found from tables or expressed in analytical form in computer simulations. L_f, L_e, c_w, e_0, p_a, σ, n and r can be considered constants. With information on the droplet size spectrum and cylinder diameter, the collection efficiency E can be found as indicated in Section 2. Using Eq. (13) the intensity of accretion in the wet growth process on the stagnation line of a cylinder of an arbitrary diameter can thus be estimated as a function of air temperature t_a, wind speed v, liquid water content W and droplet diameter d.

4. Critical conditions and maximum intensity

The conditions under which the ice accretion process changes from wet growth to dry growth or vice versa can be found by equalizing the intensities in (1) and (13). By doing so and taking into account that at the boundary between dry and wet growth $q_r = 0$, we obtain the following expression for the critical liquid water content W_c:

$$W_c = \frac{k_a}{E}\left(\frac{\rho_a}{vD\mu_a}\right)^{1/2} \frac{-t_a + \dfrac{kL_e}{c_p p_a}(e_0 - e_a) - \dfrac{rv^2}{2c_p}}{L_f + c_w t_a} - \frac{\sigma n t_a}{Ev(L_f + c_w t_a)}. \qquad (14)$$

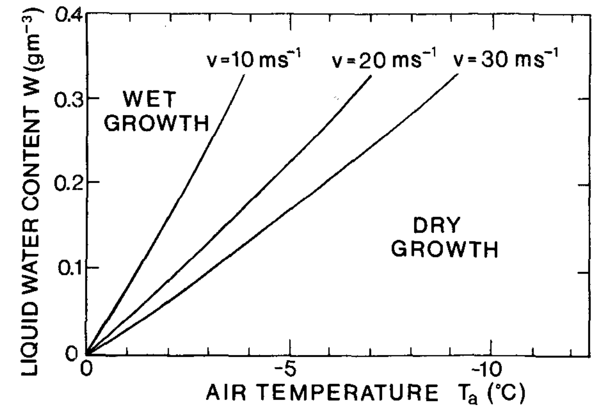

FIG. 1. The lines separating the dry-growth and wet-growth processes on a 5 cm diameter cylinder, for various values of the wind speed v. The droplet diameter is 30 μm.

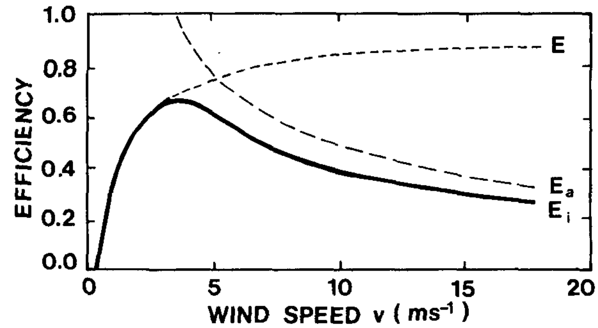

FIG. 3. Icing efficiency E_i, accretion efficiency E_a and collection efficiency E on the stagnation line of a 15 mm diameter cylinder as a function of the wind speed. The air temperature is -1°C, liquid water content 0.2 g m^{-3} and droplet diameter 30 μm.

In a similar manner other critical parameters such as $t_{a,c}$ or v_c can be solved—although not analytically—for given fixed values of other parameters. In Fig. 1 the critical line is presented for different wind speed values in t_a, W coordinates. The droplet diameter of 30 μm, used for calculations in Fig. 1, is a typical value of median volume diameter in fogs with a temperature somewhat below 0°C (Kuroiwa, 1965[4]; Kumai, 1973).

Having established the estimation formula for the icing intensity for dry growth (1), wet growth (13) and critical conditions (14), it is now possible to make estimates for the whole range of atmospheric conditions. This is done in Fig. 2 by presenting the icing intensity as a function of the air temperature for different wind-speed values, using the liquid water content value 0.3 g m^{-3}, which can be considered a practical upper limit of W in supercooled fogs near the ground (see, e.g, Kumai, 1973; Pinnick *et al.*, 1978). An example of the dependence of the accretion intensity on the wind speed, with fixed values of other parameters, is given in Fig. 3 in terms of the icing efficiency E_i. In Fig. 3 the icing efficiency E_i increases with increasing wind due to an increase in the collection efficiency $E(E_a = 1$ in dry growth) up to the wind speed value ($\sim$4 m s^{-1} in Fig. 3) where the change to the wet-growth process occurs, and then decreases due to the decreasing accretion efficiency.

