Temperature and Humidity Effects on Refractive Index Fluctuations in Upper Regions of the Convective Boundary Layer

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ABSTRACT

Here we illustrate a method which readily permits determination of the relative contributions of the individual temperature-humidity structure terms to total $C_n^2$ within the uppermost region of the clear, convective boundary layer. The relative contributions of terms involving $C_r^2$, $C_{T_q}$ and $C_{q^2}$ to acoustic, optical and microwave $C_n^2$ are shown to be functions primarily of the ratio, $\Delta q/\Delta \Theta_v$, of humidity to virtual potential temperature jump across the inversion. A graphical procedure is illustrated for quickly determining the expected degree of error if $C_r^2$ or $C_{q^2}$ are directly inferred from $C_n^2$.

1. Introduction

Often in remote sensing applications it is desirable to infer individual temperature and humidity structure parameter values ($C_r^2$, $C_{T_q}$ and $C_{q^2}$) from the intensity of the return signal. The return signal, however, is proportional to the refractive index structure parameter $C_n^2$ and generally an assumption must be introduced in order to infer $C_r^2$ and $C_{q^2}$ from the measured $C_n^2$ value. For example, temperature fluctuations are often taken to be the sole contributor to acoustic $C_n^2$, and thus $C_r^2$ is inferred directly from an acoustic sounder return (Neff, 1975; Asimakopoulos et al., 1976). As another example, moisture fluctuations are often assumed to be the sole contributor to microwave $C_n^2$, and thus clear-air radar returns are used to infer $C_{q^2}$.

A simple method of quickly determining the extent of validity of the above-mentioned inferences appears desirable. Here we describe such a method for the interfacial layer of a clear, convective boundary layer. We combine the layer-averaged expressions for individual structure parameters developed by Wyngaard and LeMone (1980) with the $C_n^2$ relationships formulated by Wesely (1976).

2. Temperature-humidity contributions to $C_n^2$

Interfacial layer-averaged structure parameters are written by Wyngaard and LeMone (hereafter WL) as

$$
\langle C_r^2 \rangle = \frac{T_r \theta_v}{Z_{T_r}^{2/3}}, \tag{1}
$$

$$
\langle C_{T_q} \rangle = \frac{q_r \theta_v}{Z_{q_r}^{2/3}}, \tag{2}
$$

$$
\langle C_{q^2} \rangle = \frac{3.9(p\Delta q)^3 \theta_v}{Z_{q^2}^{2/3} \Delta \Theta_v}, \tag{3}
$$

where

$$
q_i = \rho \Delta q \left( 2.2 - 2.4 T \frac{\Delta q}{\Delta \Theta_v} \right),
$$

$$
T_i = \Delta \Theta_v \left[ 0.5 - 2.6 T \frac{\Delta q}{\Delta \Theta_v} + 1.4 \left( T \frac{\Delta q}{\Delta \Theta_v} \right)^2 \right].
$$

When combined with the Wesely (1976) relationships, we can evaluate the percent contribution of individual structure parameter terms to total $C_n^2$. Wesely writes

$$
(C_n^2)_a = (C_r^2/4T^2)\alpha_a^2, \tag{4}
$$

$$
(C_n^2)_o = \left( \frac{A_p}{T^2} \right) \alpha_o^2, \tag{5}
$$

$$
(C_n^2)_m = \left( \frac{C_p}{\epsilon T^2} \right) \alpha_m^2, \tag{6}
$$

where the subscripts $a$, $o$ and $m$ refer to acoustic, optical and microwave, respectively, and

$$
\alpha_a^2 = 1 + \frac{2DT}{\epsilon} \frac{C_{T_q}}{C_r^2} + \left( \frac{DT}{\epsilon} \right)^2 \frac{C_{q^2}}{C_r^2}, \tag{7}
$$

$$
\alpha_o^2 = 1 + \frac{2(1-A_2/A_1)T}{\epsilon} \frac{C_{T_q}}{C_r^2}
$$

$$
+ \frac{(1-A_2/A_1)^2T^2}{\epsilon^2} \frac{C_{a^2}}{C_r^2}, \tag{8}
$$

$$
\alpha_m^2 = 1 - \left[ \frac{2A_2}{C} + \frac{4 \epsilon \epsilon'}{pT} \right] \frac{C_{T_q}}{C_{q^2}}
$$

$$
+ \left[ \frac{A_2}{C} + \frac{2 \epsilon \epsilon'}{pT} \right] \frac{C_T}{C_{q^2}}. \tag{9}
$$

We choose to use specific humidity $q$ rather than absolute humidity $Q$, but otherwise the notation is the same as in WL (see the Appendix for a list of symbols).
The assumption frequently used to infer temperature or humidity structure parameters from remote sensor measurements is that these various \(\alpha^2\) values equal unity. In that event, optical and acoustic \(C^2\) are solely dependent on \(C_T^2\), and microwave \(C^2\) is solely dependent on \(C_T^2\). Here we wish to quantitatively evaluate the extent of validity of such assumptions concerning \(\alpha^2\) values. Further, in cases where \(C^2\) is not simply dependent solely on a single structure parameter, we seek a handy method of quickly evaluating the relative contribution of each temperature-humidity structure parameter to total \(C^2\). Thus, we now focus more closely on the individual \(\alpha^2\) values.

If we take interfacial averages of Eqs. (7)–(9), and utilize the WL expressions [Eqs. (1)–(3)], we find

\[
\langle \alpha^2 \rangle = 1 + \frac{2DT}{\epsilon} \frac{\rho r [2.2 - 2.4 Tr]}{0.5 - 2.6 Tr + 1.4 (Tr)^2} \\
+ \left( \frac{DT}{\epsilon} \right)^2 \frac{3.9 (\rho r)^2}{0.5 - 2.6 Tr + 1.4 (Tr)^2}, \tag{10}
\]

\[
\langle \alpha^2 \rangle = 1 + \frac{2(1 - A_2/A_3) T}{\epsilon} \\
\times \frac{\rho r [2.2 - 2.4 Tr]}{0.5 - 2.6 Tr + 1.4 (Tr)^2} + \frac{(1 - A_2/A_3) T}{\epsilon} \right]^2 \\
\times \left[ \frac{3.9 (\rho r)^2}{0.5 - 2.6 Tr + 1.4 (Tr)^2} \right], \tag{11}
\]

\[
\langle \alpha_m^2 \rangle = 1 - \left[ \frac{2Ae}{C} + \frac{4ee}{pT} \left[ \frac{2.2 - 2.4 Tr}{3.9 \rho r} \right] + \frac{Ae}{C} + \frac{2ee}{pT} \right]^2 \left[ \frac{0.5 - 2.6 Tr + 1.4 (Tr)^2}{3.9 (\rho r)^2} \right], \tag{12}
\]

![Fig. 1. Evaluation of the interfacial layer acoustic correction factor \(\langle \alpha_a^2 \rangle\) and optical correction factor \(\langle \alpha_o^2 \rangle\) as a function of \(r = \Delta q/\Delta q_o\). Here Eqs. (10) and (11) are evaluated with \(p = 900 \text{ mb}\) for several indicated temperatures.](image1)

![Fig. 2. Ratios of third term to second term in Eqs. (10)–(12) with \(|\alpha_a^2(3-2)|\) being the absolute magnitude of this ratio in Eq. (10), \(|\alpha_o^2(3-2)|\) for Eq. (11), and \(\alpha_m^2(3-2)\) for Eq. (12). Dominance of second term involving \(C_T^2\) is indicated in each case. Selected mean temperature, pressure and vapor pressure indicated on figure.](image2)
where we have taken $r = \Delta q/\Delta \Theta_v$. The striking (and useful) feature of these expressions is that the dependence on inversion height $z_i$ and surface virtual temperature scale $\theta_e$ has canceled out. Furthermore, we will show graphically that these $\langle \alpha^2 \rangle$ expressions are functions primarily of $r$, with only relatively weak dependencies on temperature, pressure and, in the case of $\langle \alpha_m^2 \rangle$, vapor pressure. Thus, for selected values of $T$, $p$ and $e$, we may evaluate Eqs. (10)–(12) for a wide range of interfacial layer conditions by varying the ratio $r$.