[4] Kuroiwa, D., 1965: Icing and snow accretion on electric wires. Cold Regions Res. Eng. Lab., Res. Rep., 123, 10 pp.

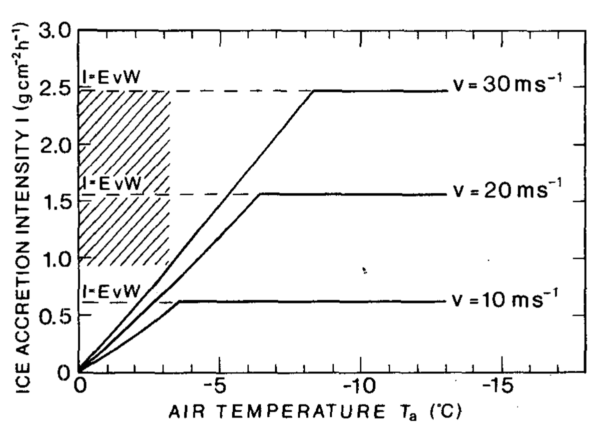

FIG. 2. Ice accretion intensity in the stagnation region of a 5 cm diameter cylinder as a function of the air temperature for various values of the wind speed v. The liquid water content is 0.3 g m^{-3} and the droplet diameter 30 μm. The hatched area represents one example of the quantity $(EvW - I)t_a$ for v = 30 m s^{-1} (see text).

The limited value of the liquid water content W in near-ground atmospheric icing can be used in further estimating the terms in the heat balance equation. On the basis of the form of the wet growth curves it can be shown that the quantity $|(EvW - I)t_a|$ represented by the hatched area in Fig. 2 has its maximum when I is approximately $EvW/2$. With this I value $|t_a|$ is <5°C for all wind speed values, and it follows that $|q_r/q_f| < 5°\text{C} \times c_w/L_f = 6 \times 10^{-2}$. For smaller values of W the fraction $|q_r/q_f|$ is even smaller. The change to smaller structures or larger droplets does not substantially increase the value q_r/q_f from that in Fig. 2. Being thus able to neglect the heat lost by runoff water in the wet-growth process in atmospheric icing on structures near the ground is of great practical importance since it allows the formulation of the intensity of wet growth accretion, which is independent of the liquid water content and the collection efficiency:

$$I = \frac{h\left[-t_a + \dfrac{kL_e}{c_p p_a}(e_0 - e_a) - \dfrac{rv^2}{2c_p}\right] - \sigma n t_a}{L_f + c_w t_a}. \quad (15)$$

The advantage of (15) is that the properties of the fog—which are usually not known, and are extremely difficult to forecast—need not be considered, apart from the fact that the existence of supercooled droplets is necessary for icing to occur. Because the maximum intensity of accretion at fixed values of v and t_a is reached in the wet-growth process, and since the solutions of (14) and (15) differ insignificantly under the conditions in question, it is concluded that the maximum intensity for known values of air temperature and wind speed can be approximated from the curves in Fig. 2. If the droplet diameter or the cylinder size were different from those in Fig. 2, the corresponding curves could be produced with Eqs. (14) and (15).

5. Discussion

The methods suggested provide a means of estimating the intensity of ice accretion on structures, trees, etc. If the liquid water content of rain or drizzle is lower than 0.3 g m^{-3} (for estimation methods see, e.g., McKay and Thompson, 1969), the theory also may be applied to the cases of freezing precipitation accompanied by strong wind. The most restricting hypothesis in the theory is probably the neglecting of the heat conducted into the structure. However, the seriousness of this for real objects is limited in view of the results for the air-temperature range where the wet-growth process is possible under near-ground icing conditions. Another source of inaccuracy may be the effect of a change in the shape and dimensions of the deposit on the collection and heat transfer coefficients during long-term icing. This can, in computerized simulation, easily be taken into account by stepwise calculation, providing that the deposit remains roughly cylindrical. As this is not always the case, data on the effects of deposition shape on the collection and heat transfer properties would be needed. The effects of surface roughness and free-stream turbulence on these properties also should be investigated, although they seem not to be critical in the stagnation region in real atmospheric conditions (Achenbach, 1977; Schlichting, 1979, p. 573).