Figs. 1–4 present plots showing the nature of these $\langle \alpha^2 \rangle$ dependencies. We examine only the typical case for a convective boundary layer in which $\Delta q$ is negative, $\Delta \Theta_v$ positive, and therefore, $r$ negative.

Fig. 1 shows that the dependence on mean temperature of $\langle \alpha_a^2 \rangle$ and $\langle \alpha_e^2 \rangle$ is not strong. This figure also shows that when the humidity jump is small and the virtual potential temperature jump is large across the interfacial layer, $\langle \alpha_a^2 \rangle$ and $\langle \alpha_e^2 \rangle$ approach unity. This is reasonable since under such
conditions the humidity fluctuations would tend to
be small, while the temperature fluctuations would
be large, making \((C_n^2)_{a}\) and \((C_{r}^2)_{a}\) primarily
dependent on \(C_{r}^2\). At the other extreme of large \(\Delta q\)
and small \(\Delta \Theta_a\), Fig. 1 indicates the importance of
accounting for the terms involving \(C_{q}^2\) and \(C_{r}^2\)
when making the transformation from \(C_{r}^2\) to acoustic or
optical \(C_n^2\).

It may appear odd that the \(a^2\) values in Eqs. (4)
and (5) turn out to be less than unity. The term
involving \(C_{r}^2\), however, is negative when \(r\) is negative
and, further, this term tends to be larger in
absolute magnitude than the positive \(C_{q}^2\) term. The
dominance of the \(C_{r}^2\) term over the \(C_{q}^2\) term in Eqs.
(10)–(11) is displayed in Fig. 2. Plotted are the
absolute magnitudes of the ratios of the third to
second term in Eq. (10) [labeled \(|\alpha_{r}^{a}(3−2)|\)] and in
Eq. (11) [labeled \(|\alpha_{r}^{a}(3−2)|\)].

These results are in qualitative agreement with the
computations of structure parameters by Burk
(1980) using a numerical boundary-layer model. (See
Burk, Figs. 6, 8 and 12 for examples of clear,
convective boundary layers in which there is con-
siderable interfacial layer cancellation of the \(C_{r}^2\)
contribution to acoustic \(C_n^2\) by the \(C_{r}^2\) term.) A more
complete comparison of the WL formulations with
predictions of a numerical turbulence closure model
are presented in Burk (1981).

Fig. 3 illustrates the magnitude of the pressure
dependence of \(\langle \alpha_{r}^{2}\rangle\) and \(\langle \alpha_{m}^{2}\rangle\). In Fig. 4 the
behavior of \(\langle \alpha_{m}^{2}\rangle\) is displayed, showing that \(\langle \alpha_{m}^{2}\rangle\)
deviates significantly from unity when \(|r| < 1.\)
Thus, when the temperature jump is large and the
humidity jump small, the \(C_{r}^2\) and \(C_{r}^2\) terms make
important contributions to total microwave \(C_n^2\) in
fact, the \(C_{r}^2\) term dominates the \(C_{r}^2\) term, as can be
seen in Fig. 2. The ratio of the third term in Eq.
(12) to the second term is labeled as \(\alpha_{m}^{a}(3−2)\) in
Fig. 2.

3. Concluding remarks

The transformations between individual tempera-
ture-humidity structure parameters and \(C_n^2\) shown
in Eqs. (4)–(6) require knowledge of the correction
factors \(\alpha_{r}^{2}\), \(\alpha_{q}^{2}\) and \(\alpha_{m}^{2}\). Using formulations
developed by WL, we show that these \(a^2\) factors have
simple dependencies on bulk properties within the
interfacial layer.