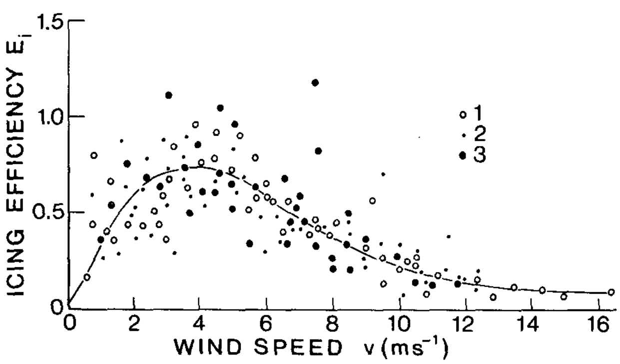

FIG. 4. Dependence of the overall icing efficiency on the wind speed for various liquid water contents W according to Glukhov (1971), where 1) W = 0.12–0.16 g m^{-3}, 2) W = 0.17–0.21 g m^{-3}, 3) W = 0.22–0.26 g m^{-3}.

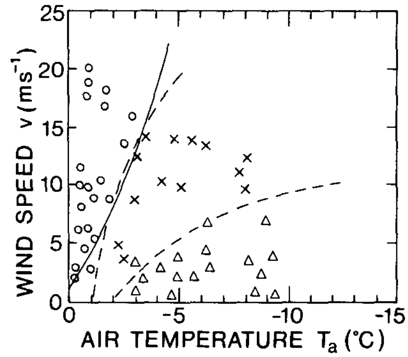

FIG. 5. Relationship between meteorological conditions and the type of icing according to observations [Kuroiwa (1965) dashed lines, (○) glaze, (×) hard rime, (△) soft rime] and the line separating the wet-growth and the dry-growth processes for D = 5 cm, d = 30 μm and W = 0.2 g m^{-3} according to the present theory (solid line).

A comparison of the theory with laboratory measurements has given some promising results, but in terms of icing under natural conditions, there is little adequate data. For quantitative comparisons the data on the intensity of accretion with simultaneous measurements of air temperature, wind speed, liquid water content and droplet size spectrum are essential. In the ice accretion measurements under natural conditions the last two parameters, and often also the exact duration of icing, are usually not observed. The comparison with observations is therefore limited mainly to qualitative relationships between the intensity and the meteorological conditions, and to the critical conditions.

The data of Glukhov (1971) in Fig. 4, used in formulating a regression model connecting the overall icing efficiency and the wind speed, may be compared with the results of the present model shown in Fig. 3. The large scatter of the points in Fig. 4 is mainly due to the differences in the air temperature, which were not taken into account by Glukhov (1971). No quantitative comparison is therefore possible, but the qualitative agreement with Fig. 3, where a constant temperature is used, is good. From the theoretical point of view it is interesting to note that Glukhov's regression formula (represented by the solid curve in Fig. 4) is independent of liquid water content, too. From this it follows that in his method —in which E_i is calculated from the regression formula and then used in Eq. (2)—the ice accretion intensity is directly proportional to the liquid water content for all the wind speed values. The present theory suggests that there is no such dependence in the wet growth process.

A test of the ability of the present theory to predict the temperature and wind conditions for glaze and rime formation is presented in Fig. 5. The data of Kuroiwa (1965) in Fig. 5 does not include the liquid water content W or the exact deposit diameter D and it is hence not directly comparable to the theoretical line with arbitrary values of W and D. However, it is interesting to note the qualitative agreement.

As for the dry growth process only, the theoretical results coincide with the data obtained from the measurements by the Finnish Meteorological Institute in Lapland (unpublished). No dependence of icing intensity on air temperature was found and the intensity was approximately linearly dependent on the wind speed, following the regression function $I = 11 \times 10^{-3}\ v$ (where I is in g cm^{-2} h^{-1} and v is in m s^{-1}). The correlation coefficient between the amount of ice on a 5 cm cylinder and the wind speed multiplied by the duration of the accretion was 0.69, indicating that a crude estimation of the icing intensity in the dry growth process is possible using the wind speed only—in spite of the variations in the liquid water content and the droplet size under natural conditions. The results for a 10 cm diameter cylinder obtained by Baranowski and Liebersbach (1977) are very similar to the Finnish results, giving $I = 7.5 \times 10^{-3}\ v$ for soft rime and $I = 15 \times 10^{-3}\ v$ for hard rime, the correlation coefficients of these regression functions being 0.60 and 0.50, respectively.

For future comparisons of a more quantitative nature, extensive ice accretion measurements including the liquid water content and droplet size spectrum determinations should be made. On this basis it may be possible to approach the problem of practical forecasting of ice loads using, say, climatological data. Before that, however, a good deal of work is needed in examining the cross correlations between the parameters involved in the ice accretion physics, for example, the dependence of the liquid water content and droplet size on the air temperature and wind speed in in the atmospheric surface layer.

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