As an example, consider a sounding in which
\(\Delta \Theta_a = 4^\circ C\), \(\Delta q = -5 \times 10^{-3}\), \(p = 900\)mb, \(e = 10\)
mb and \(T = 285\) K. From Fig. 1 we find that the
contribution of \(C_{r}^2\) to optical \(C_n^2\) is reduced by about
35% due to the terms involving \(C_{r}^2\) and \(C_{r}^2\) (and
Fig. 2 shows that in this example the optical \(C_{r}^2\)
term is about 16 times larger in absolute magnitude
than the \(C_{r}^2\) term). Also, if we were to assume
that acoustic \(C_{n}^2\) was solely dependent on \(C_{r}^2\) when
making the transformation indicated in Eq. (4), Fig. 1
shows that we would be in error by nearly a factor of
3. In this example the correction factor for micro-
waves, \(\langle \alpha_{m}^{2}\rangle\), is about 1.25 according to Fig. 4.

It should be reiterated that use of Eqs. (10)–(12),
or Figs. 1–4, should be restricted to the conditions
discussed in WL; viz., the interfacial layer of clear,
convectively driven boundary layers.

As noted by a reviewer, a word of caution is war-
ranted concerning our treatment of \(C_{r}^2\). The
Wyngaard and LeMone (1980) expression for \(\langle C_{r}^2\rangle\) has
not been verified in the inertial subrange at the
highest frequencies where wave-scattering phe-
nomena occurs. The aircraft data from which \(C_{r}^2\)
is computed are generally sampled at a consider-
ably lower frequency than that responsible for the
scattering. Direct testing using optical and other re-
 mote sensing techniques in conjunction with con-
t rional aircraft sampling appears necessary to re-
solve remaining ambiguities.

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APPENDIX

List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Representation</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Coefficient in microwave refractivity ((=77.6 \times 10^{-6}) K mb(^{-1}))</td>
</tr>
<tr>
<td>A(_1), A(_2)</td>
<td>Coefficients in optical refractivity ((A_1 = 78.7\times 10^{-6}) K mb(^{-1}), (A_2 = 66.3\times 10^{-6}) K mb(^{-1}))</td>
</tr>
<tr>
<td>C</td>
<td>Coefficient in microwave refractivity ((=0.375\ K^2\ mb^{-1}))</td>
</tr>
<tr>
<td>C(_n^2), C(_q^2), C(_r^2)</td>
<td>Structure parameters for refractive index, specific humidity and temperature, respectively</td>
</tr>
<tr>
<td>C(_{r}^2)</td>
<td>Joint temperature-humidity structure parameter</td>
</tr>
<tr>
<td>D</td>
<td>Constant in acoustic refractivity ((=0.307))</td>
</tr>
<tr>
<td>e</td>
<td>Vapor pressure</td>
</tr>
<tr>
<td>p</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>q</td>
<td>Specific humidity</td>
</tr>
<tr>
<td>q(_i)</td>
<td>Interfacial layer humidity scale [Eq. (2)]</td>
</tr>
<tr>
<td>r</td>
<td>Ratio of humidity to virtual potential temperature jump across interfacial layer (=[\Delta q/\Delta \Theta_i])</td>
</tr>
<tr>
<td>T</td>
<td>Absolute temperature</td>
</tr>
<tr>
<td>T(_i)</td>
<td>Interfacial layer temperature scale, Eq. (1)</td>
</tr>
<tr>
<td>Z(_i)</td>
<td>Inversion height</td>
</tr>
</tbody>
</table>

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Greek symbols

\(\alpha_n^2\), \(\alpha_o^2\), \(\alpha_m^2\)  correction factors in transformations of acoustic, optical and microwave \(C_n^2\) [Eqs. (4)-(6)]

\(\epsilon\)  constant appearing in vapor pressure to specific humidity conversion (= 0.622)

\(\rho\)  atmospheric density

\(\theta_v\)  mixed-layer temperature scale

\(\Theta_v\)  virtual potential temperature

Other symbols

\(\langle\;\rangle\)  virtual

\(\Delta\langle\;\rangle\)  bulk difference across interfacial layer

\(\langle\;\rangle\)  interfacial-layer average.

REFERENCES


